Evolutionary Quantum Game Theory
and Scientific Communication

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in honor of John Nash, on occasion of the 60th anniversary of Nash equilibrium

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1. Introduction

2. Classical Evolutionary Game Theory
   - Key aspects of classical evolutionary game theory
   - Different game classes

3. Quantum Game Theory
   - Key aspects of quantum game theory
   - Game classes of quantum games

4. Applications
   - Evolutionary Quantum Game Theory and Scientific Communication

5. Summary

Definition of an unsymmetric (2 player)-(2 strategy) game Γ

An unsymmetric (2 × 2) game Γ is defined as ...

(2 × 2) Game: $\Gamma := \left(\{A, B\}, \mathcal{S}^A \times \mathcal{S}^B, \hat{\$}_A, \hat{\$}_B \right)$

Set of pure strategies of player A and B: $\mathcal{S}^A = \left\{ s_1^A, s_2^A \right\}$, $\mathcal{S}^B = \left\{ s_1^B, s_2^B \right\}$

Set of mixed strategies of player A and B: $\tilde{\mathcal{S}}^A = \left\{ \tilde{s}_1^A, \tilde{s}_2^A \right\}$, $\tilde{\mathcal{S}}^B = \left\{ \tilde{s}_1^B, \tilde{s}_2^B \right\}$

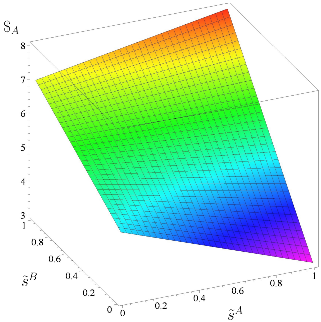
Payoff matrix for player A: $\hat{\$}_A = \begin{pmatrix} \$_{11}^A & \$_{12}^A \\ \$_{21}^A & \$_{22}^A \end{pmatrix}$

Payoff matrix for player B: $\hat{\$}_B = \begin{pmatrix} \$_{11}^B & \$_{12}^B \\ \$_{21}^B & \$_{22}^B \end{pmatrix}$

The mixed strategy payoff function $\tilde{\$}^\mu$ of player $\mu = A, B$

Mixed strategy payoff function $\tilde{\$}^A\left(\tilde{s}^A, \tilde{s}^B\right)$ of player A

$(\$^A_{11} = 8,\ \$^A_{12} = 5,\ \$^A_{21} = 7,\ \$^A_{22} = 3)$



Payoff $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$ as a function of $\tilde{s}^A, \tilde{s}^B \in [0,1]$:

$$\tilde{\$}^\mu : ([0,1] \times [0,1]) \rightarrow \mathbb{R}$$

$$\tilde{\$}^\mu(\tilde{s}^A, \tilde{s}^B) = \$^\mu_{11}\tilde{s}^A\tilde{s}^B + \$^\mu_{12}\tilde{s}^A(1-\tilde{s}^B) + \$^\mu_{21}(1-\tilde{s}^A)\tilde{s}^B + \$^\mu_{22}(1-\tilde{s}^A)(1-\tilde{s}^B)$$

, where $\tilde{s}^A := \tilde{s}^A_1$, $\tilde{s}^B := \tilde{s}^B_1$, $\tilde{s}^A_2 = 1 - \tilde{s}^A_1$ and $\tilde{s}^B_2 = 1 - \tilde{s}^B_1$

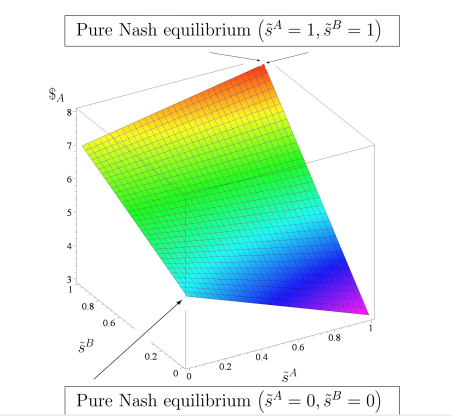
Payoff $\tilde{\$}^\mu\left(\tilde{\mathcal{S}}^A \times \tilde{\mathcal{S}}^B\right)$ as a function of the sets of mixed strategies for player A and B:

$$\tilde{\$}^\mu : \left(\tilde{\mathcal{S}}^A \times \tilde{\mathcal{S}}^B\right) \rightarrow \mathbb{R}$$

$$\tilde{\$}^\mu((\tilde{s}^A_1, \tilde{s}^A_2), (\tilde{s}^B_1, \tilde{s}^B_2)) = \$^\mu_{11}\tilde{s}^A_1\tilde{s}^B_1 + \$^\mu_{12}\tilde{s}^A_1\tilde{s}^B_2 + \$^\mu_{21}\tilde{s}^A_2\tilde{s}^B_1 + \$^\mu_{22}\tilde{s}^A_2\tilde{s}^B_2$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



A strategy combination $(\tilde{s}^{A*}, \tilde{s}^{B*})$ is called a Nash equilibrium, if:

