Opinion Dynamics

Self-Organization (summer-term 2014)

July 21, 2014

Introduction

- Consider a group of interacting agents among whom some process of opinion formation takes place
- Example: Commission of experts working for UNO is requested to estimate world population in 25 years
  - Work out own estimate
  - Meet and discuss
  - Withdraw and repeat until either consensus is achieved or it is foreseeable that none will be achieved

Modeling opinions dynamics

- Typically linear models used
- Agent takes opinions of others into account to certain extent
- Can be modeled by different weights which agent puts on opinions of other agents
- Repeat process of 'averaging' $\rightarrow$ dynamical process
- Here we consider two approaches:

  Probabilistic: choose each step two agents to interact (Deffuant-Weisbuch/ DW)
  Deterministic: all agents interact in each step (Hegselmann-Krause/ HK)
- Simple models, extend them to investigate certain subjects

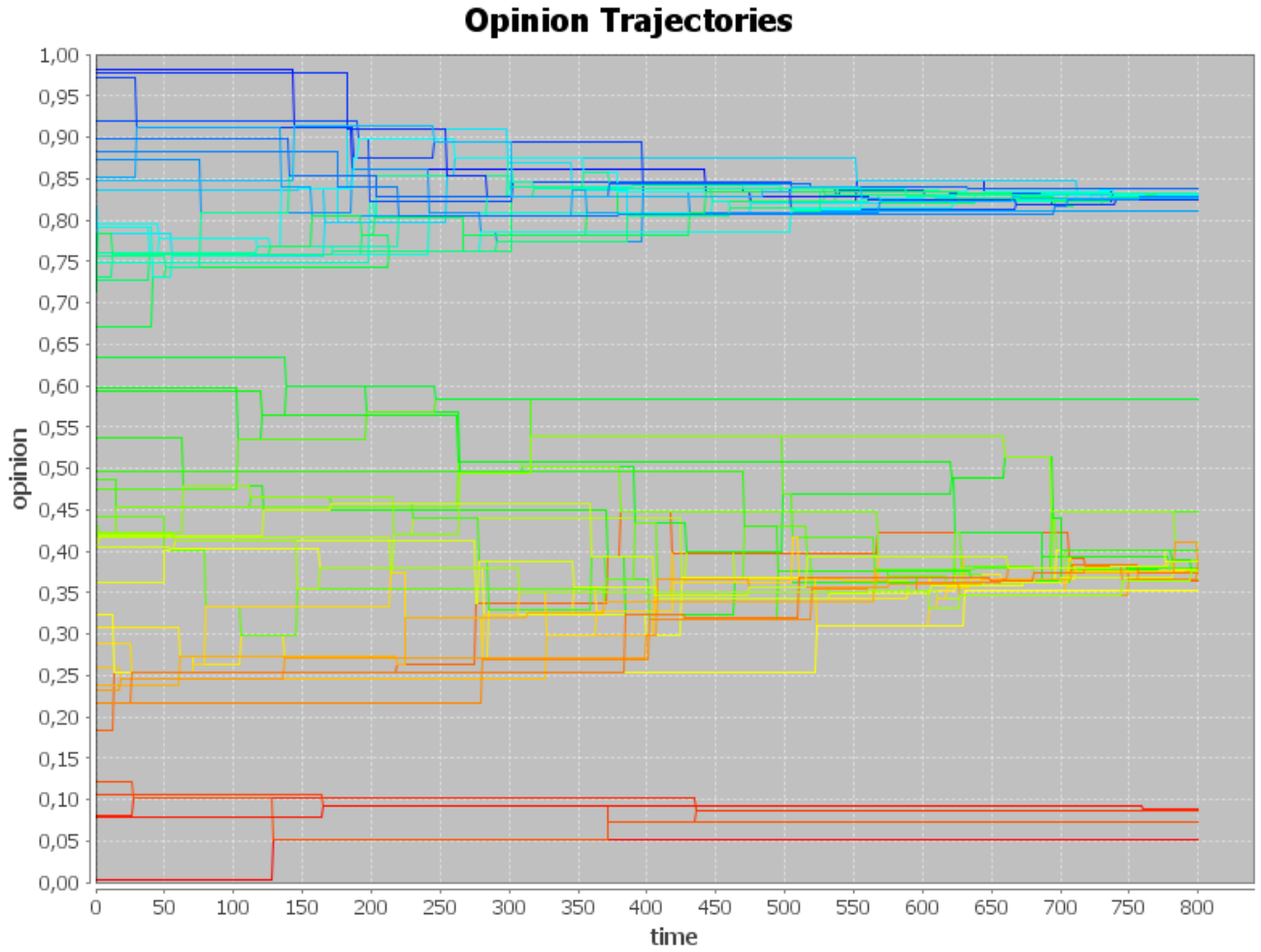
- n: number of agents
- $S = [0, 1]$: opinion space $\rightarrow$ continuous opinion dynamics
- $x(t) = (x_i(t))_{1 \leq i \leq n} \in S^n$: opinion profile
- given initial opinion profile $x(0)$ dynamics is defined by $x(t+1) = f(t, x(t))$
- Consider only agents whose opinions differ not more than a certain confidence level $\epsilon \rightarrow$ model with bounded confidence

