1 Synchronization Phenomena

Study the synchronization of coupled oscillators. The application could visualize different phenomena:

- **Standard Kuramoto Model**: Each oscillator has its own preferred frequency, but the coupling is the same between each pair of oscillators. This should yield the typical phase transition in the order parameter, indicating global synchronization (1).

- **As a possible variation**, one can study the behavior if the coupling is modified. Options are random couplings, or coupling strengths representing certain topologies, such as rings or lattices. Changing the topological structure can yield interesting dynamical states like waves or so-called *chimera states* (2).

(1) Pikovsky, A. and M. Rosenblum – *Dynamics of globally coupled oscillators: Progress and perspectives*
(2) Abrams, D. M. and S. H. Strogatz – *Chimera states for coupled oscillators*

2 Percolation in Graphs

Create an application that visualizes the results of percolation theory that was discussed in the lecture.

- Show the distribution of cluster sizes depending on the average degree for random and scale-free graphs.
- Identify the transition to a giant cluster.
- Compare both types of networks with respect to robustness against node failure (random or hubs first).
- You can also implement more complex graph models, such as the Barabasi-Albert model.

3 Dynamic Programming: Why non-rational behavior may be rational

Seemingly non-rational behaviour may arise when maximizing the chance to survive in ecological models. Dynamic programming (1) is a method to evaluate optimal strategies in complex decision environments. Show that a choice of survival strategy can violate transitivity when the available strategies are limited, as can be seen in (2).

- Examine the behaviour of an animal which has access to all three actions appearing in (2) by using dynamic programming. How does the behaviour change through time?
- Examine the choice of strategies of animals with access to only two of the choices. Show that in this case, the choice of strategy at some energy levels becomes intransitive.

(1) Complex decisions made simple: a primer on stochastic dynamic programming
(2) Violations of transitivity under fitness maximization
4 Traffic Jams in Car Following Models

Implement a single-lane traffic simulation using a simple car following controller and investigate the dynamics with respect to the model parameters, such as maximum speed, acceleration and/or braking. As an extension, you may also implement a multi-lane simulation, where lane changes are possible. In particular, this would allow the investigation of target speed variability and its effect on the traffic flow.

Treiber, M., Hennecke, A. and Helbing, D. – Congested Traffic States in Empirical Observations and Microscopic Simulations

5 Predator-Prey Models as Cellular Automata

The Idea of predator-prey systems such as the Lotka-Volterra equations can be extended to a spatial topology, leading to certain types of cellular automata (1).

- Implement different variants of a predator-prey system on a cellular grid with next-neighbor interactions. Variations of the model may concern the rule of reproduction/spreading or the rules of interaction between both populations (2).
- Investigate how the dynamics change under different parameter settings.
- How well can the aggregate dynamics be described by simpler, non-spatial population models?

(1) Cellular Automata
(2) Cattaneo, G. and Dennunzio, A. – A Full Cellular Automaton to Simulate Predator-Prey Systems

6 Fractal Dimension of Strange Attractors

Describing the geometry of chaotic attractors can be a difficult task. One quantitative measure that can be used is the fractal dimension of the attractor. In the lecture, the box counting dimension was already introduced as a way to calculate this quantity. However, this is not the only possibility to define the attractor dimensions, and different approaches might lead to different results (1).

- Simulate different chaotic attractors of your choice and calculate the attractor dimensions for different parameters.
- Can you identify transitions from non-chaotic to chaotic regimes via these measures?

(1) Scholarpedia – Attractor Dimension

Theiler, J. – Estimating the Fractal Dimension of Chaotic Time Series

7 Mass Opinion Dynamics

A very simple model of opinion dynamics among agents was introduced by Deffuant et al. (1). It describes the exchange and mixing of opinions among individuals, eventually leading to the formation of clusters of opinions, separated by each other.

- Implement the model in its most simple form and discuss the results.
- Extend the model by e.g. introducing random fluctuations or by randomizing individual agents.
- Further possible extensions include a multi-dimensional opinion space or imposing topological constraints on the agent interaction.

(1) Deffuant, G., Neau, D. Amblard, F. and Weisbruch, G. – Mixing beliefs among interacting agents
8 Attractors in Hopfield Networks

Hopfield networks are recurrent networks that can be trained to behave like an associative memory (1). A typical example would be the recognition of a certain digit in some image representing an instance of this number.

- Implement a Hopfield network including a learning algorithm such that can be trained on some given dataset.
- Download the MNIST dataset (2) and test it with your network.
- How large does your network need to be to reliably converge to the correct pattern?

(1) Hopfield Networks
The MNIST dataset