Sheet 4

Hand in via OLAT until 01.12.2020 18:00.

10) Adiabatic expansion *(2+2+4=8 Points)*

Consider the following phase space volume of a gas with constant number of particles N in a constant volume V

$$\Gamma(E,V,N) = f(N)V^N E^{\frac{3N}{2}}.$$

(i) Calculate the temperature and determine the equation of state

$$E = g(T,N).$$

(ii) Calculate the pressure and determine the equation of state

$$P = w(E,V).$$

(iii) Show that in case of an adiabatic expansion, the following relation holds

$$PV^{\frac{5}{3}} = \text{const}.$$

Hint: Make use of the 1. Law of thermodynamics.

11) Differentials *(2+2=4 Points)*

Let x, y, z be variables, satisfying an equation of state $f(x,y,z) = 0$.

(i) Show that

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}.$$

(ii) Show that

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1.$$

Hint: Consider the total differential df and pay attention, which variables are kept constant.

12) Stirling formula *(2+3+3=8 Points)*

(i) Prove that $\log(n!)$ can be approximated for very large n as follows (Stirling formula): $\ln(n!) \simeq n\ln(n) - n$.
Hint: Write $\ln(n!)$ as a sum and argue, that this sum can be identified as an integral in the limit of large n.

(ii) Consider N subsystems (e.g. N identically weighted dice), which can take m states (e.g. sides) with probability distribution $\{w_1, \dots w_m\}$. Show that the total number of states is given by $\Gamma = \frac{N!}{N_1!N_2!\dots N_m!}$, where a state of the system is characterized by the set of numbers $\{N_k\}$ with N_k the number of subsystems in state $k \in \{1, \dots m\}$ and $\sum\limits_{k=1}^{m} N_k = N$.
Hint: you may choose iteratively: N_1 from N subsystems in state 1: $\binom{N}{N_1}$,
N_2 from $N - N_1$ subsystems in state 2: $\binom{N-N_1}{N_2}$, etc.

(iii) It is possible to determine the entropy $S = \ln\Gamma$ of the total system Γ, considered in (ii). Use the Stirling formula to show, that for large N, the entropy is given as N:

$$S = -N \sum_{k=1}^{m} w_k \ln(w_k), \qquad \text{where} \qquad w_k = \frac{N_k}{N}.$$

This entropy is one representation of Shannon entropy for N independent systems. The Shannon entropy is a measure for the information of a message, measuring the amount of information per character of a source. The Shannon entropy is most often used in computer science.