  otherwise agents do not even discuss: lack of understanding, conflicts of interest or social pressure

Deffuant-Weisbuch model

- Choose pair of agents (i, j) at random
- $x_i(t+1) = \begin{cases} x_i(t) + \mu(x_j(t) - x_i(t)), & \text{if } |x_j(t) - x_i(t)| < \epsilon \\ x_i(t), & \text{otherwise} \end{cases}$
- Same for $i \leftrightarrow j$
- μ is only a convergence parameter $\rightarrow$ choose $\mu = \frac{1}{2}$
- ϵ constant for simplicity, in general: $\epsilon = \epsilon(x_i(t), x_j(t), t)$
- Average opinion conserved during dynamics in homogeneous case ($\epsilon = \text{const.}$)
- Consider example with $n = 20$, $n = 0.15$

- $n = 20$, $\epsilon = 0.15$



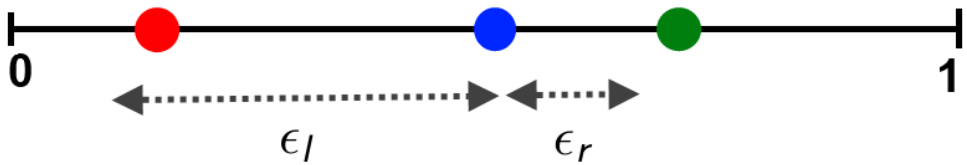
- Process always converges to a limit opinion profile
- Density of limit profile: $\rho_\infty(x) = \sum_{\alpha=1}^{K} m_\alpha \delta(x - x_\alpha)$ with $\sum_{\alpha=1}^{r} m_i = 1$ and $K \ll n$
- Minimum distance between peaks $= 2\epsilon$ in homogeneous case
- $\sum_{\alpha=1}^{r} m_\alpha x_\alpha$ equals conserved mean opinion and all cluster fulfill $|x_\alpha - x_\beta| > \epsilon$ $(\alpha \neq \beta)$ in homogeneous case

Hegselmann-Krause model

- Fix opinion profile $x(t)$ and agent i
- $I(i, x(t)) = \{1 \leq i \leq n : |x_j(t) - x_i(t)| \leq \epsilon\}$: set of interacting agents
- simple model: equal weights on all $j \in I(i, x(t))$
- $x_i(t+1) = \frac{1}{|I(i,x(t))|} \sum_{j \in I(i,x(t))} x_j(t)$
- Generalize to asymmetric confidence intervals $[-\epsilon_l, \epsilon_r]$

