Exercise 1: Dyson-Schwinger equation of the effective action ***(8 points)***
Consider the following interacting, euclidean, real scalar field theory

$$S[\phi] = \int \left[\phi \left(-\partial_\mu \partial_\mu + m^2 \right) \phi + \frac{\lambda_3}{3!}\phi^3 + \frac{\lambda_4}{4!}\phi^4 \right] dx. \tag{1}$$

Starting from the Dyson-Schwinger equation for $Z[J]$, derive the Dyson-Schwinger equation for the generating functional of the proper Green's functions $\Gamma[\varphi]$

$$-\frac{\delta S}{\delta \phi(x)}[\phi] + \frac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0, \tag{2}$$

with

$$\phi(x') = \varphi(x') + \int \frac{\delta^2 W[J]}{\delta J(x')\delta J(z)} \frac{\delta}{\delta \varphi(z)} dz, \tag{3}$$

as well as $\varphi(y) = \frac{\delta W[J]}{\delta \varphi(y)}$, and the generating functional $\Gamma[\varphi]$ is obtained by Legendre transformation.
Hint: Use the following identity, that also holds in the case of functional derivatives

$$f\left(\frac{\partial}{\partial x}\right) e^{F(x)} = e^{F(x)} f\left(\frac{\partial}{\partial x} + \frac{\partial F(x)}{\partial x}\right) \tag{4}$$

Exercise 2: Integrals of Grassmann variables ***(6+6=12 points)***
Calculate the following integrals for complex Grassmann numbers θ_i

i) An integral of two-point gaussian type

$$\int \left(\prod_i d\theta_i^\star d\theta_i \right) \theta_k \theta_l^\star e^{-\theta_i^\star A_{ij} \theta_j} \tag{5}$$

ii) Two four-point gaussian integrals

$$\int \left(\prod_i d\theta_i^\star d\theta_i \right) \theta_k \theta_l \theta_m \theta_n e^{-\theta_i^\star A_{ij} \theta_j}, \tag{6}$$

and

$$\int \left(\prod_i d\theta_i^\star d\theta_i \right) \theta_k \theta_l \theta_m^\star \theta_n^\star e^{-\theta_i^\star A_{ij} \theta_j}. \tag{7}$$