Exercise Sheet #8

Deadline: 18.12.2023, 12:00h

Problem 1 (*Hausdorff Dimension*) (10 points)

Calculate the Hausdorff dimension of

(a) a straight line,

(b) the cantor set

Hint: The Cantor ternary set is created by repeatedly deleting the open middle thirds of a set of line segments. First, one removes the open middle third (1/3, 2/3) from the interval [0, 1], leaving two line segments: [0, 1/3] and [2/3, 1]. Then, the open middle third of each of these remaining segments is also removed, leaving four line segments: [0, 1/9], [2/9, 1/3], [2/3, 7/9] and [8/9, 1]. Then the remaining four segments get their open middle third removed, and the process is repeated infinitely.,

(c) and the Koch snowflake.

Hint: Start with an equilateral triangle. In each iteration, divide every line segment into three equally long segments, draw an equilateral triangle that has the middle segment as its base and points outward and then remove the middle segment.

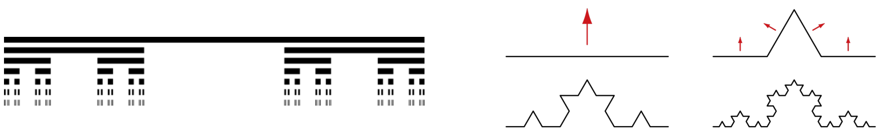

Figure 1: Illustration of the Cantor set (left) and the Koch snowflake (right).

Problem 2 (*Polymerization as a 3D Random Walk*) (10 points)

A random walk in 3D can be applied to polymerization in order to estimate the end-to-end distance of a polymer. A polymer is a material consisting of repeating subunits, called monomers. Polymerization is the process of combining many small molecules (the monomers) into a covalently bonded chain or network. For our purposes, think of a polymer as a simple chain of monomers, where each monomer has the same length a, but is oriented in a random direction.

(a) If a polymer is made up of $N \in \mathbb{N}$ monomers, calculate the expected value of the vector connecting the starting and the ending points of the polymer and the mean square distance between these points.

(b) What is the probability distribution of the distance between the starting and the ending points? What is the probability that the distance between the endpoints of the polymer is R?