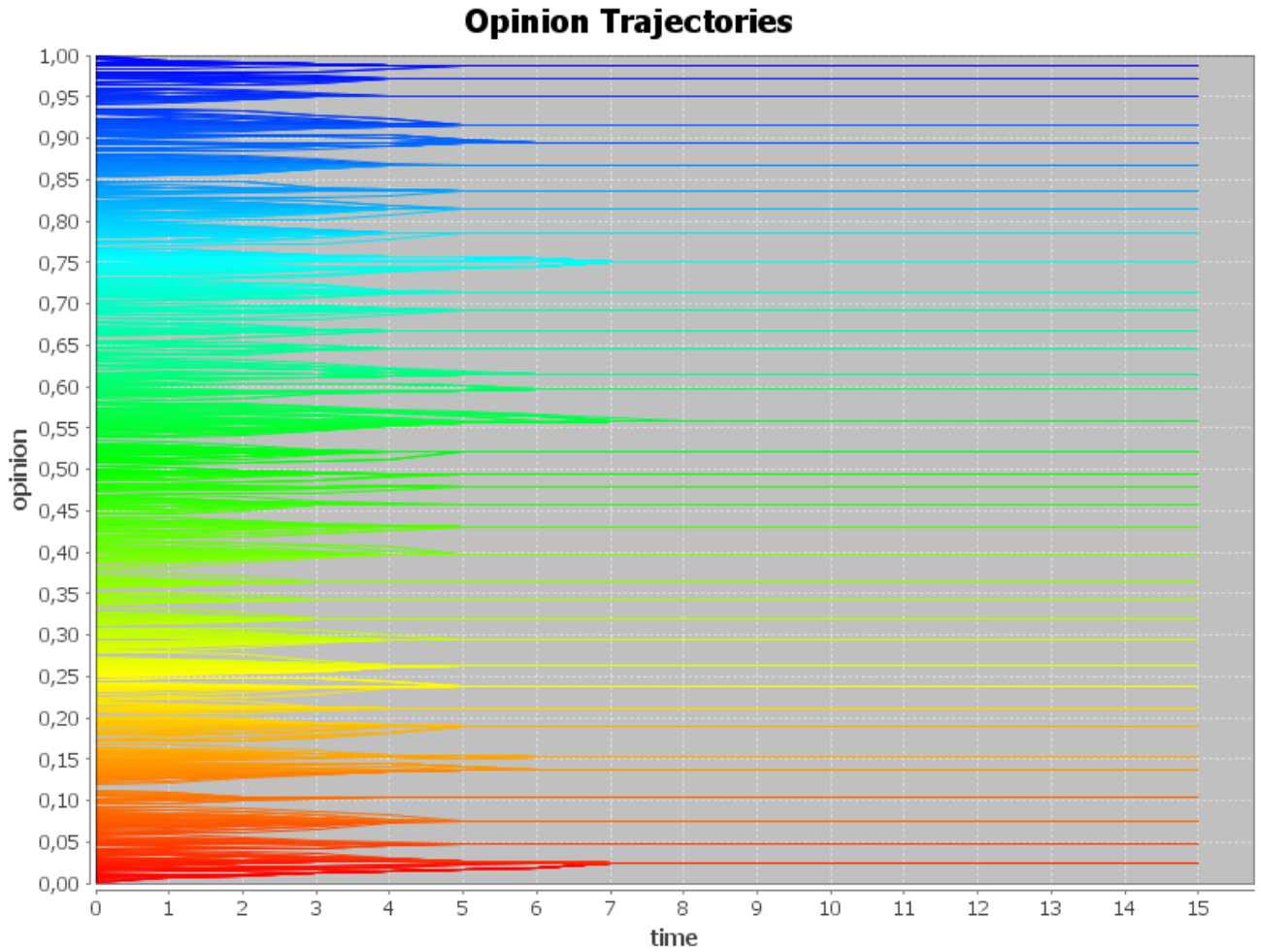
$$I(i, x(t)) = \{1 \leq i \leq n : -\epsilon_l \leq x_j - x_i \leq \epsilon_r\}$$

- $\epsilon_l > \epsilon_r$: agent has more confidence to opinions which are more left than his own

