# General Relativity in the Theater of the Absurd

Matthias Hanauske
*Institute for Theoretical Physics , Goethe University,*
*Max-von-Laue-Straße 1, 60438 Frankfurt, Germany*
*E-mail: hanauske@th.physik.uni-frankfurt.de*

Einstein's theory of spacetime curvature and its impressive astrophysical and philosophical consequences, not only since the experimental evidence of gravitational waves and the first image of a black hole, represent milestones in human knowledge, and the presentation of these insights in a popular scientific manner is an important undertaking. How can bizarre concepts such as the curvature of spacetime or the event horizon of a black hole be understood? Learned things that seem outlandish to personal experience are quickly forgotten. Simple stories and familiar images can make it easier for the layperson to retain what they have learned, and one can also use absurd analogies to confront the audience with the facts of general relativity. For example, the different phases of a neutron star collision can be illustrated as an omnium-gatherum of different ballroom dances, or the essential properties of black holes are reflected in the architecture of the German Reichstag building.

*Keywords*: Proceedings Sixteenth Marcel Grossmann Meeting, Parallel Session: Teaching Einsteinian Physics to School Students

## 1. Introduction

Albert Einstein presented his revolutionary Theory of General Relativity to the scientific public in 1915[1,2]. According to Einstein, every accumulation of energy bends the structure of spacetime and this curved spacetime is the fundamental cause of gravity. Within such a point of view, the apple falls from the tree to the ground, because the high energy content of the earth bends the spacetime structure so much that the apple has to move to the ground according to geodetic laws. Einstein derived his field equations (see left side of Eq. 1) using a general, mathematical symmetry principle of covariance and in[1] he wrote about his theory "Hardly anyone who has really grasped the theory will be able to escape the magic of it; ..."[a]. The following equations show the Einstein equation (left) and the geodesic equation (right) in natural geometrized units ($c = 1 = G$, where $c$ and $G$ denote the speed of light and the gravitational constant)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \;=\; -\,8\pi\, T_{\mu\nu} \quad , \quad \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{d\tau} \frac{dx^\rho}{d\tau} \;=\; 0 \quad , \tag{1}$$

where $T_{\mu\nu}$ is the energy-momentum tensor, $R_{\mu\nu}$ is the Ricci tensor, which contains first and second derivatives of the spacetime metric $g_{\mu\nu}$ and $R$ is the Ricci

---

[a] "Dem Zauber der Theorie wird sich kaum jemand entziehen können, der sie wirklich erfasst hat;..."

scalar. The Greek spacetime indices $\mu, \nu, \rho ...$ run from 0..3, whereby, unless otherwise stated, they correspond to the time coordinate ($x^0 = t$) and the Cartesian space coordinates ($x^1 = x, x^2 = y$ and $x^3 = z$) of the four dimensional coordinates $x^\mu$. The geodesic equation (right side in Eq. 1) describes how a test body moves in space and indicates that this movement always takes place along the shortest path in the curved spacetime described by the metric $g_{\mu\nu}$, whereby $\tau$ is an affine parameter, e.g. the proper time of the test body, and $\Gamma^\mu_{\nu\rho}$ are the Christoffel symbols, which contain first derivatives of the metric $g_{\mu\nu}$.

It did not take long to find the first analytical solution to the Einstein equations. Just a few months after Einstein's article was published, Karl Schwarzschild worked out two possible analytical solutions to the new theory of spacetime curvature[3,4]. In the first of these works ("On the gravitational field of a mass point according to Einstein's theory"), Mr. Schwarzschild considered the Einstein equation for empty space ($T_{\mu\nu} \equiv 0$), with the restriction on a time-independent and spherically symmetric metric. The mathematical basics of the solution are briefly presented in Appendix A. The Schwarzschild solution is of particular importance for astrophysical considerations, because it describes on the one hand the metric of a non-rotating black hole [b] and on the other hand, based on the Birkhoff theorem[6,7], the metric outside of a single isolated, non-rotating star.

In order to calculate the spacetime structure within a spherically symmetrical body, one has to consider the entire Einstein equation ($T_{\mu\nu} \neq 0$) and the solution depends on the equation of state of matter within the object. Assuming that the body consists of an static ideal fluid, Tolman, Oppenheimer and Volkoff were able to show in 1939 how the inner solution can be calculated and connected to the outer Schwarzschild solution[8,9]. The solution found also represents the general relativistic fundamental equation for the most compact objects in our universe - the so-called neutron stars.

In addition to white dwarfs and stellar black holes, neutron stars are the possible final states of the evolutionary process of a sun. Neutron stars are born in supernova explosions and they represent the last stable state of matter before it collapses to a black hole. These fascinating stellar objects are only 20 kilometers in diameter, but they combine a mass of 1-2 solar masses in a very small region. Of the approximately 100 million neutron stars that are believed to exist in our galaxy, around 2800 were observed by radio telescopes as pulsars. Pulsars are rapidly rotating neutron stars with a strong magnetic field (up to $10^{11}$ Tesla), which emit electromagnetic radiation along the poles and can therefore be observed with radio and x-ray telescopes. However, neutron stars cannot become arbitrarily dense and there exists a fundamental upper limit to the compactness (total mass divided by the radius) of bodies. Already in 1916, Karl Schwarzschild showed that objects with a constant density profile need to become unstable if the radius of the object is less

---

[b] The stationary and axially symmetric vacuum solution of Einstein's field equations, which describes the spacetime structure of rotating black holes was found in the year 1963 by Roy Kerr[5].

than 9/8 times twice its mass[4]. This absolute limit of stability is often referred to as the Buchdahl limit[10].

But what happens when a neutron star collapses to a black hole? In 1936, J.R. Oppenheimer and H. Snyder[11] studied a spherically symmetrical collapse of a cloud of dust and were able to show for the first time how a black hole is created due to continued gravitational contraction. Later, in 1965, R. Penrose demonstrated that black hole formation is a robust prediction of general relativity[12] and that the formation of an apparent horizon (an outermost trapped surface) is independent of the specific assumptions of the equation of state of the collapsing matter.

