**Problem 1**  \textit{(Generalised Liénard system)}  \hspace{1cm} 10 Pts

Consider the following prototype system:

\[
\ddot{x} = f(V(x)) \dot{x} - \nabla V(x),
\]

where \( f(V) \) is a generalised friction term, which depends explicitly on the mechanical potential function \( V(x) \).

\begin{enumerate}[a)]
\item Analyse the stability of the fixpoints, considering the \( V(x) = x^3/3 - x^2/2 \) potential function and \( f(V) = \mu - V \) as the friction term.
\item Defining the systems total energy as \( E = x^2/2 + V(x) \), show that the energy uptake/dissipation can be controlled by the \( f(V) \) friction term.
\item Show that the fixpoints of (1), corresponding to local minima of any general potential function \( V(x) \) undergo a Hopf bifurcation when dissipation changes to antidissipation in their neighborhood.  
\textit{Hint:} The real parts of the eigenvalues should change their sign.
\end{enumerate}

**Problem 2**  \textit{(Van der Pol Oscillator)}  \hspace{1cm} 10 Pts

Consider the Van der Pol oscillator, governed by the equation

\[
\begin{align*}
\dot{x} &= y, \\
\dot{y} &= \varepsilon \left(1 - x^2\right) y - x.
\end{align*}
\]

Estimate the period of the limit cycle in the limit \( \varepsilon \gg 1 \).
To do this, follow the analysis made in the lecture using the Liénard variables. Then, use the identity for the length \( T \) of a limit cycle

\[
T = \int_0^T dt
\]

and approximate the differential \( dt \) by only taking into account the movement along the slow branches.  
\textit{Hint:} Change \( dt \) to \( dx \) and integrate over \( x \). You need to find suitable limits for this.