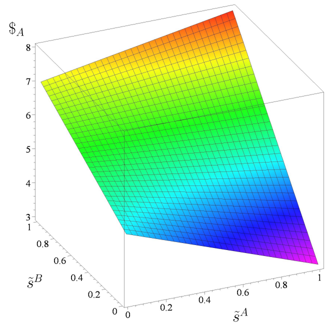
$$\tilde{\$}^A(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^A(\tilde{s}^{A}, \tilde{s}^{B*}) \quad \forall\ \tilde{s}^A \in [0,1]$$
$$\tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B}) \quad \forall\ \tilde{s}^B \in [0,1]$$

A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

$$\left.\frac{\partial \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^A}\right|_{\tilde{s}^B = \tilde{s}^{B\star}} = 0 \quad \forall\ \tilde{s}^A \in [0,1] \ ,\ \tilde{s}^{B\star} \in]0,1[$$
$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall\ \tilde{s}^B \in [0,1] \ ,\ \tilde{s}^{A\star} \in]0,1[$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



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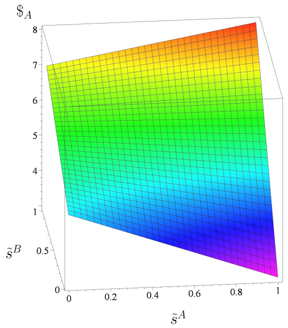
Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



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$$\begin{aligned}\tilde{\$}^A(\tilde{s}^{A*}, \tilde{s}^{B*}) &\geq \tilde{\$}^A(\tilde{s}^{A}, \tilde{s}^{B*}) \quad \forall\ \tilde{s}^A \in [0,1]\\ \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B*}) &\geq \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B}) \quad \forall\ \tilde{s}^B \in [0,1]\end{aligned}$$

A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

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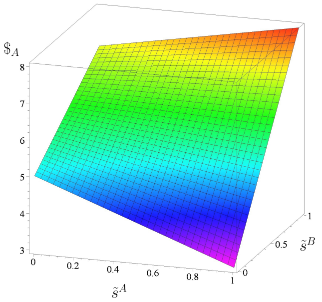
Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



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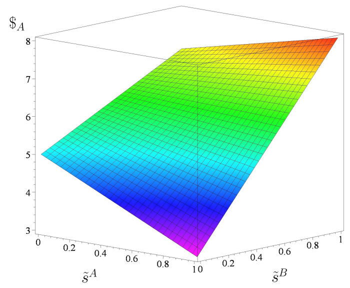
$$\left.\frac{\partial\tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial\tilde{s}^B}\right|_{\tilde{s}^A=\tilde{s}^{A\star}} = 0 \quad \forall \ \tilde{s}^B \in [0,1] \ , \ \tilde{s}^{A\star} \in]0,1[$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



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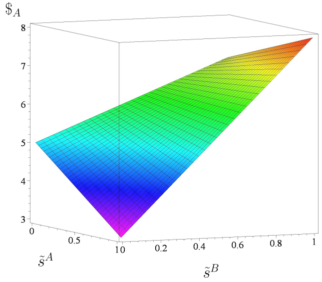
$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall \; \tilde{s}^B \in [0,1] \;\; , \;\; \tilde{s}^{A\star} \in]0,1[$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



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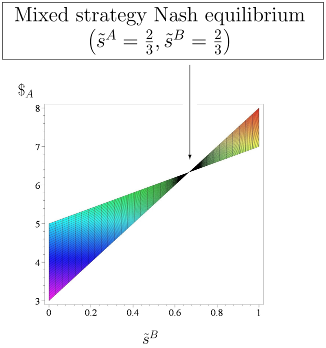
Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



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\tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B*}) &\geq \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B}) \quad \forall \ \tilde{s}^B \in [0,1]
\end{aligned}$$

A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

$$\begin{aligned}
\left.\frac{\partial \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^A}\right|_{\tilde{s}^B = \tilde{s}^{B\star}} &= 0 \quad \forall \ \tilde{s}^A \in [0,1] \ , \ \tilde{s}^{B\star} \in]0,1[\\
\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} &= 0 \quad \forall \ \tilde{s}^B \in [0,1] \ , \ \tilde{s}^{A\star} \in]0,1[
\end{aligned}$$

Replicatordynamics

Replicatordynamics: The dynamical behavior of a population of players

$$\frac{dx_i^A(t)}{dt} = x_i^A(t) \left[\sum_{l=1}^{2} \$_{il}^A \, x_l^B(t) - \sum_{l=1}^{2} \sum_{k=1}^{2} \$_{kl}^A \, x_k^A(t) \, x_l^B(t) \right]$$

$$\frac{dx_i^B(t)}{dt} = x_i^B(t) \left[\sum_{l=1}^{2} \$_{li}^B \, x_l^A(t) - \sum_{l=1}^{2} \sum_{k=1}^{2} \$_{lk}^B \, x_l^A(t) \, x_k^B(t) \right]$$

The two population vectors $\vec{x}^A$ and $\vec{x}^B$ have to fulfill the normalizing conditions of a unity vector

$$x_i^\mu(t) \geq 0 \quad \text{and} \quad \sum_{i=1}^{2} x_i^\mu(t) = 1 \quad \forall\, i = 1,2\,,\;\; t \in \mathbb{R},\;\; \mu = A, B$$