For a long time, the research of astrophysical processes was limited to events that could be seen with the eyes, and optical telescopes were not developed until the 16th century. The perception of the universe in the other frequency ranges of electromagnetic radiation was made possible by radio, infrared and X-ray telescopes, which were only constructed in the 20th century. The entire perceived picture of our universe was therefore limited to astrophysical phenomena that generate electromagnetic radiation. This fact changed in the 21st century with the detection of gravitational waves. Since then, it has been as if humanity had a new sense organ, like new wondrous glasses, with which one can perceive previously unobservable events in our universe. On September 14, 2015, almost exactly one hundred years after Albert Einstein developed the field equations of general relativity and predicted the existence of gravitational waves[13], these strange spacetime ripples were experimentally detected by a pair of merging black holes with the LIGO detectors (GW150914[14]).

By means of the gravitational wave detectors LIGO and Virgo, 11 gravitational waves have be detected within the first and second observing runs[15], whereby one of these gravitational waves (GW170817) was caused by the collision of two neutron stars[16] that merged about 130 million years ago.[c] Electromagnetic radiation in all frequency bands was also detected during this event[19,20] and an emitted gamma-ray burst (GRB 170817A[21]) hit the gamma-ray satellite telescopes with a delay of 1.7 seconds. Space-based gamma-ray telescopes (e.g., the Fermi's gamma-ray burst monitor or the Swift gamma-ray burst mission) detect on average approximately one gamma-ray burst per day - however, the gamma-ray burst[21] that had been associated with GW170817 is an outstanding event and, in addition with the observations of the electromagnetic counterparts of the associated kilonova, provides a conclusive picture of the whole merger event. This coincidence of the direct detection of a gravitational wave from a neutron star collision with the emitted short gamma-ray burst was the first observational proof that binary neutron star mergers generate short gamma-ray bursts. The detected gravitational wave GW170817 depicted, however, due to the still low sensitivity of the detectors at high frequencies,

---

[c] The evaluation of the third observing run is still ongoing[17], but it is considered largely secured that 62 more gravitational waves were detected, with two neutron star-black hole coalescences being observed for the first time[18].

only the late phase of the orbiting neutron stars shortly before the collision.

But what happened after the merger of the two neutron stars, during the 1.7 second unobserved period between their coalescence and the detection of the gamma ray burst? In order to answer this questions, due to the complexity of the problem, we rely on numerical computer simulations that solve the Einstein equation on supercomputers. The current calculations indicate that after the collision a metastable state of matter was formed for a short time, which then collapsed to a black hole and a fraction of the matter emitted during this process was then absorbed into the black hole and produced the gamma ray burst. In the following, the current results in this area of research are summarized and embedded in a personal surreal story.

## 2. General Relativity in the Theater of the Absurd

From time to time it happens to me that an emotional mood of critical questioning of my own work takes hold of me. A common mathematical theoretical researcher often succeeds easily in counteracting these unproductive phases by tightening the strings of the mind again and marveling at the infinite number of theoretical fruits of the tree of knowledge. As a mathematician interested in reality, however, one always waits for a new sign of confirmation of one's own achievements and for me, such a waiting mood manifests itself in the form of an internal companion to whom the boots of the mathematically constriction are too tight.

This is what happened to me a few years ago, in times of the repetitive tiring

Fig. 1. "Waiting for Godot" is a play by Samuel Beckett that was published in 1952. It is considered as the prime example of the "Theater of the Absurd". The leading character and his companion are waiting for "Godot" to appear. Since the wait never ends, they pass the time with games of diversion, which on the one hand save their minds from the abyss, but on the other hand represent a denial of responsibility in times of crisis and lead to a moral "night of unfathomable depths" [22].

waiting, sitting at the barren tree of knowledge while my companion was stripping off his boots. It happened in the years when mankind was still in a partially blinded state. Human perception and knowledge was still reduced to light-generating events and ghostly neutrinos, but the really great energy-moving processes and the spacetime catastrophes associated with them could not be observed directly. Oh, if only we could catch a glimpse of the ultimate ending, it would also give us a meaning for our own existence. Of course, there were indirect signs and the scenarios simulated on supercomputers inevitably pointed to an imminent occurrence and an end to the wait. Although the laser cannons had been precisely adjusted, the state of waiting for the new phenomenon dragged on and made the tree barren. Then, suddenly it seemed as if the phantom would show up and the first gravitational waves from merging black holes and neutron stars were detected.

The hope for the longed for end of the waiting volatilized shortly after the GW170817 news was announced, as the measured spacetime distortion of the event could only depict the phase shortly before the merger of the two neutron stars. Well, this phase of circling and cautious approach of the two stars has been known since the discovery of the Hulse–Taylor binary pulsar through radio wave observations in 1975 and the "Double Pulsar" found in 2003 (see lower right part of Fig. 2 for an illustration) provides even more precise results here. The actual merging and the struggle of matter with the all-devouring nothingness remained hidden from the observer. The gamma ray flash observed shortly afterwards and the kilonova that followed, naturally provided a convincing indication that the collapsing matter lost the battle with "space and time" and a black hole formed - but it would have been a satisfactory confirmation of the theory if this catastrophic process had been observed directly! Nevertheless, nowadays we are able to imagine the exact course of this event by means of complex parallel supercomputer simulations. The numerical results of a neutron star collision are actually quite simple to understand and one can regard the whole process as an omnium-gatherum of different ballroom dances[23,24]. First the stars orbit around each other and come closer (Viennese waltz phase), then the wild Discofox phase, then the differentially rotating Merengue phase and finally, at the end, the collapse of matter to a black hole (tango phase) in which the event horizon is formed.

