Lösungen der Übungsaufgaben — Theoretische Physik I

WS 04/05 — Prof. Dr. C. Gros

2. Übung

Aufgabe 5

Bemerkung:

$$\dot{\underline{\omega}} = 0 = \mathring{\underline{\omega}} \qquad \underline{g} = (0,0,g)^T \qquad \underline{\omega} = (0, \omega\cos\phi, \omega\sin\phi)^T \qquad \underline{R} = (0,0,R)^T$$

a.)

$$m\underline{g} = m(\underline{r} + \underline{R})^{\bullet\bullet} = m\,\ddot{\underline{R}} + m\,\ddot{\underline{r}}$$

a1.) $m\,\ddot{\underline{R}} = m(\underline{\omega} \times \dot{\underline{R}}) = m(\underline{\omega} \times (\underline{\omega} \times \underline{R}))$

a2.)

$$\begin{aligned} \dot{\underline{r}} &= \mathring{\underline{r}} + \underline{\omega} \times \underline{r} \\ \ddot{\underline{r}} &= (\dot{\underline{r}})^\circ + (\underline{\omega} \times \underline{r})^\circ = \mathring{\mathring{\underline{r}}} + \underline{\omega} \times \mathring{\underline{r}} + \underline{\omega} \times \mathring{\underline{r}} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) \end{aligned}$$

$$\begin{aligned} \Rightarrow m\underline{g} &= m\underline{\omega} \times (\underline{\omega} \times \underline{R}) + m\,\mathring{\mathring{\underline{r}}} + 2m\underline{\omega} \times \mathring{\underline{r}} + m\underline{\omega} \times (\underline{\omega} \times \underline{r}) \\ \Leftrightarrow m\,\mathring{\mathring{\underline{r}}} &= \underbrace{m\underline{g} - m\underline{\omega} \times (\underline{\omega} \times \underline{R})}_{=\tilde{\underline{g}}} - 2m\underline{\omega} \times \mathring{\underline{r}} - m\underline{\omega} \times (\underline{\omega} \times \underline{r}) \end{aligned}$$

b.)

$$\begin{aligned} \Delta\underline{g} &= \underline{\omega} \times (\underline{\omega} \times \underline{R}) = \underline{\omega} \cdot (\underline{\omega} \cdot \underline{R}) - \underline{R} \cdot (\underline{\omega} \cdot \underline{\omega}) \\ &= \begin{pmatrix} 0 \\ \omega^2 R\cos\phi\sin\phi \\ \omega^2 R\sin^2\phi \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ \omega^2 R \end{pmatrix} = \begin{pmatrix} 0 \\ \omega^2 R\cos\phi\sin\phi \\ -\omega^2 R\cos^2\phi \end{pmatrix} \end{aligned}$$

c.)

$$\begin{aligned} F_c &= -2m\underline{\omega} \times \mathring{\underline{r}} \qquad \text{mit } \underline{r} = (r_1, r_2, r_3)^T \\ &= -2m\omega \begin{pmatrix} \mathring{r}_3\cos\phi - \mathring{r}_2\sin\phi \\ \mathring{r}_1\sin\phi \\ -\mathring{r}_1\cos\phi \end{pmatrix} \end{aligned}$$

d.)

$$\begin{aligned}
\overset{\circ\circ}{r}_1 &= -2\omega(\overset{\circ}{r}_3 \cos\phi - \overset{\circ}{r}_2 \sin\phi) \\
\overset{\circ\circ}{r}_2 &= -2\omega \overset{\circ}{r}_1 \sin\phi \\
\overset{\circ\circ}{r}_3 &= -\underline{\tilde{g}} + 2\omega \overset{\circ}{r}_1 \cos\phi
\end{aligned}$$

e.) $\overset{\circ}{r}_1 \approx 0 \approx \overset{\circ}{r}_2 \Rightarrow$

$$\begin{aligned}
\overset{\circ\circ}{r}_1 &= -2\omega \overset{\circ}{r}_3 \cos\phi \\
\overset{\circ\circ}{r}_2 &= 0 \\
\overset{\circ\circ}{r}_3 &= -\underline{\tilde{g}}
\end{aligned}$$

