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Sheet 6

Hand in via OLAT until 15.12.2020 18:00.

16) Heat pump *(6=2+4 points)*

A room at temperature T_2 looses heat to its environment. The temperature of the environment is $T_1 < T_2$ and the heat loss rate is $\frac{dQ}{dt} = C(T_2 - T_1)$. At the same time, the room is heated by a heat pump, which is described by an inverse Carnot process, operating between T_1 and T_2.

(i) The inverse Carnot process is a Carnot process that operates in the inverse order. Show that $\Delta W > 0$, i.e. the work has to be put into the system. The heat effectivity is definined as

$$\eta_H = -\frac{\Delta Q_2}{\Delta W}.$$

Show that $\eta_H > 1$.

(ii) Derive an expression for the equilibrium temperature T_2 of the room as a function of C, T_1 and the power $P = \frac{dW}{dt}$.

17) Specific heat C_V *(4 Points)*

Derive an expression for the specific heat C_V in terms of the variables T, μ und V.

Hint: Use the trick with the functional determinant from the lecture.

18) Absolute zero *(4 points)*

In the lecture you showed, that as a consequence of the 3. law of thermodynamics the heat capacities C_V and C_P have to vanish at the absolute zero $T = 0$. The same is true for the isobar coefficient of thermal expansion

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p .$$

Measuring the heat capacity for low temperatures experimentally suggests the following Ansatz that satisfies the necessary conditions

$$C_P = T^x\Big(a(P) + b(P)T + c(P)T^2 + ...\Big),$$

with $x > 0$ and coefficients $a(P)$, $b(P)$, $c(P)$ depending on the material.
Show that using this Ansatz for the heat capacity, leads to a finite result for the ratio

$$\lim_{T\to 0}\frac{\alpha V}{C_P},$$

in the limit $T \to 0$.
Hint: Express αV as a function of (S, P, T) and connect it to the heat capacity. Writing the entropy as

$$S = \int_0^T (dS)_P,$$

integrating along a path of constant pressure P, will be of great use.

19) Fluctuations in the canonical ensemble *(4+2=6 Points)*

Consider a system at constant volume ($dV = 0$) which is in thermal contact with a heat bath of temperature T.

(i) Prove that the fluctuations of the energy E in the canonical ensemble is $\langle(\Delta E)^2\rangle = T^2 C_V$.

(ii) Show that the fluctuations vanish in the thermodynamic limit: $\frac{|\Delta E|}{E} \sim \frac{1}{\sqrt{N}}$.