Problem 1  \textit{(Jacobian and Lyapunov exponents)}  \hspace{1cm} 10 \text{Pts}

For the dynamical system defined by the equations:

\[
\frac{dx}{dt} = \alpha x + \beta y \\
\frac{dy}{dt} = \alpha y - \beta x
\]

1. Calculate the Jacobian matrix and deduce from it the Lyapunov exponents of the fixpoint.

2. Draw a trajectory map of the system for positive/negative values of $\alpha$ and $\beta$ (all four combinations). What is the connection between the values of the exponents and the behaviour of this system close to the fixpoint?

Problem 2  \textit{(Bifurcation potentials)}  \hspace{1cm} 10 \text{Pts}

A dynamical system is defined by:

\[
\frac{dx}{dt} = -\frac{d}{dx}U(x)
\]

With the potential:

\[
U(x) = -\frac{a}{2}x^2 - \frac{b}{3}x^3 + \frac{1}{4}x^4
\]

1. Draw the system’s phase diagram with respect to $a$ (assume $b$ is constant). For each fixpoint in the diagram, determine whether it is stable or unstable.

2. Identify the bifurcations in the phase diagram and determine their type.