Replicatordynamics of (2×2) games

Replicatordynamics of unsymmetric (2×2) games

$$\frac{dx(t)}{dt} = \left(\left(\$^A_{11} + \$^A_{22} - \$^A_{12} - \$^A_{21}\right)\left(x(t) - (x(t))^2\right)\right) y(t) + \left(\$^A_{12} - \$^A_{22}\right)\left(x(t) - (x(t))^2\right) =: g_A(x,y)$$

$$\frac{dy(t)}{dt} = \left(\left(\$^B_{11} + \$^B_{22} - \$^B_{12} - \$^B_{21}\right)\left(y(t) - (y(t))^2\right)\right) x(t) + \left(\$^B_{12} - \$^B_{22}\right)\left(y(t) - (y(t))^2\right) =: g_B(x,y)$$

Replicatordynamics of symmetric (2×2) games

$$\frac{dx}{dt} = x\left[\$_{11}(x - x^2) + \$_{12}(1 - 2x + x^2) + \$_{21}(x^2 - x) + \$_{22}(2x - x^2 - 1)\right]$$

$$= x\left[(\$_{11} - \$_{21})(x - x^2) + (\$_{12} - \$_{22})(1 - 2x + x^2)\right] =: g(x)$$

with: $x = x(t) := x_1(t) \quad \rightarrow \quad x_2(t) = (1 - x(t))$

Payoff transformation and Game classes

Nash equivalent games

The set of Nash equilibria, the dynamical behavior of evolutionary games and the existence of evolutionary stable strategies (ESS) are unaffected by positive affine payoff transformations and by additionally added constants, where the strategy choice of the other players are fixed (see e.g. Weibull(1995)[17]). In the following the second kind of payoff transformation will be used to transform the payoff matrices in order to classify the games into different categories.

Symmetric payoff matrix after payoff transformation

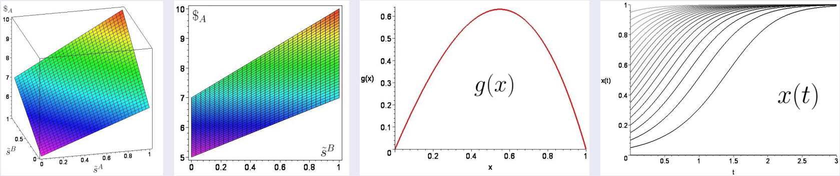
A\B	s_1^B	s_2^B
s_1^A	$(\$_{11},\$_{11})$	$(\$_{12},\$_{21})$
s_2^A	$(\$_{21},\$_{12})$	$(\$_{22},\$_{22})$

$\Longrightarrow$

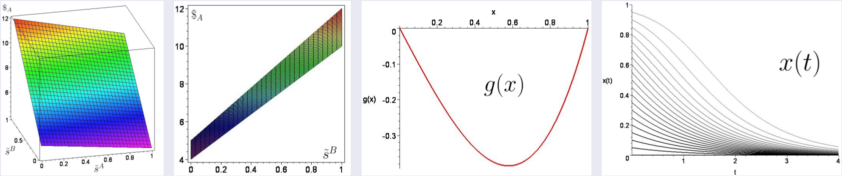
A\B	$Trafo\, s_1^B$	$Trafo\, s_2^B$
$Trafo\, s_1^A$	$(\underbrace{\$_{11}-\$_{21}}_{:=a},\underbrace{\$_{11}-\$_{21}}_{:=a})$	$(0,0)$
$Trafo\, s_2^A$	$(0,0)$	$(\underbrace{\$_{22}-\$_{12}}_{:=b},\underbrace{\$_{22}-\$_{12}}_{:=b})$

Symmetric (2×2) games: Dominant Class $(a < 0, b > 0)$ or $(b < 0, a > 0)$

Dominant Game: a=3, b=-2, one pure NE and one ESS (s_1^A, s_1^B)

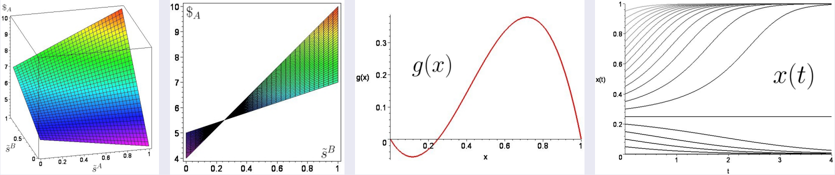


Prisoner's Dilemma: a=-2, b=1, one pure NE and one ESS (s_2^A, s_2^B)

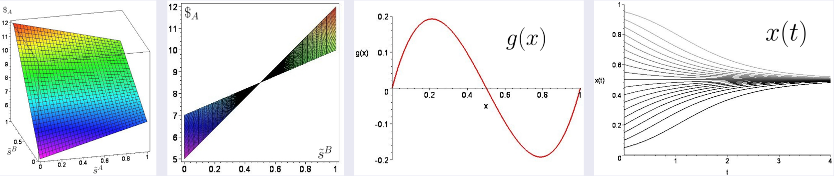


Coordination $(a, b > 0)$ and Anti-Coordination $(a, b < 0)$ Class

Coordination game: a=3, b=1, two pure and one interior NE at $\tilde{s}^{\star} = \frac{1}{4}$, two ESS $((s_1^A, s_1^B)$ and $(s_2^A, s_2^B))$