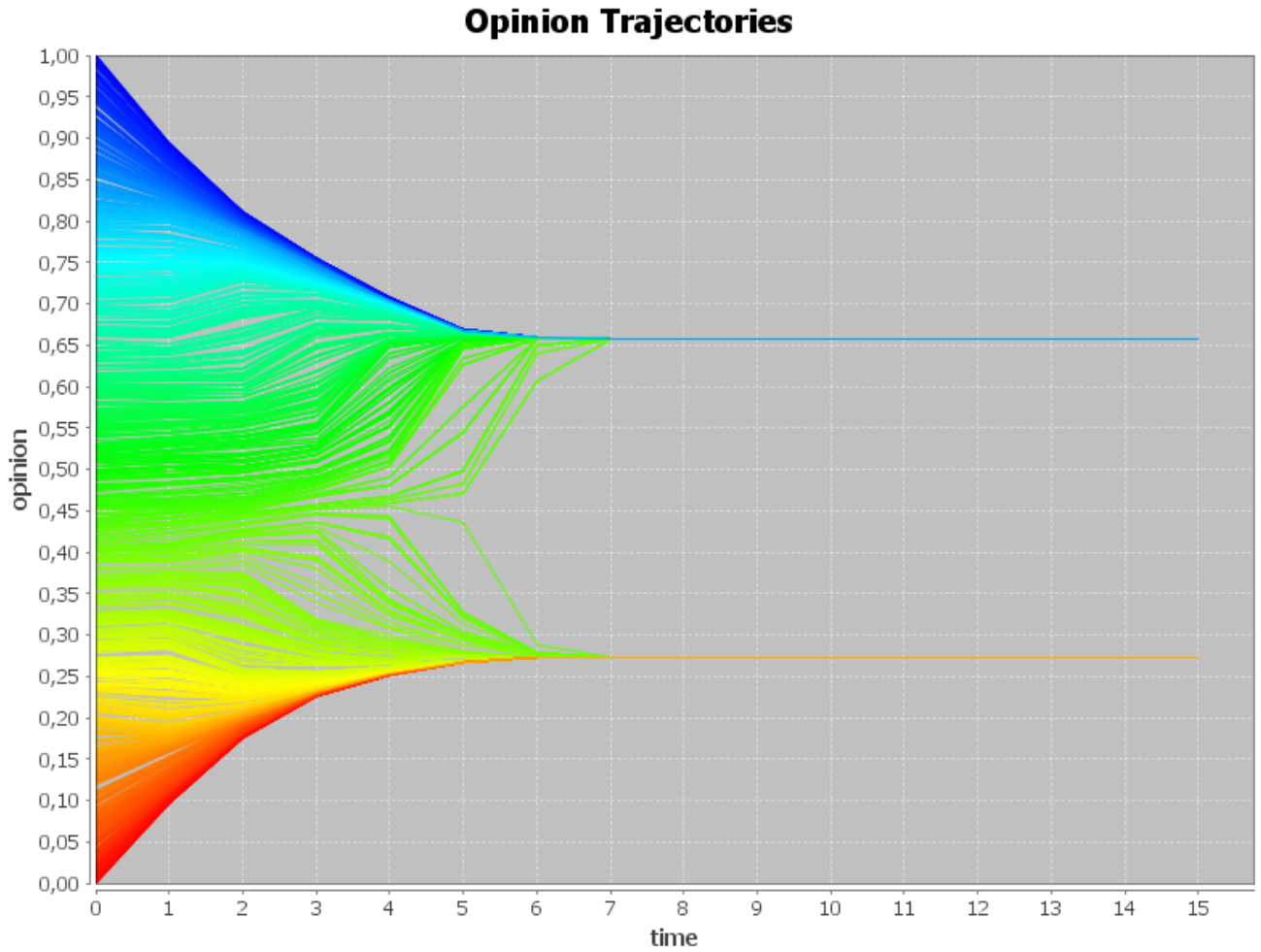


- First consider symmetric confidence intervals, i.e. $\epsilon_l = \epsilon_r$
- Generate 1000 opinions at random and use this profile