Exercise Sheet #5

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Problem 1 (*Jacobian and Lyapunov exponents*) 10 Pts

For the dynamical system defined by the equations:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha x + \beta y$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha y - \beta x$$

1. Calculate the Jacobian matrix and deduce from it the Lyapunov exponents of the fixpoint.

2. Draw a trajectory map of the system for positive/negative values of α and β (all four combinations). What is the connection between the values of the exponents and the behaviour of this system close to the fixpoint?

Problem 2 (*Bifurcation potentials*) 10 Pts

A dynamical system is defined by:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}x}U(x)$$

With the potential:

$$U(x) = -\frac{a}{2}x^2 - \frac{b}{3}x^3 + \frac{1}{4}x^4$$

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1. Draw the system's phase diagram with respect to a (assume b is constant). For each fixpoint in the diagram, determine whether it is stable or unstable.

2. Identify the bifurcations in the phase diagram and determine their type.