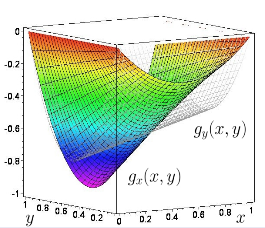
Anti-Coordination game: a=-2, b=-2, two pure asymmetric NE and one interior NE at $\tilde{s}^{\star} = \frac{1}{2}$, one ESS $(\tilde{s}^{A\star} = \frac{1}{2}, \tilde{s}^{B\star} = \frac{1}{2})$



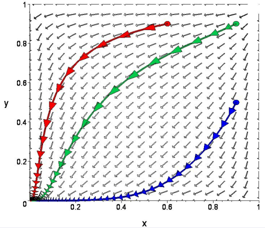
Game classes of unsymmetric (2 player)-(2 strategy) games

Corner Class (one ESS)

$g_x(x,y)$ (colored) and $g_y(x,y)$ (wired):

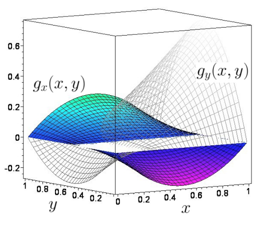


Phase diagram of xy-trajectories:

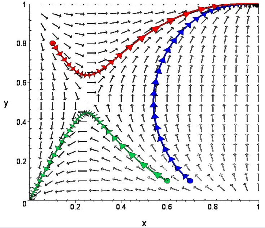


Saddle Class (two ESS)

$g_x(x,y)$ (colored) and $g_y(x,y)$ (wired):

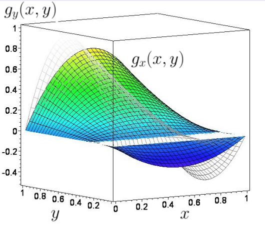


Phase diagram of xy-trajectories:



Center Class (no ESS)

$g_x(x,y)$ (colored) and $g_y(x,y)$ (wired):



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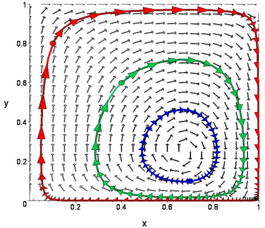
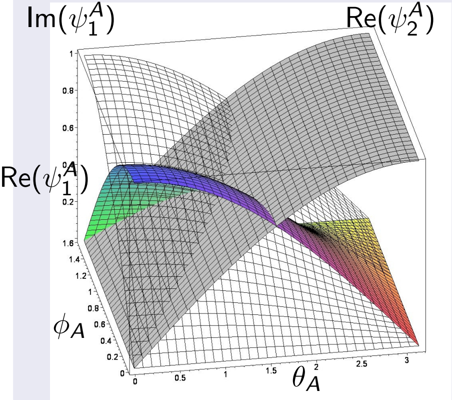


The quantum decision state $|\psi\rangle_\mu$ of player $\mu = A, B$

Real and imaginary parts of $|\psi\rangle_A$



Quantum state of player A:

$$|\psi\rangle_A = \psi_1^A \left| s_1^A \right\rangle + \psi_2^A \left| s_2^A \right\rangle = \begin{pmatrix} \psi_1^A \\ -\psi_2^A \end{pmatrix} \in \mathcal{H}_A$$

$$\text{with:} \quad \left| s_1^A \right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \left| s_2^A \right\rangle = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

s_1-quantum strategies and the decision operator $\widehat{\mathcal{U}}(\theta, \varphi)$:

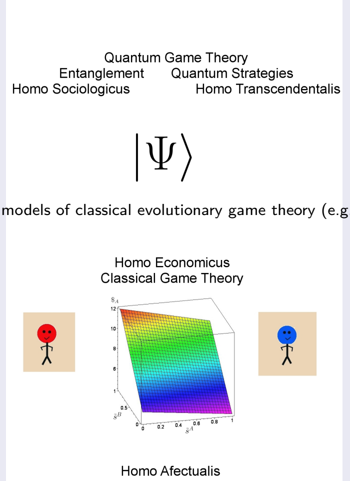
$$|\psi\rangle_A = \widehat{\mathcal{U}}(\theta_A, \varphi_A) \left| s_1^A \right\rangle = \begin{pmatrix} e^{i\,\varphi_A} \cos(\frac{\theta_A}{2}) \\ -\sin(\frac{\theta_A}{2}) \end{pmatrix}$$

$$\widehat{\mathcal{U}}(\theta, \varphi) := \begin{pmatrix} e^{i\,\varphi} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\,\varphi} \cos(\frac{\theta}{2}) \end{pmatrix}$$

$$\forall \quad \theta \in [0, \pi] \ \wedge \ \varphi \in [0, \frac{\pi}{2}]$$

The 2-player state $|\Psi\rangle$ and the entangling operator $\widehat{\mathcal{J}}(\gamma)$

