Exercise 1: Renormalizability (12 points)

Calculate the divergence index of the following interaction terms and determine the spacetime dimensions for which the interaction becomes renormalizable

a)
$$\bar{\psi}\psi\phi$$
, b) ϕ^6 , c) $\bar{N}N$, d) $\pi^T \cdot \pi$, e) $\epsilon^{abc}\pi^b\partial^\mu\pi^c$, f) $\bar{N}\gamma^5\gamma^\mu T^aN$,

where ϕ , ψ , π and N represent scalar bosons, fermions, pions (mesons) and nucleons (baryons).

Exercise 2: QED β -function (3+3+2 points)

The QED beta-function to two loop order is given as

$$\beta(e) = \frac{e^3}{12\pi^2} + \frac{e^5}{64\pi^4} + \mathcal{O}(e^7).$$
(1)

We want to study the behavior of the coupling for $\mu \to 0$.

i) Use separation of variables to solve the differential equation

$$\mu \frac{de}{d\mu} = \beta(e), \tag{2}$$

and obtain a solution of the form

$$e^2 = f(e^2 \ln e^2).$$
 (3)

Hint: When performing the separation of variables, expand the inverse beta-function $1/\beta(e)$ at e = 0, neglecting all terms $\mathcal{O}(e)$.

ii) To obtain an equation of the form $e^2 = f(\mu)$, where no coupling is appearing on the right hand side anymore, we need to iteratively solve the equation from i). Iterate the equation, using the approximation

$$e^2 \approx -\frac{1}{\frac{1}{12\pi^2} \ln\left(\frac{\mu^2}{\mu_0^2}\right)}$$
 (4)

in the second iteration.

iii) Perform the limit $\mu \to 0$ and comment on the behavior of the coupling.