Sheet 11

Hand in via OLAT until 09.02.2020 18:00.

30) Fermi velocity *(5 Points)*

Compute for a particle of mass m_0 and spin S in a Fermi-gas of density $\frac{N}{V}$ the speed of at the Fermi surface and determine in what regime the Fermi speed can still be treated non-relativistic.
Also compute the Fermi energy for the relativistic case.

31) Relativistic degenerate Fermi-Gas *(1+2+2=5 Points)*

Consider a Fermi-Gas of high density in the limit $T \to 0$. We assume, that the energy of the gas particles is large, so that relativistic effects become essential. Furthermore, the rest energy mc^2 can be neglected, so that the energy of the particles is given by the ultrarelativistic limit

$$\epsilon = cp.$$

(i) Determine the Fermi-energy ϵ_F in the ultrarelativistic limit.

(ii) Determine the total energy E of the gas, as a function of the density N/V.

(iii) Determine the pressure of the gas and show, that one obtains the following equation of state

$$PV = \frac{1}{3}E.$$

32) Magnetic susceptibility at large temperatures *(5+3+2=10 Points)*

We want to calculate the contribution of the orbit movement of the electrons to the magnetic susceptibility of an electron gas in a weak field B, at large temperature.
In the lecture, the following grandcanonical potential of the system for arbitrary temperatures has been derived

$$\Omega = -2\mu_B B \frac{V 2mT}{4\pi^2\hbar^3} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \ln\left(1 + z e^{-\frac{\epsilon_n}{T}}\right) dp_3,$$

with energy levels

$$\epsilon_n = \frac{p_3^2}{2m} + \hbar\omega_c\left(n + \frac{1}{2}\right).$$

(i) Show, that in the limit of high temperatures, Ω is given as

$$\Omega = -\frac{4V\mu_B z B}{\lambda^3} \frac{e^{-\frac{\hbar\omega_c}{2T}}}{1 - e^{-\frac{\hbar\omega_c}{T}}},$$

with the thermal wavelength $\lambda = \sqrt{\frac{2\pi\hbar^2}{mT}}$.
Hint: Series expand the logarithm, use the geometrical sum and the integral

$$\int_{-infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}.$$

(ii) Series expand Ω in the limit of high temperatures $T \ll \frac{\hbar\omega_c}{2}$ up to order $\mathcal{O}\big(\big(\frac{\hbar\omega_c}{2T}\big)^2\big)$ and calculate the magnetic susceptibility χ.

(iii) Express the magnetic susceptibility in terms of the density N/V and show, that the result resembles the Curie law for ideal gases

$$\chi = \frac{C}{T},$$

with C a temperature independent constant.
Hint: Use the expression of the density N/V at high temperatures, familiar from the lecture.