Beyond Homo Economicus



Extended models of classical evolutionary game theory (e.g. [16, 15])

The final 2-player quantum state:

$$|\Psi\rangle = \widehat{\mathcal{J}}^{\dagger} \left(\widehat{\mathcal{U}}_A \otimes \widehat{\mathcal{U}}_B \right) \widehat{\mathcal{J}} \, |s_1^A s_1^B\rangle$$

$\widehat{\mathcal{J}}(\gamma)$: Entangling operator

$\widehat{\mathcal{J}}^{\dagger}(\gamma)$: Disentangling operator

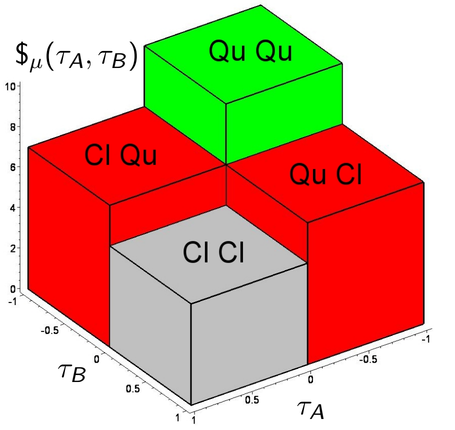
$\gamma \in [0, \pi]$: Strength of entanglement

$\widehat{\mathcal{U}}_A$: Decision Operator for player A

$\widehat{\mathcal{U}}_B$: Decision Operator for player B

The extended payoff $\$_\mu(\tau_A, \tau_B)$ of player $\mu = A, B$

Visualisationspace of $\$_\mu(\tau_A, \tau_B)$



The expected payoff within a quantum version of a general 2-player game:

$$\begin{aligned} \$_A &= \$^A_{11}\, P_{11} + \$^A_{12}\, P_{12} + \$^A_{21}\, P_{21} + \$^A_{22}\, P_{22} \\ \$_B &= \$^B_{11}\, P_{11} + \$^B_{12}\, P_{12} + \$^B_{21}\, P_{21} + \$^B_{22}\, P_{22} \\ \text{with:} \quad P_{\sigma\sigma'} &= |\,\langle \sigma\sigma' | \Psi \rangle\,|^2 \quad , \quad \sigma, \sigma' = \{s_1, s_2\} \end{aligned}$$

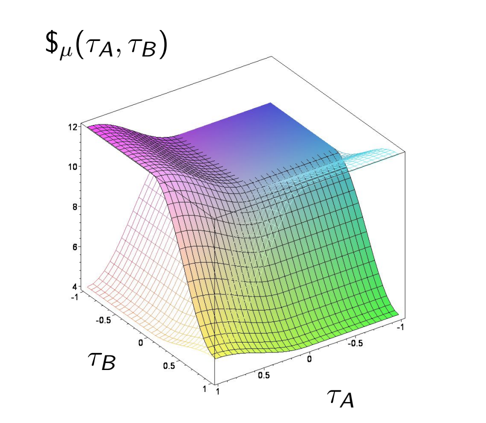
Reduction of quantum strategies:

$$|\Psi\rangle = |\Psi(\theta_A, \varphi_A, \theta_B, \varphi_B)\rangle \rightarrow |\Psi(\tau_A, \tau_B)\rangle$$

$$\underbrace{\{(\tau\,\pi, 0) \mid \tau \in [0,1]\}}_{\text{classical region } Cl} \;\wedge\; \underbrace{\{(0, \tau\,\frac{\pi}{2}) \mid \tau \in [-1, 0[\}}_{\text{quantum region } Qu}$$

Quantum extension of dominant class games

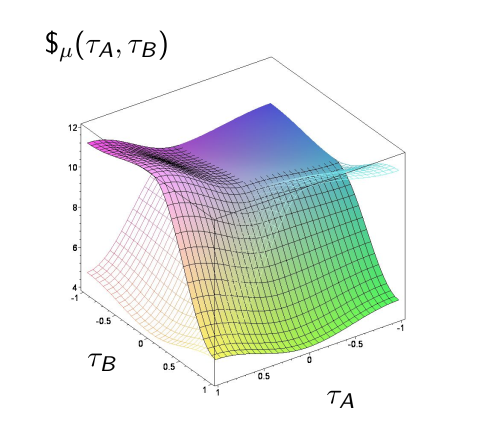
Payoff of player A (colored) and player B (wired) for $\gamma = 0$ (no entanglement)



The diagram clearly exhibits that the non-entangled quantum game simply describes the classical version of the prisoner's dilemma game. For the case, that both players decide to play a quantum strategy ($\tau_A < 0 \wedge \tau_B < 0$) their payoff is equal to the case where both players choose the classical pure strategy s_1 ($\$_A(\tau_A = 0, \tau_B = 0) = 10$). The classical Nash equilibrium ((s_2^A, s_2^B), the dominant strategy) corresponds to the following τ-values:$(s_2^A, s_2^B) \hat{=} (\tau_A = 1, \tau_B = 1)$.

