Exercise Sheet #11

Deadline: 29.01.2024, 12:00h

Problem 1 (*Time Series with Noise*) (10 points)

Consider the time series generated by a logical OR ($\vee$) operation,

$$\sigma_{t+2} = \sigma_{t+1} \vee \sigma_t$$

where $\sigma \in [0, 1]$.

(a) Evaluate the probability of $0, 1$ to occur given random initial conditions.

(b) Now add noise to the series, assuming that each bit may flip with probability w. With $w^2 \to 0$, find the frequency of $0, 1$ by writing down an equation for the time dynamics and finding a steady solution (write an equation for the probabilities of each steady state of the noiseless case).

(c) Validate your results by calculating the frequencies numerically. You will need to set $w < 0.0001$.

Problem 2 (*Information Content*) (10 points)

Two labs want to communicate the results of an experiment between them. In the experimental lab, two fair coins with sides $0, 1$ are tossed each time, and their results are summed together. After a long day of running these experiments, the results are sent to the theoretical lab for analysis.

(a) Calculate the binary entropy of a single coin toss, and of the sum of two coin tosses.

(b) If the string of results is compressed optimally, how long would the string of toss sums be compared to a string containing the information of all individual tosses?

(c) Calculate the same for the sum of three coin tosses, and for the sum of two six-sided dice.