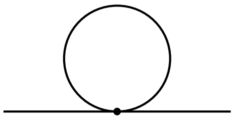
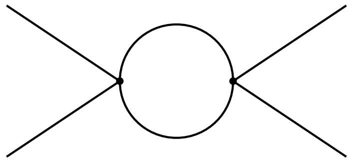
Exercise 1: Feynman rules in momentum space

Give the integral expressions of the following Feynman diagrams in momentum space using the Feynman rules of ϕ^4-theory given in the lecture script.





Integrate out all the δ-functions but do not perform the remaining integrals (if you do it, it is at your own risk...).

Exercise 2: Fermion propagator

The propagator of spinors is defined as

$$\begin{aligned} S_{F,mn}(x-y) &= \langle 0| T(\psi_m(x)\bar{\psi}_n(y)) |0\rangle \\ &= \Theta(x_0-y_0) \langle 0| \psi_m(x)\bar{\psi}_n(y) |0\rangle - \Theta(y_0-x_0) \langle 0| \bar{\psi}_n(y)\psi_m(x) |0\rangle \end{aligned} \tag{1}$$

Comment on the additional minus sign in the time ordering and calculate the fermion propagator in momentum space by inserting the Fourier representation of the field operators, to obtain

$$\tilde{S}_F(p) = \mathrm{i}\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}. \tag{2}$$

Exercise 3: Compton-scattering

Again consider the theory of Quantum Electrodynamics for electrons, positrons and photons,

$$\mathcal{L}_{QED} = \bar{\psi}\left[\mathrm{i}\not{D}-m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \bar{\psi}\left[\mathrm{i}\gamma^\mu\left(\partial_\mu+\mathrm{i}eA_\mu\right)-m\right]\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{3}$$

Compton-scattering is the process of electron-photon scattering

$$e^-(p_1,r_1)+\gamma(p_2)\to e^-(p_1',s_1)+\gamma(p_2'). \tag{4}$$

i) Identify all tree level Feynman diagrams of the scattering process and calculate the matrix elements in momentum space using the Feynman rules.

ii) Compute the squarred matrix element in the massless limit and average over spins and polarization states by calculating the corresponding sums, leading to

$$|\bar{\mathcal{M}_f}|^2 = 2e^4\left(-\frac{u}{s}-\frac{s}{u}\right), \tag{5}$$

with Mandelstam variables s and u.

iii) Use your result to calculate the differential cross section in the center-of-mass frame. Why is it not possible to integrate the cross section and obtain the total cross section? How can you fix the problem?

Hint: For each process involving two external photons the matrix element can be written as

$$\mathcal{M}_{f_i} = \mathcal{M}_{\alpha\beta}(k, k', ...)\epsilon^{\alpha}_{\lambda}(k)\epsilon^{\beta\star}_{\lambda'}(k'). \tag{6}$$

Gauge invariance enforces that replacing any polarization vector with the corresponding 4-momentum leads to a vanishing matrix element (as it will be discussed in the lecture)

$$k^{\alpha}\mathcal{M}_{\alpha\beta} = k^{\beta}\mathcal{M}_{\alpha\beta} = 0. \tag{7}$$

Therefore the second term in the completeness relation of the polarization vectors

$$\sum_{\lambda=1}^{2} \epsilon^{\alpha\star}_{\lambda}(k)\epsilon^{\beta}_{\lambda}(k) = -\left(g^{\alpha\beta} - \frac{k^{\alpha}k^{\beta}}{k^2}\right) \tag{8}$$

will not contribute to the matrix element $\mathcal{M}$ *and can be discarded.*