

Exercise Sheet #6

Problem 1 (*The umbrella dilemma*)

I own a total of 4 umbrellas which are stored either in my home or in the office. I keep moving between home and office, only carrying an umbrella with me if it rains. If it does not rain, I leave the umbrella behind (at home or in the office). It may happen that all umbrellas are in one place and I am at the other, it starts raining, so I get wet.

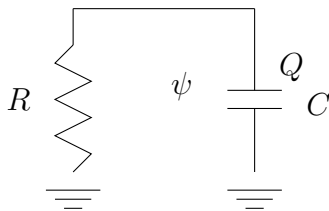
- (a) Construct a Markov chain that models this process.
- (b) Let p be the probability of rain for one of my transitions between home and the office. What is the probability I get wet?
- (c) I am umbrella shopping. Current estimates show that $p = 0.6$ in Frankfurt. How many umbrellas should I own so that, following the same strategy described above, the probability that I get wet is less than 1%?

Problem 2 (*Thermal fluctuations*)

Consider the electrical circuit illustrated below with a resistor R and a capacitor C . The charge Q in the capacitor disappears on average, but is subject to thermal fluctuations, as described by

$$R\dot{Q} + \frac{Q}{C} - \psi_R(t) = 0, \quad (1)$$

where $\psi_R(t)$ is the fluctuating voltage.



- (a) Compare the above equation with the Langevin equation for a Brownian particle and show that

$$\langle \psi_R(t) \psi_R(0) \rangle = 2RkT \delta(t).$$

- (b) Calculate the autocorrelation function $\langle Q(t)Q(0) \rangle$.