A merger of two neutron stars is like a great love story. It begins with the first face-to-face encounter of the two lovers (neutron stars), which is probable the greatest serendipity. During this inspiral phase (Viennese waltz phase) the two stars are separated by a certain distance and orbit around each other. Due to the emission of gravitational waves their separating distance decreases with time. Among the currently known 2800 neutron stars (pulsars), there are some which are in binary systems where the companion of the neutron star is either a normal star, a planet, a white dwarf or again a neutron star. The Hulse–Taylor binary (PSR B1913+16) is the first binary pulsar ever discovered and its properties have been observed with ever increasing accuracy over the past 46 years[25,26]. However, one

Fig. 2. The Viennese waltz phase of a neutron star collision embedded in an illustration of the "Double Pulsar" (PSR J0737-3039).

of the most impressive binary neutron star system is the so called "Double Pulsar" (PSR J0737-3039), which has been discovered in 2003 [27–29]. The two neutron stars in this binary system, which are only separated by 800000 km, orbit around each other with an orbital period of 147 minutes and a mean velocities of one million km/h. Additionally, each neutron star rotates around its rotational axis and the whole movement looks similar to a Viennese waltz dance, where each neutron star corresponds to an individual dancer who dances with an invisible companion (see Fig. 2). Due to the emission of gravitational waves, the distance between the two neutron stars decreases with time and finally the two objects need to merge and the two lovers touch each other for the first time and become a couple. The measured gravitational wave GW170817 just showed the last few seconds of this Viennese waltz dance, but the subsequent wild dance of the amalgamating lovers could not be observed due to the still low sensitivity of the detectors at high frequencies.

At merger time, where the emitted gravitational wave of united couple reaches its maximum value, the temperature inside the newly born remnant (hypermassive neutron star) reaches enormous values (up to 80 MeV $\sim 10^{12}$ K). The density maxima are almost at the center and the hot areas are at the couple's points of contact (for details see [30]). The following vibrant, early post-merger phase (Discofox phase) is characterized by a pronounced density double-core structure of the matter and the high temperature regions are smeared out in areas between the double-core density maxima. The movement of advanced Discofox dancers consist of two separate motions: A coming closer and a subsequently removing of the two bodies and a shared rotation with respect to the static dance floor. The two dancers correspond now to the two, still separately visible double-core density maxima of the hypermassive neutron star and the movement of the double-core structure within the first approximately 5 ms after the merger looks quite similar to a Discofox (see

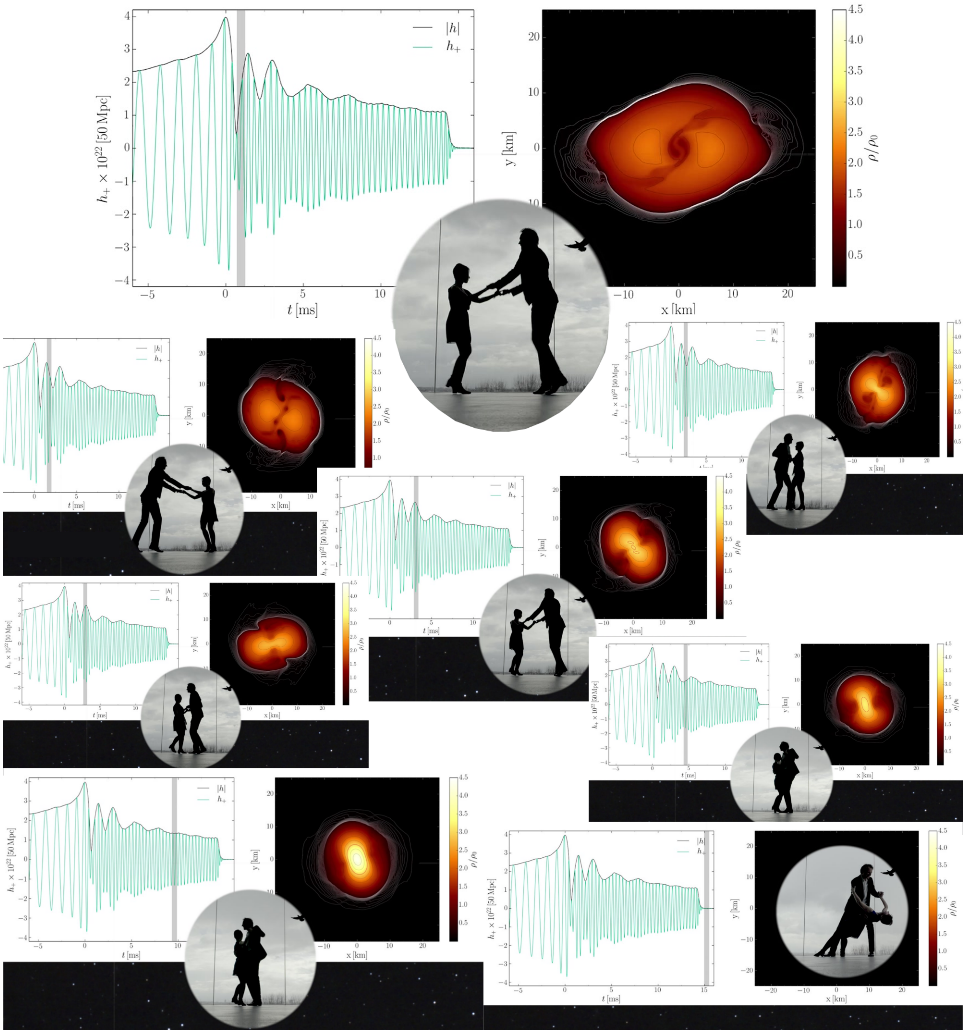


Fig. 3. The several images show selected time snap-shots within the post merger phase of a neutron star collision and illustrate the Discofox, Merengue and Tango phase. The green curve depicts the amplitude of the gravitational wave emitted by the hypermassive neutron star, with the vertical gray bar marking the point in time considered. The corresponding density profile of the star at this point in time is shown on the right and the illustration of this density structure in the form of a dance is shown within the gray ellipse. The corresponding movie can be viewed at `http://itp.uni-frankfurt.de/~hanauske/TanzNeutronensterne.mp4`.

upper half of pictures in Fig. 3).

The movement of the dancers can also be viewed in a so-called co-rotating system[31,32]. In a "corotating frame", each grid point is rotating at a frequency that is half the angular frequency of the instantaneous emitted gravitational wave and it corresponds to the collective rotation of the whole hypermassive neutron star.

After transforming the observer into such a frame of reference the movement of the two density double-cores (dancers) takes place on a straight line. In a corotating frame the Discofox looks more like a West Cost Swing dance and the merger snapshot can be regarded as the first "sugar push" of the West Cost Swing couple.

