

Exercise Sheet #0

Problem 1 (*Probability Distributions*)

Consider a course of N students taking an exam. The professor is evil and wants to make sure that not too many students pass the course. On the day of the exam he hands each student an exam, however, with a probability of p he hands out an exceptionally hard exam that an average student only passes with a probability of $p_h = 0.1$. A ‘normal’ exam, on the other hand, can be passed by an average student with a probability of $p_n = 0.7$.

- What is the probability for a given average student to pass the exam he is handed?
- What is the probability distribution p_k that $k \leq N$ students pass the exam?
- Derive the expected number of students that will pass the exam $\langle k \rangle$ from the probability distribution p_k .
- On the day of the exam, the professor gets up on the wrong foot and decides that 80% of his students should fail the exam. Determine the probability p with which he should hand out hard exams.

Problem 2 (*Exploring a simple nonlinear system*)

Consider the differential equation

$$\dot{x} = x(1 - x), \quad x(0) = x_0,$$

where $x(t)$ is a real-valued function of time.

- Compute all constant solutions (i.e., solutions where $x(t)$ does not change in time).
- Solve the differential equation explicitly.
- Write a Python program that:
 - numerically solves the differential equation for several initial values x_0 ,

- plots the solutions $x(t)$ over time,
- compares the numerical solutions with the explicit formula from part (b).

Let a chatbot help you if you are not familiar with Python programming.