

Exercise Sheet #10

Deadline: 22.01.2024, 12:00h

Problem 1 (*Central Limit Theorem*) (10 points)

Two friends play board games together every night. At the end of the night the loser has to pay the winner 10 Euros. Both of them are equally skilled players and therefore their odds of winning are the same. One of the friends is worried that he might lose too much money over time. Use the central limit theorem to calculate the probability that he loses more than 5000 Euros after 10 years that the friends play together.

Problem 2 (*Bayesian Inference*) (10 points)

Being the hard-working student that you are, you head home from the library pretty late at night. On your way home you meet a fellow student that instead spent the night drinking. You observe him tumbling along the street and start hypothesizing about the distribution of the probability that he takes a step to the right, p , or a step to the left, $(1 - p)$. After a while of stalking your classmate you have gathered more data: You counted that he walked one step to the left, but then four consecutive steps to the right. In your head you construct the dataset

$$D = \{\Delta x_0 = -1, \Delta x_1 = 1, \Delta x_2 = 1, \Delta x_3 = 1, \Delta x_4 = 1\}.$$

You want to improve your hypothesis on the probability p with the newly gathered data using Bayesian inference.

The probability distribution for the parameter p is $P(p)$. The initial hypothesis is that the probability is symmetric around $1/2$, and therefore you assume $P(p) \propto p(1 - p)$.

- (a) Something in your hypothesis for the probability is missing. Compute the full probability distribution.
- (b) Use Bayesian inference,

$$P(p | D) = \frac{P(D | p)P(p)}{P(D)},$$

to improve your hypothesis with the gathered data D , that is, compute $P(p | D)$.

- (c) Using your enhanced distribution, find the probability that your classmate takes the next step, Δx_5 , to the right.
- (d) A naive approach to estimate p from the data would be to divide the number of steps to the right by the number of steps to the left. Compare this to your result from part (c). Discuss the reason for possible differences.