After this rocking, early post merger phase the Merengue phase begins (see lower left picture in Fig. 3). The double-core structure disappears and the maximum density value shifts to the central region of the hypermassive neutron star. The high temperature regions transform to two temperature hot spots and they move further out. The density distribution at this post merger time has been named 'peanut' shape but the highest value of the density is located in the center of the hypermassive neutron star. In a Merengue dance the dancers are so close together that distinguishing the individuals is difficult (no double-core structure). In close embrace, the united couple quickly rotate around each other and this movement describes in a clear way the motion of the hypermassive neutron star at this post merger time. The high temperature values are reached in regions where the density is in a range of twice normal nuclear matter, while the maximum density values are always at moderate temperatures[30,31,33,34]. The inner area of the star now becomes so dense that matter discloses its fundamental elementary structure. The individual quarks trapped in the neutrons by means of the confinement are now released and the previously unobserved color degree of freedom of the gluons and strange quarks comes to light. Whether this inner ultimate liberating fusion of the dancers takes place and whether they can keep this state alive depends on their inner being (equation of state of elementary matter). Our first simulation results indicated that just when the matter wants to reveal its internal structure, the star becomes unstable, collapses and the strange matter is engulfed by the formation of a black hole[35,36]. However, more recent calculations, which are not the subject of this report, give hope for stabilization by the formation of a metastable hypermassive hybrid star[37–39].

The event horizon of a black hole marks a certain threshold of knowledge (see Appendix A) and the collapse of a star to a black hole looks for an outside observer like a frozen picture of the collapsing star.[d] The collapse of the hypermassive hybrid star to a black hole is the last dance of the two lovers and can be described by the end phase of a Tango, where the motion of the dancers is suddenly freezing, and the emission of gravitational wave and the music stops (see lower right picture in Fig. 3 and Fig. 4). During the collapse of the hypermassive hybrid star to a black hole (Tango phase) the density in the inner region increases rapidly, the quarks get free and the color charge of the deconfined pure quark phase becomes visible - however, the formation of an apparent horizon prevents that this new degree of color can be observed from the outside and the whole deconfined quark phase gets macroscopically confined by general relativity. Fig. 4 shows the last simulation snap-

---

[d] However, the color of this frozen picture will be infinitely red-shifted quite rapidly.

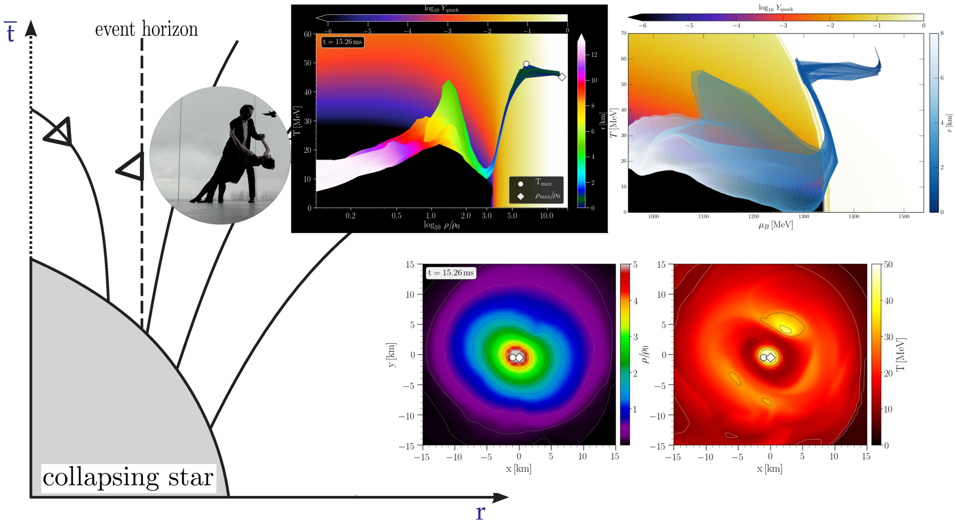


Fig. 4. The last simulation snap-shot of the late Tango phase, before the formation of a Kerr black hole. When the density and temperature profile within the collapsing hybrid star (see lower middle and right images) are visualized in the manner of a phase diagram of quantum chromodynamics, the image of a strange bird appears (density-temperature diagram, upper middle picture) or the image of a swan shows up (chemical potential-temperature diagram, upper right picture).

shot before the apparent horizon finder of the simulation program has detected the formation of a Kerr black hole (for details see[35,36,40]). Such a gravitational collapse and the formation of an outermost trapped surface (apparent horizon) inside the star was already discussed by R. Penrose in 1965[12], however, how the matter behaves during this process has so far remained widely unexplored.

The two lower middle and right images of Fig. 4 depict the density and temperature profiles of the collapsing star and the upper middle image shows these two quantities again in a density-temperature diagram. The shape of the populated area in the density-temperature diagram looks similar to a shape of a strange bird. The color-coding of the bird indicates the radial distance of the corresponding fluid element measured from center of the star, while the color-gradient in the background of the image displays the fraction of free quarks. The hot head of the strange bird contains a high amount of deconfined strange quark matter, its thin neck is composed of mixed phase matter and follows the phase transition boundary, while its hot wings (local temperature maxima) contain mostly hadronic matter at much lower densities[36]. While the quarks in the bird's head have already rescued themselves from their confinement cage, its body still largely consists of hadronic particles. It is precisely at this point in time that the apparent horizon is formed around the dense and hot head of the strange bird and the free strange quark matter is macroscopically confined by the formation of the black hole. By using the baryon chemical potential instead of the density, the shape of the bird changes and with the help of a blue color scale the shadowy blue image appears as a swan[40]. This swan image within the phase diagram of quantum chromodynamics represents the last picture what an outside observer sees: A frozen picture of a dying swan.

We have not been able to observe this magnificent spectacle by means of a gravitational wave so far and our wait will probably have to continue until the Einstein telescope[41], deep underground on the granite plates with squeezed laser light, will make this drama visible. However, the end product of this whole dance hodgepodge, the singularity and the associated properties of black holes, have been studied in detail for some time. Probably the best known example of a black hole is in the center of our galaxy[42], although the much more massive black hole in our neighboring galaxy M87* has already overtaken it by being photographed by the "Event Horizon Telescope"[43]. But one must honestly note that the promising name of the telescope is based on a hoax and unfortunately one was not able to observe the event horizon of M87* with it, but only saw the radiation of the matter falling into the black hole in the area of the photon sphere (see Appendix A). But we are lucky that by a subconsciously guided fate, the most accurate illustration of the essential properties of a black hole has been constructed within an architectural glass dome in the newly constructed core of the Reichstag building in Berlin.

