Project #1: Kessler Syndrome

Deadline: 18.05.2024, 12:00h

The Kessler Syndrome [1, 2] describes a scenario in which the density of objects in low Earth orbit (LEO) becomes so high that collisions between satellites and debris trigger a self-sustaining cascade. Each collision generates fragments, which in turn increase the probability of further collisions. In extreme cases, this runaway process may render orbital regions unusable for satellites and severely impact communication, navigation, and Earth observation systems.
In this project, we study the Kessler Syndrome from a dynamical systems perspective. We analyze simplified mean-field models and compare them with numerical many-body simulations.

Part 1: Mean-field models for space debris

For a simple mean-field theory of the space debris in Earth's orbits we consider a *one-bucket* model where the number of objects $N(t)$ is the relevant dynamical variable.

Model without collisions As a first step we neglect the Kessler Syndrome and consider a model without collisions. Assume that objects are added at a constant launch rate L and removed at a rate proportional to their number (e.g. due to atmospheric drag or de-orbiting):

$$\dot{N}(t) = L - \lambda N(t).$$

Discuss this model from a dynamical systems perspective using the methods you learned in lecture.

Model with collisions To model the Kessler effect, we include collision-induced fragmentation. Since collisions scale with the number of possible pairs, we add a quadratic term:

$$\dot{N}(t) = L - \lambda N(t) + \alpha N^2(t).$$

Here $\alpha > 0$ measures the rate at which collisions produce new debris.
Discuss this model using the methods from lecture. How does the additional term change the dynamics compared to the collision-free model? Does this model capture the Kessler Syndrome? Are there bifurcations?

Extensions (optional) If you are interested, modify the model further. You might, for example, introduce a second compartment for intact satellites which might be populated according to a launch rate. A source in the debris compartment could include collisions between intact satellites and debris.

Part 2: Simulating the many-body system of space debris

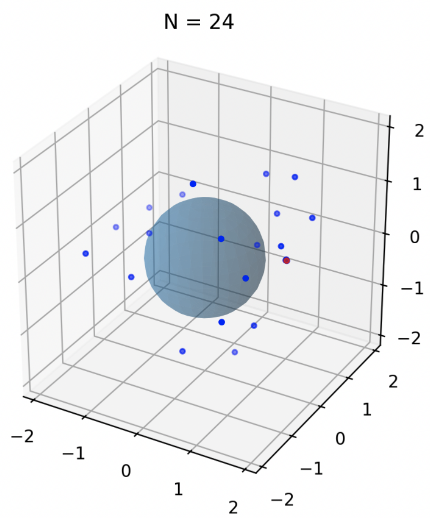
Now we complement the mean-field description with an explicit many-body simulation of the system in three-dimensional space.

Toy model: planet Thrae To keep this computationally manageable, we introduce a simplified planet *Thrae* with:

- smaller radius and orbital volume than Earth,
- fewer satellites / debris particles,
- identical satellite / debris size and collision properties.

The goal is to reduce the computational cost while preserving the physical phenomenon.

Simulation Your simulation should feature N debris particles. Their dynamics is governed by gravitational forces leading to elliptical orbits around Thrae with different altitudes and inclinations. Below is an example, how this *could* look like:



As a first step, ignore collisions to get the basic simulation running. Then include collisions. At each stage, compare you simulation to the respective mean-field model. Make some statistics of your simulation (e.g. plot the time-average of the number of debris particles or plot the number of collisions over the number of particles, ...).
You are free to choose the programming language for implementing the simulation. If you are unsure, using Python with Matplotlib for the simulation is recommended.
It is not only allowed, but encouraged to heavily rely on AI chatbots to implement a simulation.

Hint:

- When prompting the chatbot, try different approaches: You might just upload this project sheet and tell the chatbot to create the simulation, but you could also try a very detailed and structured prompt. Using a chatbot to construct a good prompt for itself is also a good idea. If you are unfamiliar with terminal coding agents, you could take this project as an opportunity to try them out, e.g. OpenCode is a free and open-source coding agent.
- Be careful with accidental collision cascades. After a collision, you might need to include a cooldown such that you do not get an exploding number of particles in a single position.
- Play around with the model parameters and the collision cross-sections.
- Try to improve the performance of the simulation and increase the scaling.

Minimum requirements and format

The project is deliberately open – you choose the format and the details of your project. Nevertheless, there are some minimum requirements:

- Hand in the project by the deadline (18.05.26, 12:00h) via email to `nevermann@itp.uni-frankfurt.de`
- Present your project in the tutorial session on 20.05.26 and be prepared to answer questions.
- Submit a discussion of the mean-field models and a simulation (with code), as described in the main text. You choose the format (e.g. written report, interactive HTML site, slides, ...).

References

[1] D. J. Kessler and B. G. Cour-Palais, "Collision Frequency of Artificial Satellites: The Creation of a Debris Belt", Journal of Geophysical Research, 83(A6):2637–2646, 1978.

[2] F. Moorhouse, "Space Debris and Launch Denial", Forethought, 2025, https://www.forethought.org/research/space-debris-and-launch-denial.