Quantum extension of dominant class games

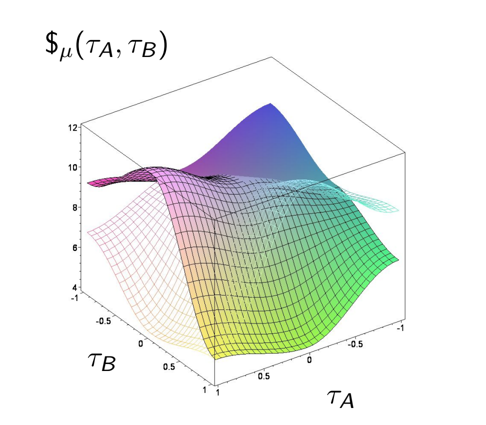
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



For the absolute classical region *ClCl* the shape of the surfaces does not change, whereas for the partially classical-quantum (*ClQu* and *QuCl*) and absolute quantum region regions *QuQu* the payoff structure changes due to a possible interference of quantum strategies within Hilbertspace. The structure of Nash-equilibria does not change for the left picture, whereas for the following pictures the previously present dominant strategy of the prisoner's dilemma game has disappeared and a new, advisable quantum Nash-equilibrium will appear at $(\widehat{Q}, \widehat{Q} \hat{=} (\tau_A = -1, \tau_B = -1))$. During the transition from this figure to the next picture two separate phenomena occur. At first, for an entanglement value $\gamma_1 \approx 0.37$, the best response for player A to the strategy $s_2^B \hat{=} \tau_B = 1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = -1, \tau_B = 1) \approx 5.05$ is now higher than $\$_A(\tau_A = 1, \tau_B = 1) = 5$. Secondly, for an entanglement value $\gamma_2 \approx 0.53$, the best response for player A to the strategy $\widehat{Q}_B \hat{=} \tau_B = -1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = 1, \tau_B = -1) \approx 9.96$ is for $\gamma_2 = 0.53$ lower than $\$_A(\tau_A = -1, \tau_B = -1) = 10$.

Quantum extension of dominant class games

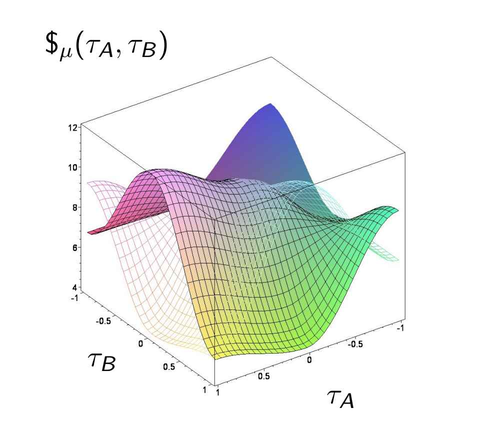
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{8} \approx 0.52$



For the absolute classical region *ClCl* the shape of the surfaces does not change, whereas for the partially classical-quantum (*ClQu* and *QuCl*) and absolute quantum region regions *QuQu* the payoff structure changes due to a possible interference of quantum strategies within Hilbertspace. The structure of Nash-equilibria did not change for the last figure, whereas for this and thee following pictures the previously present dominant strategy of the prisoner's dilemma game has disappeared and a new, advisable quantum Nash-equilibrium has appeared $(\widehat{Q}, \widehat{Q} \hat{=} (\tau_A = -1, \tau_B = -1))$. During the transition from the last picture to this figure two separate phenomena occurred. At first, for an entanglement value $\gamma_1 \approx 0.37$, the best response for player A to the strategy $s_2^B \hat{=} \tau_B = 1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = -1, \tau_B = 1) \approx 5.05$ is now higher than $\$_A(\tau_A = 1, \tau_B = 1) = 5$. Secondly, for an entanglement value $\gamma_2 \approx 0.53$, the best response for player A to the strategy $\widehat{Q}_B \hat{=} \tau_B = -1$ is no longer the strategy $s_2^A \hat{=} \tau_A = 1$, as $\$_A(\tau_A = 1, \tau_B = -1) \approx 9.96$ is for $\gamma_2 = 0.53$ lower than $\$_A(\tau_A = -1, \tau_B = -1) = 10$.

Quantum extension of dominant class games

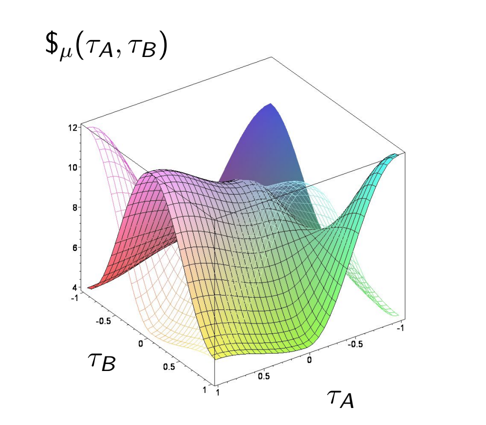
Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{6} \approx 0.94$



The results show, that a quantum extension of a classical prisoner's dilemma game is able to change the structure of Nash-equilibria, and even previously present dominant strategies could become nonexistent, if the value of entanglement increases further than a defined γ-threshold. Players with a higher strategic entanglement value γ escape the dilemma as they see the advantage of the quantum strategy combination $(\widehat{Q}_A, \widehat{Q}_B)$, which is measured as if both are playing the classical strategy s_2.

