Exercise Sheet #10

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Problem 1 (*Maximal Entropy Distribution Function*) 7 Pts

Determine the probability density $p(x)$ that, for a given average value μ and variance σ^2, maximizes its differential entropy.

Problem 2 (*Time Series with Noise*) 7 Pts

Consider the time series generated by a logical OR ($\vee$) operation,

$$\sigma_{t+2} = (\sigma_{t+1} \vee \sigma_t) \ ,$$

where $\sigma = [[0,1]]$ (please notice that the exclusive XOR and inclusive OR operations are different). Evaluate the probability $p(1)$ of occurrence of the value 1 given all possible initial conditions are equally probably. Then, calculate this probability in the presence of noise, i.e. include a probability w of the bit to be randomly flipped. Validate your analytical results with a numerical calculation.

Problem 3 (*Weighing Problem*) 6 Pts

Consider the following Situation: You are given twelve balls that all have the same weight except one, which is slightly heavier. The difference in weight is not noticeable by holding them in your hands, but you have a weighing pan that can tell you if items placed on either side of the device have the same weight or, if not, which side is heavier. Your task is to find the a strategy that will require the least number of weighing procedures. Note that all balls are equally likely to be the heavier one.

(a) Think of this problem as a process of information transmission. Doing so, using Shannon's source coding theorem, determine the number of weighings necessary (you don't need to state the strategy explicitly at this point!).

(b) Find an optimal strategy and draw a decision tree of this process. For each decision step, calculate the amount of information that you gained.