
Exercise 1: Bhabha-scattering

Consider the theory of Quantum Electrodynamics for electrons, positrons and photons,

$$\mathcal{L}_{QED} = \bar{\psi}\left[\mathrm{i}\not{D} - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \bar{\psi}\left[\mathrm{i}\gamma^{\mu}\left(\partial_{\mu} + \mathrm{i}eA_{\mu}\right) - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{1}$$

Electron (e^-) - Positron (e^+) scattering is referred to as Bhabha-scattering

$$e^+(p_1, r_1) + e^-(p_2, r_2) \to e^+(p_1', r_1') + e^-(p_2', r_2'). \tag{2}$$

i) Identify all the Feynman diagrams contributing at tree level ($\mathcal{O}(e^2)$) and write down the corresponding matrix elements in momentum space using the Feynman rules familiar from the lecture.

ii) Compute the spin averaged squared absolut value of the amplitudes (keep in mind that there could be interference terms!) in the high energy limit and in the center of mass frame using Feynman gauge ($\xi = 1$).

************************************Bonus************************************

iii) Recall from sheet 9 that the differential cross section of a 2 → 2 particle process was given as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{f_i}|^2, \tag{3}$$

in the case of equal masses for incoming and outgoing particles. Use your previous result to show that the differential cross section in the case of Bhabha-scattering is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left(\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} + \frac{1+\cos^2(\theta)}{2} - 2\frac{\cos^4(\theta/2)}{\sin^2(\theta/2)}\right), \tag{4}$$

where $\alpha = \frac{e^2}{4\pi}$ is the fine-structure constant, θ the scattering angle and E is the respective energy of electron and positron in the center-of-mass system.

Hint: Use Mandelstam variables.

Exercise 2: Compton-scattering

Again consider the theory of Quantum Electrodynamics for electrons, positrons and photons,

$$\mathcal{L}_{QED} = \bar{\psi}\left[\mathrm{i}\not{D} - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \bar{\psi}\left[\mathrm{i}\gamma^{\mu}\left(\partial_{\mu} + \mathrm{i}eA_{\mu}\right) - m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{5}$$

Compton-scattering is the process of electron-photon scattering

$$e^{-}(p_1, r_1) + \gamma(p_2, \lambda) \rightarrow e^{-}(p_1', r_1') + \gamma(p_2', \lambda'). \tag{6}$$

i) Identify all tree level Feynman diagrams of the scattering process and calculate the matrix elements in momentum space using the Feynman rules.

ii) Compute the squared matrix element in the massless limit and average over spins and polarization states by calculating the corresponding sums, leading to

$$|\bar{\mathcal{M}_f}|^2 = 2e^4\left(-\frac{u}{s} - \frac{s}{u}\right), \tag{7}$$

with Mandelstam variables s and u.

***********************************Bonus***********************************

iii) Use your result to calculate the differential cross section in the center-of-mass frame. Why is it not possible to integrate the cross section and obtain the total cross section? How can you fix the problem?

Hint: For each process involving two external photons the matrix element can be written as

$$\mathcal{M}_{f_i} = \mathcal{M}_{\alpha\beta}(k, k', ...)\epsilon^{\alpha}_{\lambda}(k)\epsilon^{\beta\star}_{\lambda'}(k'). \tag{8}$$

In the lecture you have seen, gauge invariance enforces that replacing any polarization vector with the corresponding 4-momentum leads to a vanishing matrix element

$$k^{\alpha}\mathcal{M}_{\alpha\beta} = k^{\beta}\mathcal{M}_{\alpha\beta} = 0. \tag{9}$$

Therefore the second term in the completeness relation of the polarization vectors

$$\sum_{\lambda=1}^{2}\epsilon^{\alpha\star}_{\lambda}(k)\epsilon^{\beta}_{\lambda}(k) = -\left(g^{\alpha\beta} - \frac{k^{\alpha}k^{\beta}}{k^2}\right) \tag{10}$$

will not contribute to the matrix element $\mathcal{M}$ and can be discarded.