for different values of $\epsilon_l = \epsilon_r$

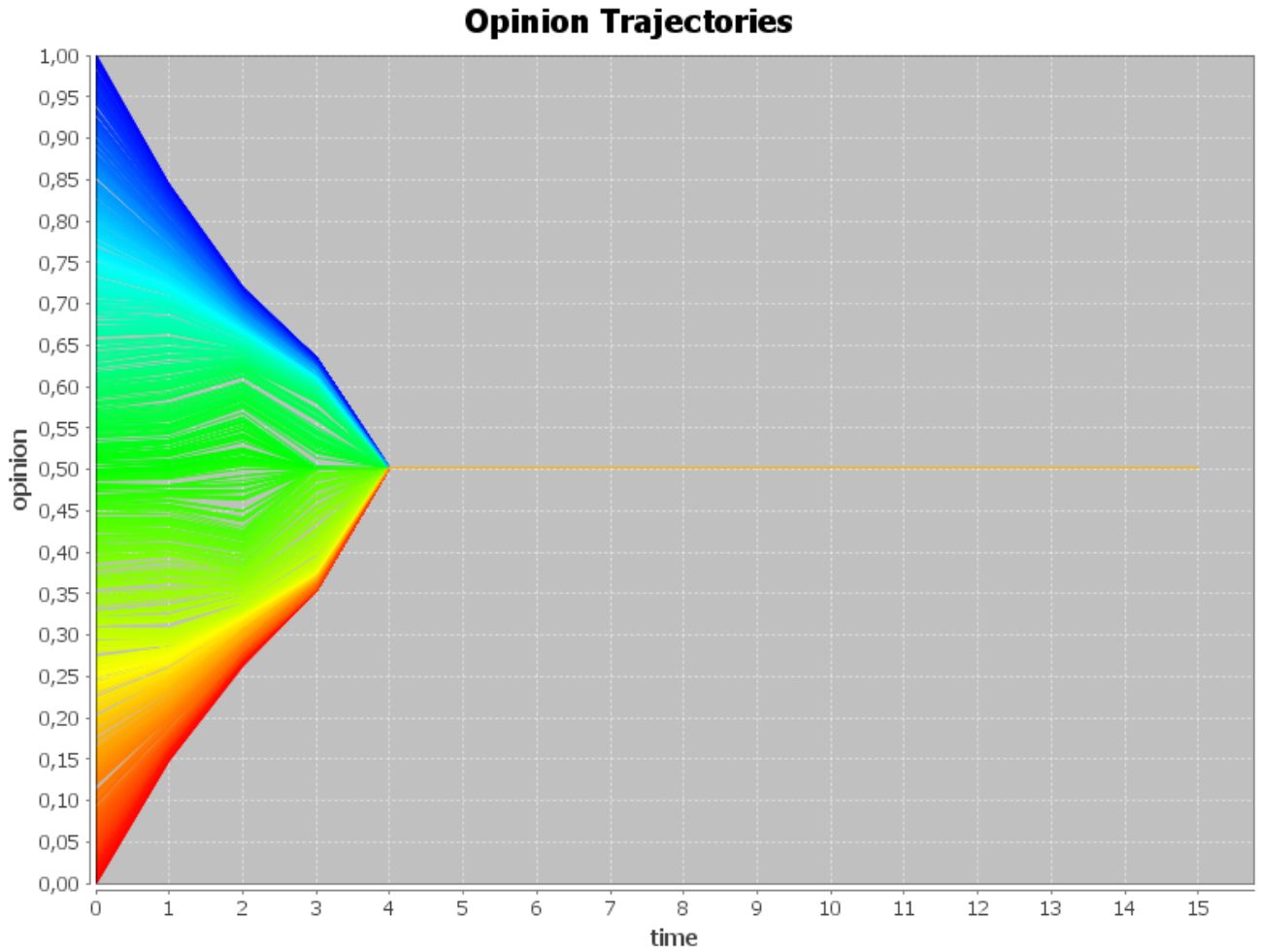
- small $\epsilon_l = \epsilon_r = 0.01$: exactly 37 different opinions survive