It happened to me some time ago, during a meeting of the German Astronomical Society, that I could no longer keep my internal companion in check and he impatiently forced me to leave the worthy rooms of knowledge of the Technical University Berlin. Before we left, however, I insisted on putting up the poster we had made for the conference, entitled "A Theory of modern ART"[44] (for details see[45]). After leaving the university buildings, we first soaked up the summer sunshine in the zoological gardens before heading towards the Reichstag – the home of the German parliament. Unfortunately, in the machinery of one's own doing, one forgets the really important things all too often and I was glad that my thoughts were now allowed to wander a little more freely. However, the paths of thoughts are not free and if they revolve around a central object for a long time, it is not so easy to leave this space of one's own thinking: How can bizarre concepts such as the curvature of spacetime or the event horizon of a black hole be understood? What possible imagery could help non-scientists to grasp the significance and vital importance of some of the major insights of theoretical physics?

Lost in thought, I looked up and realized that we had almost reached the German Reichstag. I saw for the first time that a huge glass bead had been added to the newly constructed core of the historic Reichstag building and I noticed it contained a huge funnel on the inside (see upper left picture in Fig. 5). To my amazement, the funnel looks exactly like the diagrams used in textbooks to illustrate the curvature of a black hole (see the "spacetime funnel" in Fig. A1(a) in Appendix A). In order to be able to pursue this singularness more closely, we decided to join the queue of visitors and enter the Reichstag itself. While my companion watched the play of

Fig. 5. Images taken during the day when I examined the inner core of the Reichstag – the home of the German parliament.

colors on the dome created by light from the setting sun, my thoughts wandered back into the well-trodden paths of my self-chosen existential duty. Although spacetime funnels are often used to visualize black holes, they do not actually shed much light on the really important properties of these strange entities. For instance, if you remain far from a black hole and watch something falling in to it, you will notice that the object never reaches the hole's center. It appears to come to a stop, to freeze at a certain distance from the center, known as the event horizon (see Fig. 4 and Appendix A).

We finally reached the inner, for an outer observer accessible, zone of the Reichstag, and my companion led me to a circular barrier from where one can peer down to the bottom of the funnel (see central picture in Fig. 5). Along the barrier are displayed various photographs of decisive events in German history that are designed to remind visitors of their responsibilities to the future. Fateful events etched into the memory of mankind forever - the event horizon of German history. Warnings against forgetfulness and against the repression of the Nazi era. How could it come to that and what do we have to learn so that this ineffable never happen again.

Humanity is of a contradictory nature. On the one hand, humans are able to understand and analyze the evolution of the whole universe and brilliant ideas like the prediction of gravitational waves by Albert Einstein and black holes have been recently observed. In respect to this ability, human nature crows over all other species living on our planet. On the other hand, the dominant nature of man manifests itself in a stupidity that screams to heaven. Blind, self-centered actions, which only strives for one's own short-term benefit, still often lead to violent fights and warlike conflicts, to species extinction, and to an exploitation of others. The climate crises is one example of a situation where humanity is trapped in a dilemma-like situation[46–48] - everyone sits on his rolling cart of self-centered existence even though one knows that the evolutionary path of the caravan is heading towards the abyss.

My companion then led me away from the circular barrier and we went to the elevator that was just a few steps from the borderline of the event horizon. The inside of this elevator is lined with mirrors, so that passengers can see themselves infinitely reflected (see upper middle picture in Fig. 5). This space of infinitely recurring images curves toward the center of the Reichstag and it must therefore be located at the "photon sphere" of the black hole, which is the last metastable orbit of light (see Appendix A). However, light does not circle on this sphere forever and a tiny change in the conditions can cause the trajectory of the light ray towards the black hole or away from the singularity.

This glass bead game[49]-like absurd theater of general relativity theory presented in this report cannot of course be transferred "one to one" to school lessons. Teaching laypeople and students Einstein's general theory of relativity is an important undertaking, as the fundamental knowledge it contains can change the perception of one's own existence. Existence, the ability of a person to interact with physical or mental reality, depends on the acquired understanding of the world and how one understand one's own position in our universe. Society has the educational responsibility towards young people to teach knowledge and the evolutionary learned truth. However, the question generally arises as to whether the findings of the general theory of relativity can help mankind to better master current and future challenges. Does it make sense in the current development phase of mankind to elicit more and more precisely the last secrets of nature, to explore the universe for more black holes by means of the Einstein Telescope[41] or the Cosmic Explorer[50]. Does it make sense to get closer to the essence of the dying swan (see Fig. 4), or would it perhaps be more appropriate in this century that humanity learns to be able to live sustainably and peacefully on this planet and shouldn't we put this in the foreground in the teaching institutions, so that the thoughts of the growing population revolve around it?[51] How will the all-embracing state of the world develop in the space of possible strategies?

We then left the Reichstag building and headed for the Brandenburg Gate, the former dividing line between east and west Berlin. As we sauntered through the gate, I noticed an area of land to my right that was entirely vacant except for some statues of bears, which are the symbol of the city of Berlin (see lower left picture in Fig. 5). About 60 bears were arranged in a circle, each representing a country of the world by virtue of the images printed on it. One bear, however, stood off to one side, apart from all the others. Its surface bore an image of Einstein and a quotation by the great physicist (see lower middle and right picture in Fig. 5). As a guide explained to a group of tourists that the American embassy would soon be built on the site, I read Einsteins timeless words:
"Peace cannot be kept by force. It can only be achieved by understanding."

