Exercise Sheet #11

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Problem 1 (*Bayes' rule*) 7 Pts

You want to escape from the bad weather and plan to do vacations in south-western Greece. Before preparing your luggage, you hear in the news that there might be an earthquake the day you arrive.

After doing some research, you realise that in the past few years there have been in average five earthquakes per year. Moreover, you find out that when there is an earthquake, the forecast service has predicted it 90% of the time. However, 10% of the alarms are wrong and there is no earthquake.

What is the probability that there will be an earthquake the day you arrive? Will you cancel your flight?

Problem 2 (*Bayesian inference*) 8 Pts

We are observing again a drunk person who has difficulties in walking only in one direction. Looking at him, in the beginning we believed that the probability of a step forwards (p) equals that of a step backwards ($1-p$). However, looking at him again, we see that after an initial step back, he has walked straight forward for the last 4 steps (that is, you have the evidence $D = \{\Delta x_0 = -1, \Delta x_1 = 1, \Delta x_2 = 1, \Delta x_3 = 1, \Delta x_4 = 1\}$, where Δx_i is the direction of each step).

We can still assume that each step has two possible outcomes, with a fixed probability p of forward and $1-p$ of backwards, but now with an unknown p value.

As a model of the world, we consider the $P(p)$ prior probability distribution of the parameter p. According to our expectations, this should be symmetric with respect to $\frac{1}{2}$.

(a) Compare the two different prior distributions $P_1(p) = \alpha p(1-p)$ and $P_2(p) = 1$:

   (i) Calculate the distributions $P_{1,2}(p|D)$ for the parameter p, given the assumed prior distributions $P_{1,2}(p)$ and the evidence D.

   (ii) Using the chain rule and the previous result, calculate the probability that the next step will be forward, i.e. $P(\Delta x_5 = 1|D)$.

(b) How can you enhance your prediction of Δx_{n+1} after you watch the person making one more step?

Problem 3 (*OR series with noise*) 5 Pts

Show that the statistical properties of a time series generated by an OR process in the presence of weak noise ($0 < \xi \ll 1$), i. e.:

$$\sigma_{n+1} = \begin{cases} \mathrm{OR}(\sigma_n, \sigma_{n-1}) & \text{with probability } 1-\xi \\ \neg\,\mathrm{OR}(\sigma_n, \sigma_{n-1}) & \text{with probability } \xi \end{cases}$$

do not depend on the initial conditions. Note that for a low level of noise, $\xi \to 0$, we have a discontinuity, i.e. the deterministic case is not recovered.