Quantum extension of dominant class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{2} \approx 1.57$



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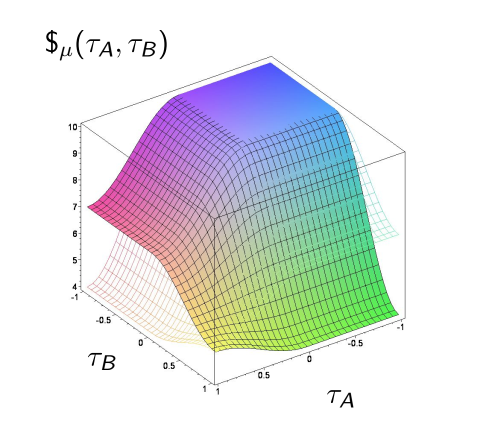
Quantum extension of coordination class games

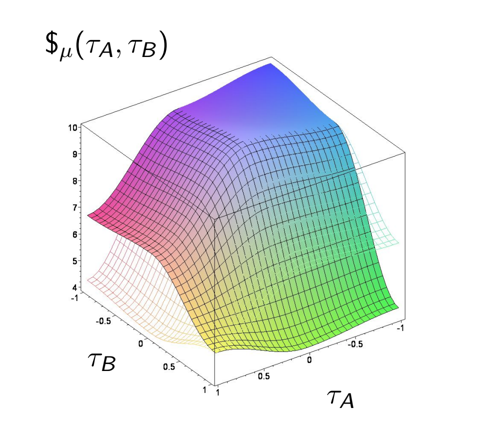
Payoff of player A (colored) and player B (wired) for $\gamma = 0$ (no entanglement)



Again, the diagram clearly indicates that the non-entangled quantum game is identical to the classical version of the underlying coordination game. For the case, that both players decide to play a quantum strategy $(\tau_A < 0 \wedge \tau_B < 0)$ their payoff is equal to the case where both players choose the classical pure strategy s_1 $(\$_A(\tau_A = 0, \tau_B = 0) = 10)$, with the overall highest possible payoff. The classical pure Nash equilibria correspond to the following τ-values: $(s_1^A, s_1^B) \hat{=} (\tau_A = 0, \tau_B = 0)$ and $(s_2^A, s_2^B) \hat{=} (\tau_A = 1, \tau_B = 1)$, whereas the classical mixed strategy equilibrium is at: $\tau^\star = \frac{2}{\pi} \arccos(\sqrt{\frac{1}{4}}) = \frac{2}{3}$.

Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{10} \approx 0.31$



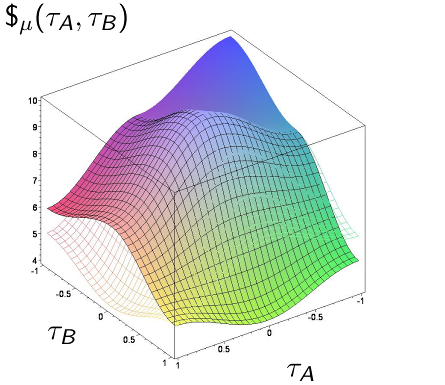
Even for tiny values of γ a new quantum Nash-equilibrium appears $(\tau_A = -1, \tau_B = -1)$.

At moderate values of γ the low payoff evolutionary stable strategy $(\tau_A = 1, \tau_B = 1)$ disappears.

The specific γ-value at which this disappearance happens, depends on the whole set of payoff parameters and not only on a and b.

Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{8} \approx 0.52$



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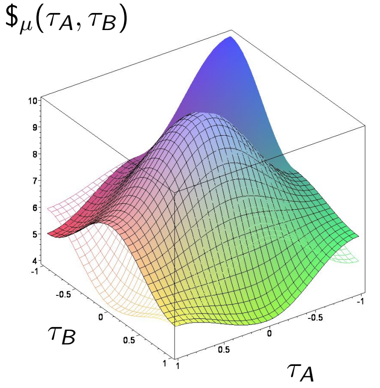
Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{6} \approx 0.94$



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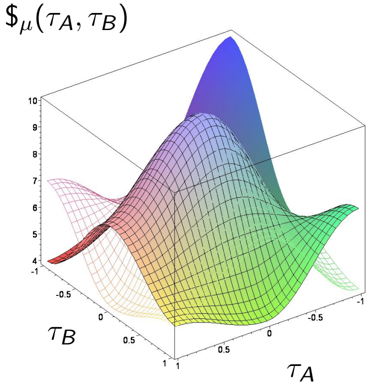
Quantum extension of coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = \frac{\pi}{2} \approx 1.57$



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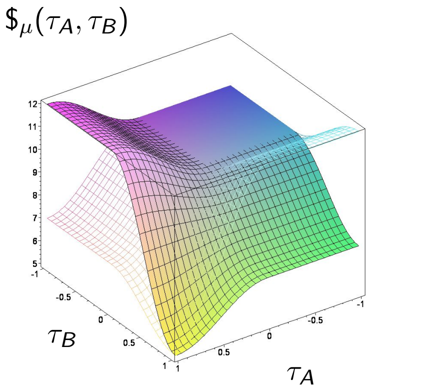
Quantum extension of anti-coordination class games

Payoff of player A (colored) and player B (wired) for $\gamma = 0$



Beside the mixed strategy evolutionary stable strategy, a new quantum ESS appears at a specific γ-value.