- $\epsilon_l = \epsilon_r = 0.2$: agents end up in two camps

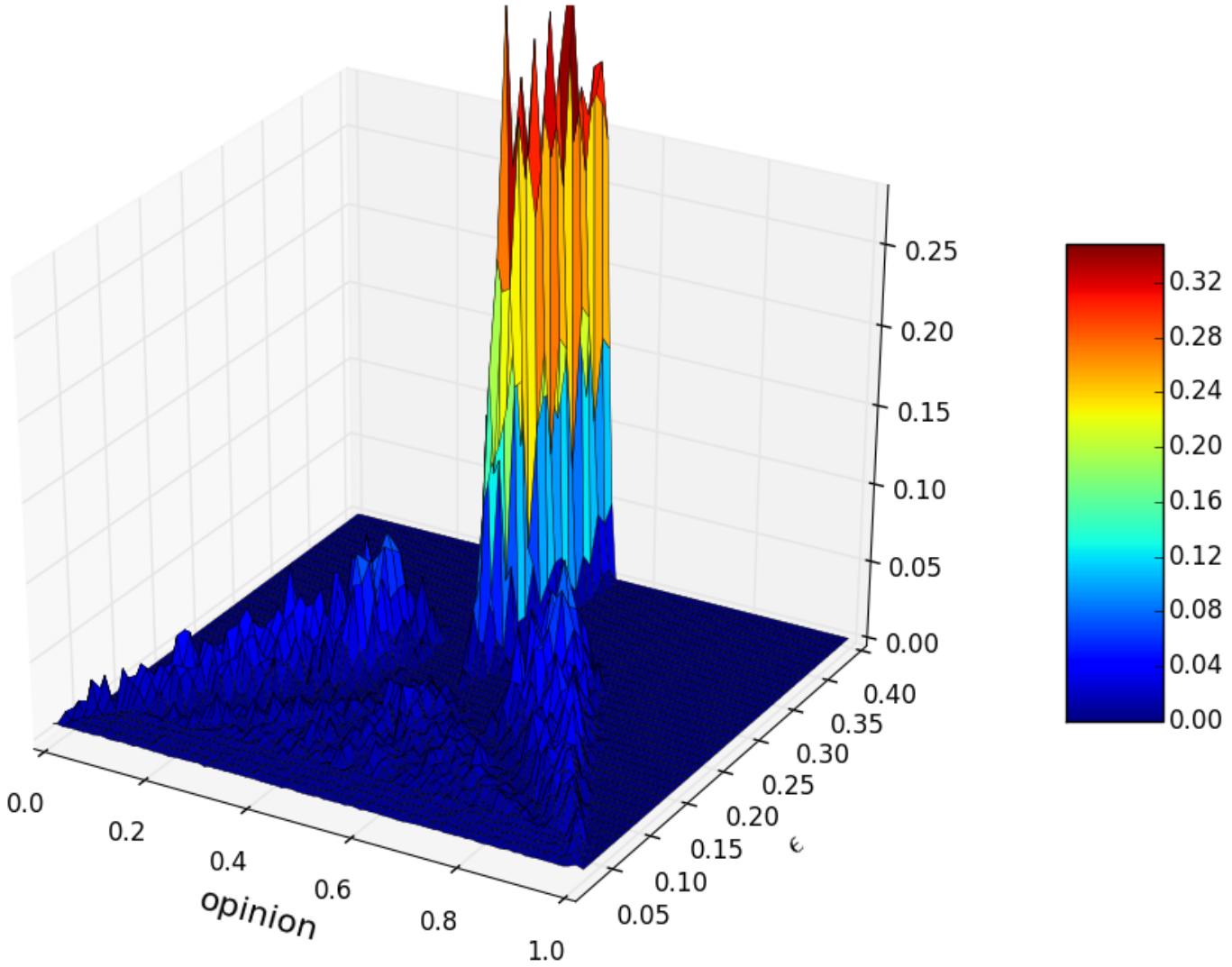


- $\epsilon_l = \epsilon_r = 0.3$: agents reach consensus



- Obviously fast convergence: less than 15 time steps for stable pattern
- Size of confidence interval matters
- Split sub-profiles do no longer interact
- Again convergence to δ-distributions
- Opinion trajectories never cross
- Extreme opinions under a one sided influence $\rightarrow$ range of the profile shrinks
- At the extremes opinions condense

- Average properties of limiting opinion profiles
- Begin with $\epsilon_l = \epsilon_r = 0.01$, then $\epsilon_l = \epsilon_r = 0.02$, ...
- In each ϵ-step:
  - Generate 1000 opinions at random
  - Simulate until convergence
  - Repeat 100 times
- Divide opinion space in 100 intervals and calculate average densities