Anfangsbedingungen: $\underline{r}(t=0) = (0,0,h)^T$, $\overset{\circ}{\underline{r}}\,(t=0) = (0,0,0)^T$

$$\Rightarrow \underline{r}(t) = \begin{pmatrix} \frac{1}{3}\omega\tilde{g}\cos\phi t^3 \\ 0 \\ -\frac{1}{2}\tilde{g}t^2 + h \end{pmatrix}$$

Die Fallzeit t_F bestimmt sich aus $r_3(t_F) = 0$ zu

$$t_F = \sqrt{\frac{2h}{\tilde{g}}}.$$

Damit ergibt sich eine Ortsabweichung von

$$r_1(t_F) = \frac{1}{3}\omega\tilde{g}\cos\phi \left(\frac{2h}{\tilde{g}}\right)^{(3/2)}$$

Aufgabe 6

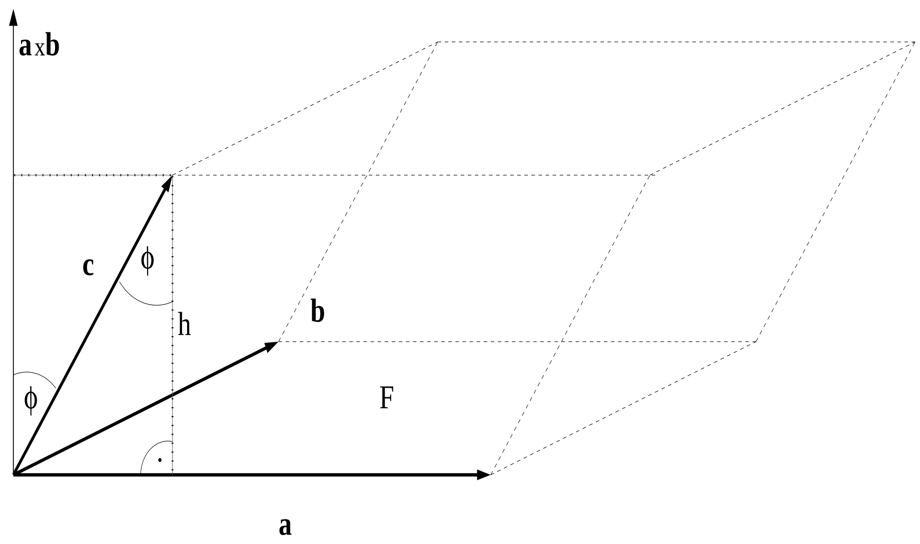
a.)

$$\begin{aligned}
\left[\vec{a} \times (\vec{b} \times \vec{c})\right]_\alpha &= \epsilon^{\alpha\beta\gamma} a_\beta \left(\vec{b} \times \vec{c}\right)_\gamma \\
&= \epsilon^{\alpha\beta\gamma}\epsilon^{\mu\nu\gamma} a_\beta b_\mu c_\nu \\
&= \left(\delta^{\alpha\mu}\delta^{\beta\nu} - \delta^{\alpha\nu}\delta^{\beta\mu}\right) a_\beta b_\mu c_\nu \\
&= a_\beta b_\alpha c_\beta - a_\beta b_\beta c_\alpha \\
&= \left[\vec{b} \cdot (\vec{a} \cdot \vec{c}) - \vec{c} \cdot (\vec{a} \cdot \vec{b})\right]_\alpha
\end{aligned}$$

b.) Nach Anwendung der "BAC-CAB" Relation und Symmetrie des Skalarproduktes folgt unmittelbar:

$$\begin{aligned} & \vec{b}\cdot(\vec{a}\cdot\vec{c}) - \vec{c}\cdot(\vec{a}\cdot\vec{b}) \\ + \; & \vec{c}\cdot(\vec{b}\cdot\vec{a}) - \vec{a}\cdot(\vec{b}\cdot\vec{c}) \\ + \; & \vec{a}\cdot(\vec{c}\cdot\vec{b}) - \vec{b}\cdot(\vec{c}\cdot\vec{a}) \quad = \quad 0 \end{aligned}$$

c.)



$$\begin{aligned} \text{Volumen} \quad &= \quad \text{Grundfläche } F \cdot \text{ Höhe } h \\ &= \quad |\vec{a}\times\vec{b}|\cdot c\cdot\cos\varphi \\ &= \quad \left(\vec{a}\times\vec{b}\right)\cdot\vec{c} \end{aligned}$$

Aufgabe 7

Lösung der Programmieraufgabe entfällt.