Exercise Sheet #4

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Problem 1 (*Preferential Attachment and Internal Growth*) 7 Pts

In networks such as the internet and social communities new connections are not only created when new nodes are added, but also between already existing nodes – a phenomenon called internal growth of a network. To model this, generalise the preferential attachment model in the following way:

(i) at each time step one new vertex and m new edges are added

(ii) with probability $\rho \in [0, 1]$ one of the new edges connects the new vertex and an existing vertex i, which is selected with probability $\Pi_i \propto k_i + C$

(iii) with probability $1-\rho$ one of the new edges connects the existing vertices j and l, which are selected with probability $\pi_{jl} \propto \Pi_j \Pi_l$

Show that the degree distribution p_k follows a power law,

$$p_k \propto k^{-\gamma},$$

with the exponent $\gamma = 1 + \frac{1}{1-\rho/2}$. What happens in the limit $\rho \to 1$?

Problem 2 (*Dynamical system*) 7 Pts

Consider the following two dimensional dynamical system in $(x, y) \in \mathbb{R}^2$:

$$\begin{aligned} \dot{x} &= x\,(a - 2x - \;\; y) \\ \dot{y} &= \quad\; a - \;\; x - 2y\ , \end{aligned}$$

with the real paramter a.

(a) Find all fixpoints of the system. (2 Pts)

(b) Linearise the system around the fixpoints in order to determine their stability and find the stable/unstable manifolds. (2 Pts)

(c) Sketch the flow of the system once for $a > 0$ and for $a < 0$. (2 Pts)

(d) What kind of bifurcation do you observe when a changes its sign? (1 Pts)

Problem 3 (*Driven harmonic oscillator – revised*) 6 Pts

Investigate the dynamics of the damped driven harmonic oscillator

$$\ddot{x}(t) = -\omega_o^2\, x(t) - \alpha\, \dot{x}(t) + f_o \cos\left(\Omega t + \varphi_o\right), \tag{1}$$

as a dynamical system, where ω_o is the natural frequency of the oscillator and α is the damping factor. The oscillator is driven by an external harmonic force with amplitude f_o, driving frequency Ω and initial phase φ_o.

Calculate the fixed point(s) of the system, examine their stability and make a sketch of the flow in the phase space for each of the following cases:

(i) weak damping without driving: $\alpha/2 < \omega_o, f_o = 0$ (1.5 Pts)

(ii) strong damping without driving: $\alpha/2 > \omega_o, f_o = 0$ (1.5 Pts)

(iii) negative damping, i. e. energy up-take, without driving: $\alpha < 0, f_o = 0$ (1.5 Pts)

(iv) very weak damping in presence of ext. force: $\alpha/2 \ll \omega_o, f_o \neq 0$ (1.5 Pts)

Compare these results to the ones obtained in the first problem session.