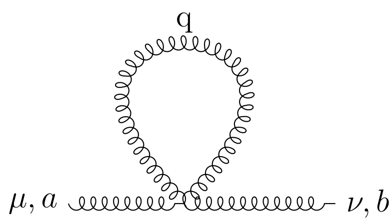
Exercise 1: 1-loop correction to the gluon propagator *(6+6+4+4 points)*

In the lecture you discussed the 1-loop correction to the gluon propagator in QCD. You explicitly calculated the 1-loop correction including gluon 3-vertices and ghost field vertices. In this exercise we are going to extend this calculation to the gluon 4-vertex and the quark loop.

i) Show that the 1-loop correction to the gluon propagator caused by the 4-gluon interaction

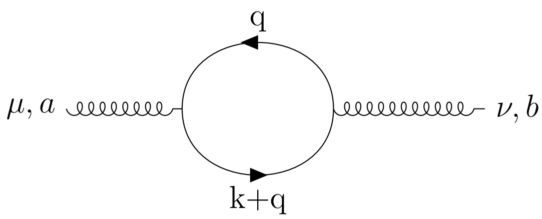


in d-dimensions is given as

$$\mathrm{i}\Pi^{(c),ab}_{\mu\nu} = -g_\epsilon^2 \delta^{ab} N_c (d-1) g_{\mu\nu} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 + \mathrm{i}\epsilon} \tag{1}$$

and comment on the divergence.

ii) Show that the fermion 1-loop correction to the gluon propagator



in d-dimensions is given as

$$\mathrm{i}\Pi^{(d),ab}_{\mu\nu} = -\frac{g_\epsilon^2}{2} \delta^{ab} 4 \sum_{f=1}^{N_f} \int \frac{d^d q}{(2\pi)^d} \frac{2q_\mu q_\nu - g_{\mu\nu}\left(g^2 + k\cdot q - m_f^2\right) + k_\mu q_\nu + k_\nu q_\mu}{(q^2 - m_f^2 + \mathrm{i}\epsilon)((q+k)^2 - m_f^2 + \mathrm{i}\epsilon)} \tag{2}$$

and comment on the divergence.

iii) Consider massless quarks. Use a Wick rotation to change to euclidean space for the fermion loop contribution and use the $J(k)$, $J^\mu(k)$, $J^{\mu\nu}(k)$ familiar from the lecture to regularize your result.

iv) Show that the regularized result corresponds to that one given in the lecture, when setting $m_f = 0$.