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Exercise Sheet #7

Deadline: 11.12.2023, 12:00h

Problem 1 (*Time-Delayed Differential Equation*) (5 points)

Consider a car following a truck. The driver of the car should accelerate or break depending on the velocity of the truck that is in in front. As the driver has a certain non-zero reaction time $T > 0$, the acceleration of the car is given by

$$\dot{v}(t+T) = \alpha\left(v_{\text{truck}}(t) - v(t)\right) \ ,$$

where $v(t)$ is the velocity of the car, v_{truck} is the velocity of the truck and $\alpha > 0$ is an acceleration parameter of the car.
Assume that the truck has the constant speed $v(t) \equiv v_0$. Analyze the solution for the velocity $\dot{x}(t)$ for this case. Study the stability of this solution as function of the parameters α and T.

Problem 2 (*Lorenz System*) (15 points)

(a) Analyze the fixpoints of the Lorenz system given by

$$\dot{x} = s(y - x)$$
$$\dot{y} = x(r - z) - y$$
$$\dot{z} = xy - bz$$

and their stability as a function of the Rayleigh number $r > 0$ with parameters $s = 10$ and $b = 8/3$ fixed. Make a sketch of the $x(r)$ bifurcation diagram.
Hint: You may calculate the eigenvalues numerically.

(b) Show analytically that all fixpoints are unstable for

$$r > r_{\text{H}} = s(s + b + 3)/(s - b - 1).$$

Hint: You do not necessarily need a full solution for the eigenvalue spectra, since you are only looking for this particular transition point.

(c) What kind of bifurcation does the system undergo at $r = r_{\text{H}}$ and what kind of dynamics are present in the system for $r > r_{\text{H}}$?

(d) Evaluate numerically the Pointcare map of the Lorenz model with $\sigma = 10$ and $b = 8/3$, for:

- $r = 22$ (regular regime) and the plane $z = 21$,
- $r = 28$ (chaotic regime) and the plane $z = 27$.

Make a plot of the maps and mark the fixpoints. What is the behavior of the points in each map as they evolve? (particularly in relation to the fixpoints).