## Appendix A. Spacetime properties of black holes

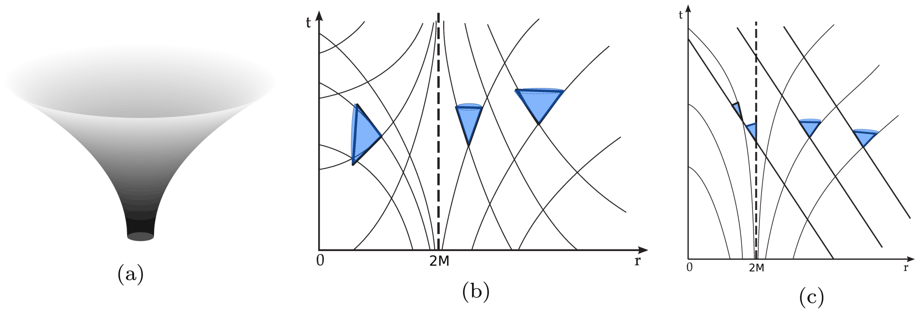
Just a few months after Einstein's article on general relativity was published, Karl Schwarzschild worked out two possible analytical solutions to the new theory of spacetime curvature[3,4]. In the first of these works Schwarzschild considered the Einstein equation for a free, empty space ($T_{\mu\nu} \equiv 0 \; \rightarrow \; R_{\mu\nu} \equiv 0$), whereby he used a spherically symmetrical coordinate system ($x^\mu = (t, r, \theta, \phi)$) and assumed that all matter/energy is located in a singular point at the origin (point like object of mass $M$). The solution he found for the resulting field equations is now known as the Schwarzschild metric:

$$g_{\mu\nu} = \begin{pmatrix} 1 - \frac{2\,M}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{2\,M}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2(\theta) \end{pmatrix} \quad \begin{matrix} \text{Central singularity} \\ \lim\limits_{r\to 0} g_{tt} \rightarrow -\infty \\ \text{Schwarzschild singularity} \\ \lim\limits_{r\to 2M} g_{rr} \rightarrow \pm\infty \end{matrix} \qquad \text{(A.1)}$$

The Schwarzschild metric describes the spacetime structure of a spherically symmetrical, non-rotating and non-charged black hole of mass $M$, whereby the entire mass of the black hole is melded in a singular point in the center. The scalar invariant of the fully contracted square of the Riemann curvature tensor, the so-called Kretschmann scalar $K = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$, becomes singular at the origin ($K = \frac{48M^2}{r^6}$), the time-time component of the metric diverge ($\lim\limits_{r\to 0} g_{tt} \to -\infty$) and the Schwarzschild metric therefore has a central singularity. In addition to this real singularity, the Schwarzschild metric has a second singularity at the so-called Schwarzschild radius $R_S = 2M$ and the radius component of the metric diverge on this sphere with radius $R_S$ ($\lim\limits_{r\to 2M} g_{rr} \to \pm\infty$).

The structure of the spacetime of the Schwarzschild metric can be visualized in different ways. Fig. A1(a) shows the so-called embedded diagram of the spatial hypersphere of the Schwarzschild manifold and the figures A1(b),(c) depict the spacetime diagrams of the Schwarzschild metric in Schwarzschild coordinates (b) and Eddington-Finkelstein coordinates (c). The embedded diagram in Fig. A1(a) shows the curvature of space caused by the Schwarzschild metric compared to a flat Euclidean space. Due to the time-independence of the metric, one considers spacetime at a fixed, arbitrary point in time, and compares the line element $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ with a flat space. The deviations of the curved spacetime from the flat Euclidean geometry shows a funnel shape with a coordinate singularity at the Schwarzschild radius $R_S = 2M$, which is known as the event horizon.

Fig. A1(b) depicts the spacetime diagram of the Schwarzschild metric and describes the spacetime behavior of a spherically symmetric black hole from the point of view of an observer resting at infinity (Schwarzschild coordinates). Several radially ingoing and outgoing light rays are shown in an area around the black hole. For the outside observer it appears as if spacetime is divided into two regions: A region outside the event horizon $r > R_S$ and an area inside $r < R_S$. If one approaches the Schwarzschild radius from $r > R_S$, the cone of light narrows so much that one would need an infinitely long time to be able to reach the event horizon. Inside the black



(a) (b) (c)

Fig. A1. (a) Embedded diagram of the Schwarzschild manifold. (b) Spacetime diagram of the Schwarzschild metric in Schwarzschild coordinates and (c) in Eddington-Finkelstein coordinates.

hole, on the other hand, the space and time coordinates have apparently changed their meaning - the future light cones inevitably point in the direction of the real singularity at $r = 0$. A particle that is at a location inside the black hole must therefore necessarily move in the direction of the real singularity. To an outside observer, the spherical surface of the Schwarzschild radius appears to be the limit of an event horizon. No information can cross the event horizon and A. S. Eddington referred to this limit already in 1920 as the "magic circle" of the Schwarzschild solution ("There is a magic circle which no measurement can bring us inside."[52]). To an observer who is at rest at infinity, it appears as if a body falling into the black hole, shortly before reaching the event horizon, is getting slower and slower and the body will not cross the "magic circle" in a finite time. It's movement in the radial direction becomes infinitely slow near the event horizon and an outside observer then only sees the frozen image of the body on the event horizon. One can additionally show that the perceived image is simultaneously infinitely red-shifted so that it slowly leaves the visible frequency band near the event horizon.

In general relativity, however, many predications depend on the point of view of the observer, i.e. on the underlying coordinates. The coordinate singularity of the Schwarzschild metric, the event horizon at $r = R_S$ can be eliminated by means of special coordinate transformations. For example, if one transforms into a coordinate system that moves with a photon falling into the black hole, one arrives at the so-called ingoing Eddington-Finkelstein coordinates and Fig. A1(c) shows the spacetime diagram of the Schwarzschild metric in this coordinate system. It can be seen that a radially infalling photon can cross the event horizon, but after passing the barrier, the object has no longer any possibility of sending information to the outside. The theoretical properties of black holes are therefore sometimes compared to death[53]. The dying person/soul crosses the boundary of living (the event horizon) and after this crossing there is no more a possibility of transmitting information to the outer, earthly area. For the external observer (the human beings who experience the death of the person) the image of his clinically dead body freezes and as a living person, one has no possibility to receive information from the beyond (inner area of the black hole) - this is only possible if one himself dies and crosses the barrier of the event horizon.