0.25
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0.05
ε
0.0
0.2
0.4
0.6
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opinion
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0.20
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0.12
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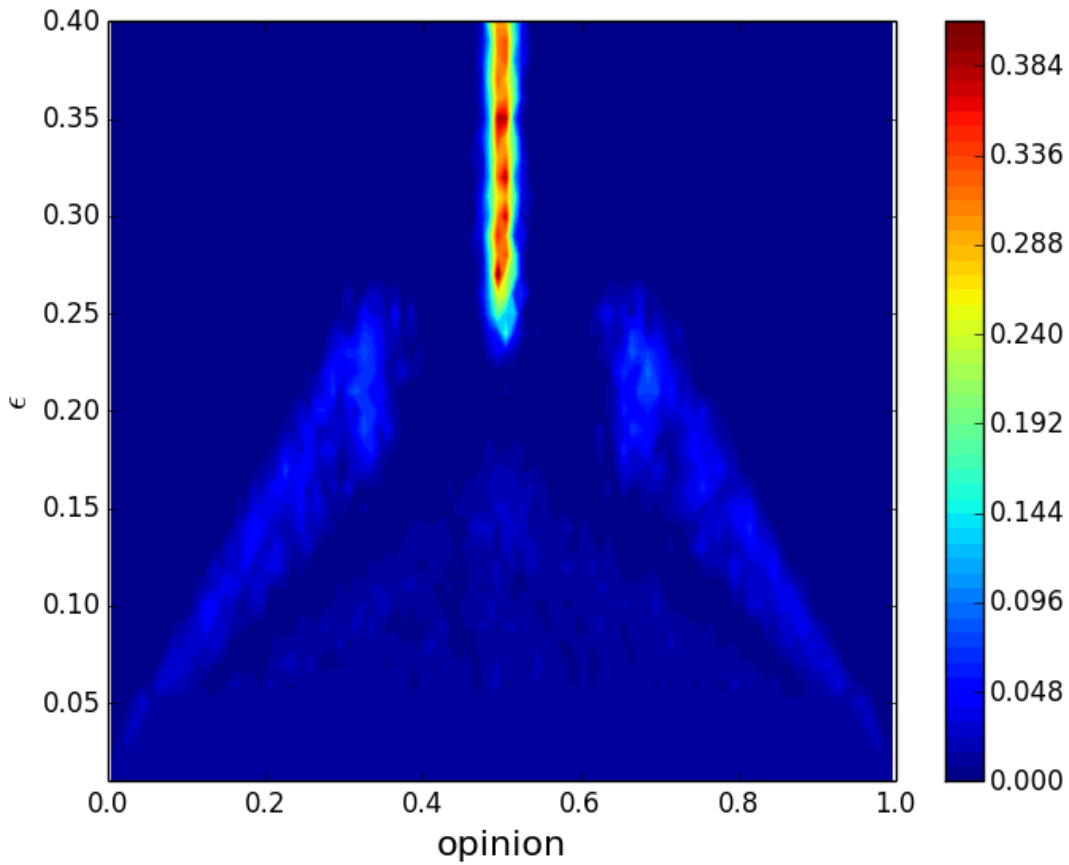
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opinion
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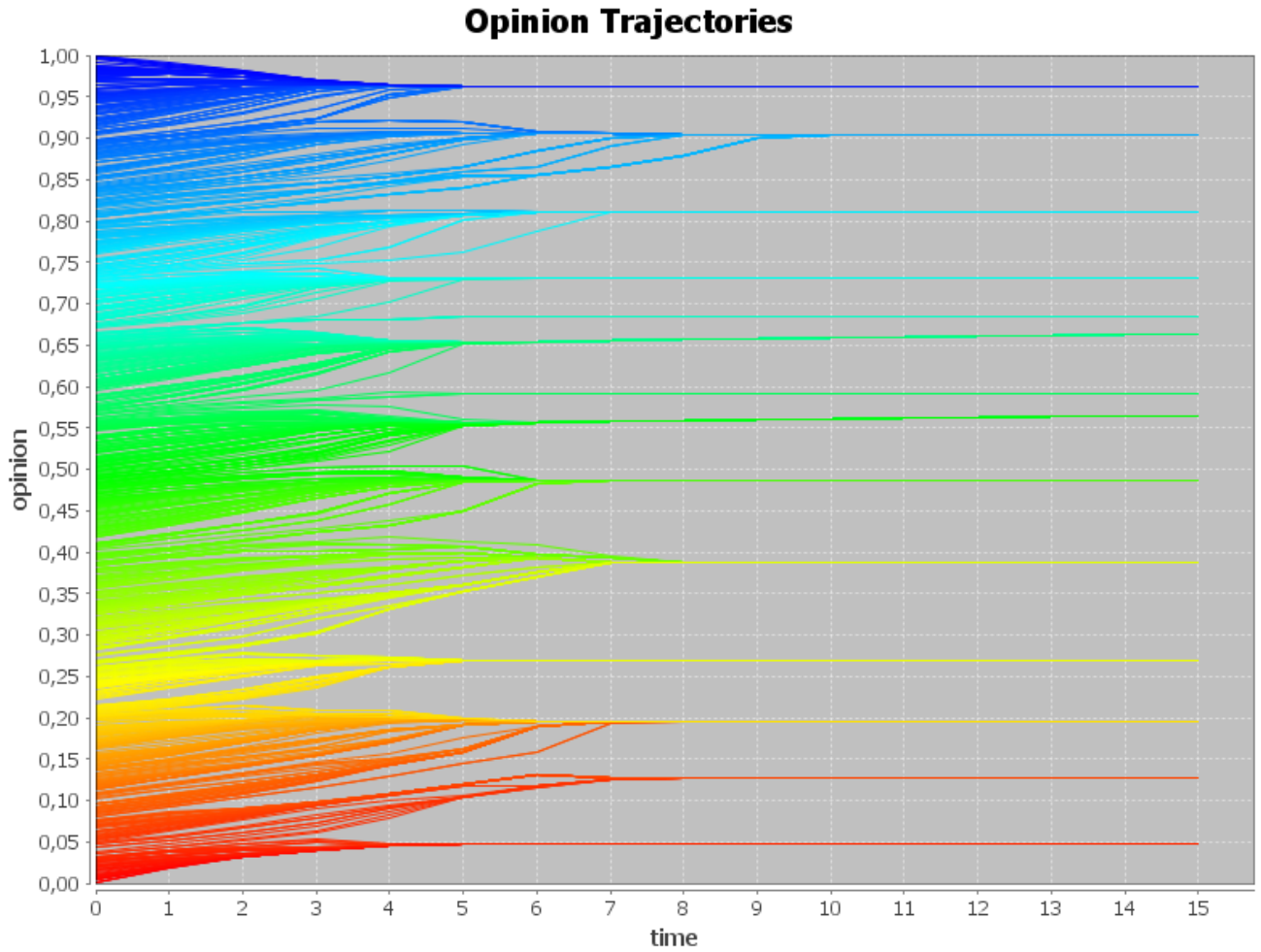
0.096
0.048
0.000

