Complex and Adaptive Dynamical Systems SS 17
to hand in: At the first tutorial (2nd week) C.Gros

Exercise Sheet #1

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Problem 1 (*Random graphs 1*) (0 P.)

For the random graph given in Fig.1 evaluate:

(a) coordination number z,

(b) connection probability p,

(c) average distance l,

(d) and clustering coefficient C.

Problem 2 (*Random graphs 2*) (0 P.)

Use a random number generator (dice, coin, roulette wheel, software...) to generate five random graphs with $N = 5$ vertices and connection probability $p = 2/3$.

(a) Evaluate the degree distribution for each graph and the degree distribution of the ensemble average.

(b) Derive a formula for the probability that a node has k edges.

Compare the simulation results (a) with the prediction via (b).

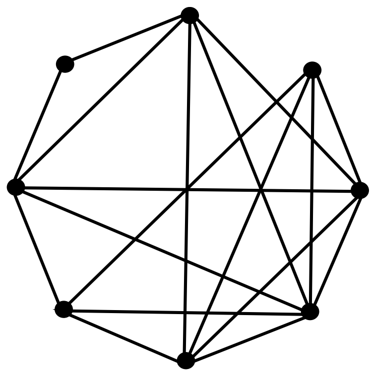

Figure 1: Random graph with N=8 vertices

Problem 3 (*Driven harmonic oscillator*) (0 P.)

Investigate the dynamics of the damped driven harmonic oscillator

$$\ddot{x}(t) = -\omega_0^2 x(t) - \alpha\dot{x}(t) + f_0 cos(\Omega t + \phi_0), \tag{1}$$

where ω_0 is the natural frequency of the oscillator and α is the damping factor.
The oscillator is driven by an external harmonic force with amplitude f_0, driving frequency Ω and initial phase ϕ_0.

Solve Eq.(1) for some initial conditions $x(t_0) = x_0, \dot{x}(t_0) = v_0$ analytically. Then sketch the system's long time evolution, i.e. $x(t)$ for $t \gg t_0$, for each of the following cases:

(a) weak damping without driving: $\frac{\alpha}{2} < \omega_0, f_0 = 0$

(b) strong damping without driving: $\frac{\alpha}{2} > \omega_0, f_0 = 0$

(c) very weak damping in presence of ext. force: $\frac{\alpha}{2} \ll \omega_0, f_0 \neq 0$

(d) very weak damping, i.e. energy up-take, with or without driving: $\alpha < 0$

Compare these four qualitatively different long-term dynamics!