In addition to the event horizon, there are two other interesting limiting values for non-rotating black holes. Both limits can be derived from the geodesic equation using the Schwarzschild metric. At a distance of $r = 3M$, the so-called photon sphere, light can move around the black hole on quasi stable circular paths. Massive bodies, with a speed that is less than the speed of light, can however only travel in stable circular orbits around the black hole if they are further away than $r = 6M$ from the center of the singularity - the so-called "Innermost Stable Circular Orbit (ISCO)" (details and visualizations see e.g.[54]).

## References

1. A. Einstein, Zur allgemeinen Relativitätstheorie, *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin* , 778 (January 1915).
2. A. Einstein, Die feldgleichungen der gravitation, *Preuss. Akad. Wiss. Berlin, Sitzungsber.* , 844 (1915).
3. K. Schwarzschild, Über das Gravitationsfeld eines Massenpunktes nach der Einsteinchen Theorie, *Sitzungsber. Dtsch. Akad. Wiss. Berlin, Kl. Math. Phys. Tech.* **1**, 189 (1916).
4. K. Schwarzschild, Über das Gravitationsfeld einer Kugel aus inkompressibler Flüssigkeit nach der Einsteinschen Theorie, *Sitzungsber. Dtsch. Akad. Wiss. Berlin, Kl. Math. Phys. Tech.* **1**, 424 (1916).
5. R. P. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics, *Physical Review Letters* **11**, 237 (1963).
6. J. T. Jebsen, On the General Spherically Symmetric Solutions of Einstein's Gravitational Equations in Vacuo., *Arkiv for Matematik, Astronomi och Fysik* **15**, p. 18 (1921).
7. G. D. Birkhoff, *Relativity and Modern Physics* (Harvard University Press, Cambridge, Mass., U.S.A., 1923).
8. R. C. Tolman, Static solutions of Einstein's field equations for spheres of fluids, *Physical Review* **55**, p. 364 (1939).
9. J. R. Oppenheimer and G. Volkoff, On massive neutron cores, *Physical Review* **55**, p. 374 (1939).
10. H. A. Buchdahl, General relativistic fluid spheres, *Physical Review* **116**, p. 1027 (1959).
11. J. R. Oppenheimer and H. Snyder, On continued gravitational contraction, *Physical Review* **56**, p. 455 (1939).
12. R. Penrose, Gravitational collapse and space-time singularities, *Physical Review Letters* **14**, p. 57 (1965).
13. A. Einstein, Über Gravitationswellen, *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin* , 154 (January 1918).
14. The LIGO Scientific Collaboration and the Virgo Collaboration, Observation of Gravitational Waves from a Binary Black Hole Merger, *Physical Review Letters* **116**, p. 061102 (February 2016).
15. The LIGO Scientific Collaboration, the Virgo Collaboration *et al.*, GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs, *Physical Review X* **9**, p. 031040 (July 2019).
16. B. P. Abbott *et al.*, Gw170817: Observation of gravitational waves from a binary neutron star inspiral, *Physical Review Letters* **119**, p. 161101 (Oct 2017).
17. The LIGO Scientific Collaboration, the Virgo Collaboration *et al.*, GWTC-2.1: Deep Extended Catalog of Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run, *arXiv e-prints* , p. arXiv:2108.01045 (August 2021).
18. K. The LIGO Scientific Collaboration, the Virgo Collaboration, Observation of Gravitational Waves from Two Neutron Star-Black Hole Coalescences, *Astrophysical Journal, Letters* **915**, p. L5 (July 2021).
19. The LIGO Scientific Collaboration, the Virgo Collaboration, B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. aAdams, T. Adams, P. Addesso and et al., Multi-messenger observations of a binary neutron star merger, *Astrophys. J. Lett.* **848**, p. L12 (2017).
20. E. Troja, L. Piro, H. van Eerten, R. T. Wollaeger, M. Im, O. D. Fox, N. R. Butler, S. B.

Cenko, T. Sakamoto, C. L. Fryer, R. Ricci, A. Lien, R. E. Ryan, O. Korobkin, S. K. Lee, J. M. Burgess, W. H. Lee, A. M. Watson, C. Choi, S. Covino, P. D'Avanzo, C. J. Fontes, J. B. González, H. G. Khandrika, J. Kim, S. L. Kim, C. U. Lee, H. M. Lee, A. Kutyrev, G. Lim, R. Sánchez-Ramírez, S. Veilleux, M. H. Wieringa and Y. Yoon, The X-ray counterpart to the gravitational-wave event GW170817, *Nature* **551**, 71 (Nov 2017).