For details see:

- M. Hanauske, *Advances in Evolutionary Game Theory*, 2009, *Lecture at the 'Université Lumière Lyon 2' in Lyon, France (MINERVE Exchange Program); Slides and additional material*
- *M. Hanauske, J. Kunz, S. Bernius, and W. König. ,Doves and hawks in economics revisited: An evolutionary quantum game theory-based analysis of financial crises. , 2009, to appear in Physica A, arXiv:0904.2113, RePEc:pra:mprapa:14680 and SSRN$_{id}$:1597735 .*

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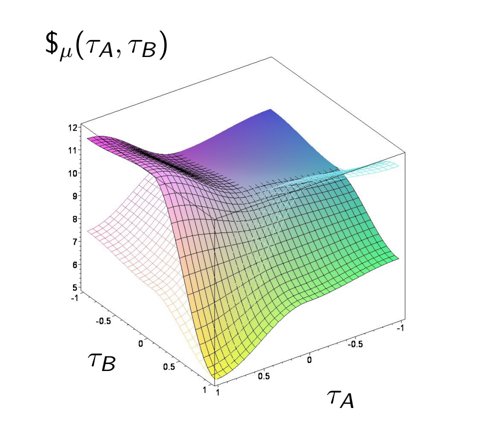

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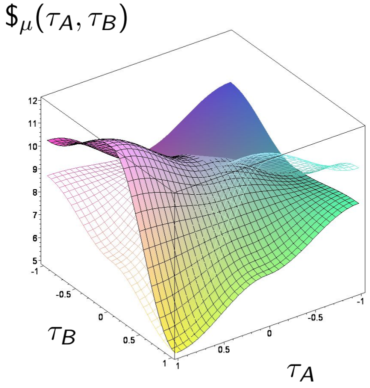

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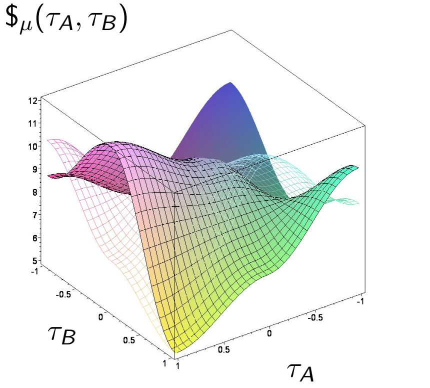

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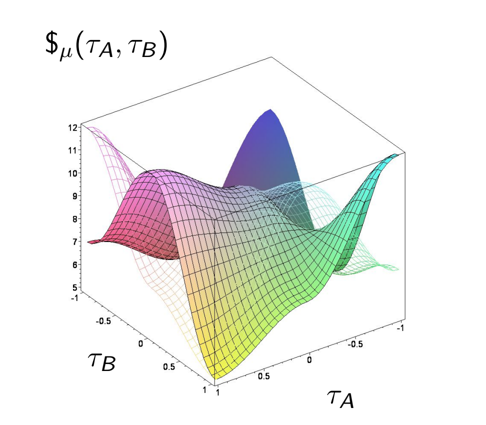

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Applications

Evolutionary Quantum Game Theory and Scientific Communication

See article [8, 13] and presentation [1, 3, 6, 4, 5, 2]

Doves and hawks in economics revisited: An evolutionary quantum game theory-based analysis of financial crises

See article [11]

Quantum Game Theory and the Evolution of Social Norms in Firms

See article [12]

Evolutionary Quantum Game Theory and Hubs- and Spoke-Networks

See article [10]

Evolutionary Quantum Game Theory and Socio-Economic Systems

See article [9, 14] and presentation [7]

Summary

Summary of the talk

In the underlying presentation, the framework of evolutionary game theory (EGT) has been described in detail. After a general introduction the formal mathematical model, the different concepts of equilibria and the various classes of evolutionary games have been defined and visualized to understand the main ideas of classical evolutionary game theory. After a general introduction into quantum game theory, the formal mathematical model was explained and visualized and the different quantum game classes where discussed. Possible applications have been discussed at the end of the talk.

Quantum game theory

Quantum game theory is a mathematical and conceptual amplification of classical game theory. The space of all conceivable decision paths is extended from the purely rational, measurable space in the Hilbertspace of complex numbers. Trough the concept of a potential entanglement of the imaginary quantum strategy parts, it is possible to include corporate decision path, caused by cultural or moral standards. If this strategy entanglement is large enough, then, additional Nash-equilibria can occur, previously present dominant strategies could become nonexistent and new evolutionary stable strategies can appear.

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