21. LIGO Scientific Collaboration, Virgo Collaboration, F. Gamma-Ray Burst Monitor and INTEGRAL, Gravitational waves and gamma-rays from a binary neutron star merger: Gw170817 and grb 170817a, *Astrophys. J. Lett.* **848**, p. L13 (2017).
22. S. Beckett, *Waiting for Godot: Tragicomedy in 2 acts* (Grove Press, 1954).
23. M. Hanauske, L. Bovard, J. Steinheimer, A. Motornenko, V. Vovchenko, S. Schramm, V. Dexheimer, J. Papenfort, E. R. Most and H. Stöcker, *MAGIC - how MAtter's extreme phases can be revealed in Gravitational wave observations and in relativistic heavy Ion Collision experiments*, Journal of Physics Conference Series, Vol. 1271 July 2019.
24. M. Hanauske and H. Stöcker, Binary compact star mergers and the phase diagram of quantum chromodynamics, *Discoveries at the Frontiers of Science* , p. 107 (2020).
25. R. A. Hulse and J. H. Taylor, Discovery of a pulsar in a binary system, *The Astrophysical Journal* **195**, L51 (1975).
26. J. M. Weisberg and Y. Huang, Relativistic measurements from timing the binary pulsar psr b1913+16, *The Astrophysical Journal* **829**, p. 55 (2016).
27. A. Lyne, M. Burgay, M. Kramer, A. Possenti, R. Manchester, F. Camilo, M. McLaughlin, D. Lorimer, N. D'Amico, B. Joshi *et al.*, A double-pulsar system: A rare laboratory for relativistic gravity and plasma physics, *Science* **303**, 1153 (2004).
28. M. Kramer and I. Stairs, The double pulsar, *Annu. Rev. Astron. Astrophys.* **46**, 541 (2008).
29. M. Burgay, M. Kramer and M. McLaughlin, The double pulsar j0737–3039a/b: a decade of surprises., *Bulletin of the Astronomical Society of India* **42**, 101 (2014).
30. M. Hanauske and L. Bovard, Neutron star mergers in the context of the hadron-quark phase transition, *Journal of Astrophysics and Astronomy* **39**, p. 45 (August 2018).
31. M. Hanauske, K. Takami, L. Bovard, L. Rezzolla, J. A. Font, F. Galeazzi and H. Stöcker, Rotational properties of hypermassive neutron stars from binary mergers, *Phys. Rev. D* **96**, p. 043004 (August 2017).
32. M. G. Alford, L. Bovard, M. Hanauske, L. Rezzolla and K. Schwenzer, Viscous Dissipation and Heat Conduction in Binary Neutron-Star Mergers, *Physical Review Letters* **120**, p. 041101 (January 2018).
33. M. Hanauske, L. Bovard, E. Most, J. Papenfort, J. Steinheimer, A. Motornenko, V. Vovchenko, V. Dexheimer, S. Schramm and H. Stcker, Detecting the hadron-quark phase transition with gravitational waves, *Universe* **5** (2019).
34. M. Hanauske, J. Steinheimer, A. Motornenko, V. Vovchenko, L. Bovard, E. Most, L. Papenfort, S. Schramm and H. Stöcker, Neutron star mergers: Probing the eos of hot, dense matter by gravitational waves, *Particles* **2**, 44 (2019).
35. E. R. Most, L. J. Papenfort, V. Dexheimer, M. Hanauske, S. Schramm, H. Stöcker and L. Rezzolla, Signatures of Quark-Hadron Phase Transitions in General-Relativistic Neutron-Star Mergers, *Physical Review Letters* **122**, p. 061101 (February 2019).
36. M. Anton, H. Matthias, W. Lukas, S. Jan and S. Horst, Magic: Matter in astrophysics, gravitational waves, and ion collisions, *Nuclear Physics Review* **37**, 272 (2020).
37. L. R. Weih, M. Hanauske and L. Rezzolla, Postmerger Gravitational-Wave Signatures of Phase Transitions in Binary Mergers, *Physical Review Letters* **124**, p. 171103 (May 2020).

38. M. Hanauske, L. R. Weih, H. Stöcker and L. Rezzolla, Metastable hypermassive hybrid stars as neutron-star merger remnants, *European Physical Journal Special Topics* **230**, 543 (May 2021).
39. M. Hanauske and L. R. Weih, Neutron star collisions and gravitational waves, *Astronomische Nachrichten* **342**, 788 (June 2021).
40. E. R. Most, L. Jens Papenfort, V. Dexheimer, M. Hanauske, H. Stoecker and L. Rezzolla, On the deconfinement phase transition in neutron-star mergers, *European Physical Journal A* **56**, p. 59 (February 2020).
41. M. Maggiore, C. Van Den Broeck, N. Bartolo, E. Belgacem, D. Bertacca, M. A. Bizouard, M. Branchesi, S. Clesse, S. Foffa, J. García-Bellido, S. Grimm, J. Harms, T. Hinderer, S. Matarrese, C. Palomba, M. Peloso, A. Ricciardone and M. Sakellariadou, Science case for the Einstein telescope, *Journal of Cosmology and Astroparticle Physics* **2020**, p. 050 (March 2020).
42. R. Genzel, F. Eisenhauer and S. Gillessen, The Galactic Center massive black hole and nuclear star cluster, *Reviews of Modern Physics* **82**, 3121 (October 2010).
43. Event Horizon Telescope Collaboration, K. Akiyama, and et al., First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole, *Astrophysical Journal, Letters* **875**, p. L1 (April 2019).
44. M. Hanauske, A. Bhattacharyya, I. N. Mishustin, L. M. Satarov, J. Schaffner-Bielich, D. Zschiesche, H. Stöcker and W. Greiner, *A Theory of Modern ART* (Poster presented at the meeting of the German Astronomical Society, see `http://itp.uni-frankfurt.de/~hanauske/BerlinPoster2002.pdf`, 2002).
45. M. Hanauske, Black holes and the german reichstag, *Physics World* **18**, p. 64 (2005).
46. C. S. Gokhale and C. Hauert, Eco-evolutionary dynamics of social dilemmas, *Theoretical Population Biology* **111**, 28 (2016).
47. K. Sigmund and R. D. Holt, Toward ecoevolutionary dynamics, *Proceedings of the National Academy of Sciences* **118** (2021).
48. R. J. Hewitt, R. Cremades, D. V. Kovalevsky and K. Hasselmann, Beyond shared socioeconomic pathways (ssps) and representative concentration pathways (rcps): climate policy implementation scenarios for europe, the us and china, *Climate Policy* **21**, 434 (2021).
49. H. Hesse, *Das Glasperlenspiel: Versuch einer Lebensbeschreibung des Magister Ludi Josef Knecht samt Knechts hinterlassenen Schriften* (Suhrkamp Verlag, 2012).
50. D. Reitze, R. X. Adhikari, S. Ballmer, B. Barish, L. Barsotti, G. Billingsley, D. A. Brown, Y. Chen, D. Coyne, R. Eisenstein, M. Evans, P. Fritschel, E. D. Hall, A. Lazzarini, G. Lovelace, J. Read, B. S. Sathyaprakash, D. Shoemaker, J. Smith, C. Torrie, S. Vitale, R. Weiss, C. Wipf and M. Zucker, *Cosmic Explorer: The U.S. Contribution to Gravitational-Wave Astronomy beyond LIGO* September 2019.
51. M. Hanauske, *Physics of Socio-Economic Systems on the Computer* (Online-Lecture `https://itp.uni-frankfurt.de/~hanauske/VPSOC/VPSOC2021.html`, 2021).
52. A. S. Eddington, *Space, time and gravitation: An outline of the general relativity theory* (University Press, 1920).
53. C. Nordmann, Einstein expose et discute sa théorie, *Revue des Deux Mondes (1829-1971)* **9**, 129 (1922).
54. M. Hanauske, *General Theory of Relativity on the Computer* (Online-Lecture `http://itp.uni-frankfurt.de/~hanauske/VARTC/VARTC2021.html`, 2021).