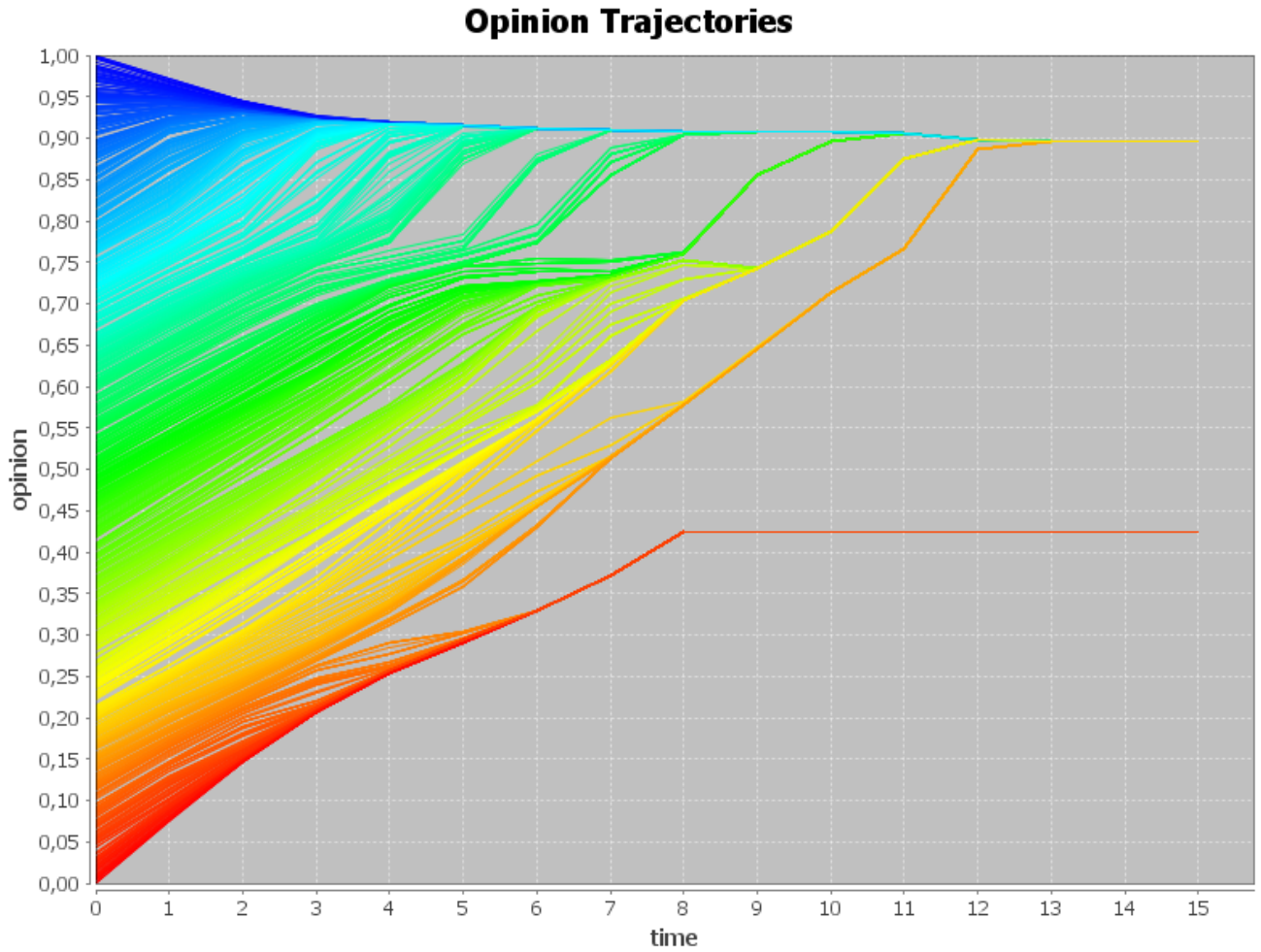
- Previous examples are very typical
- Under little confidence small fraction of opinions in any interval
- To left and right of center mountains are build
- Sudden end at $\epsilon_l = \epsilon_r = 0.25$: new and steep center mountain emerges
- From **fragmentation** (plurality) over **polarization** (polarity) to **consensus** (conformity)

- Now consider asymmetric case. Here: opinion-independent
- Generate 1000 opinions at random and use this profile for different values of $\epsilon_l \neq \epsilon_r$

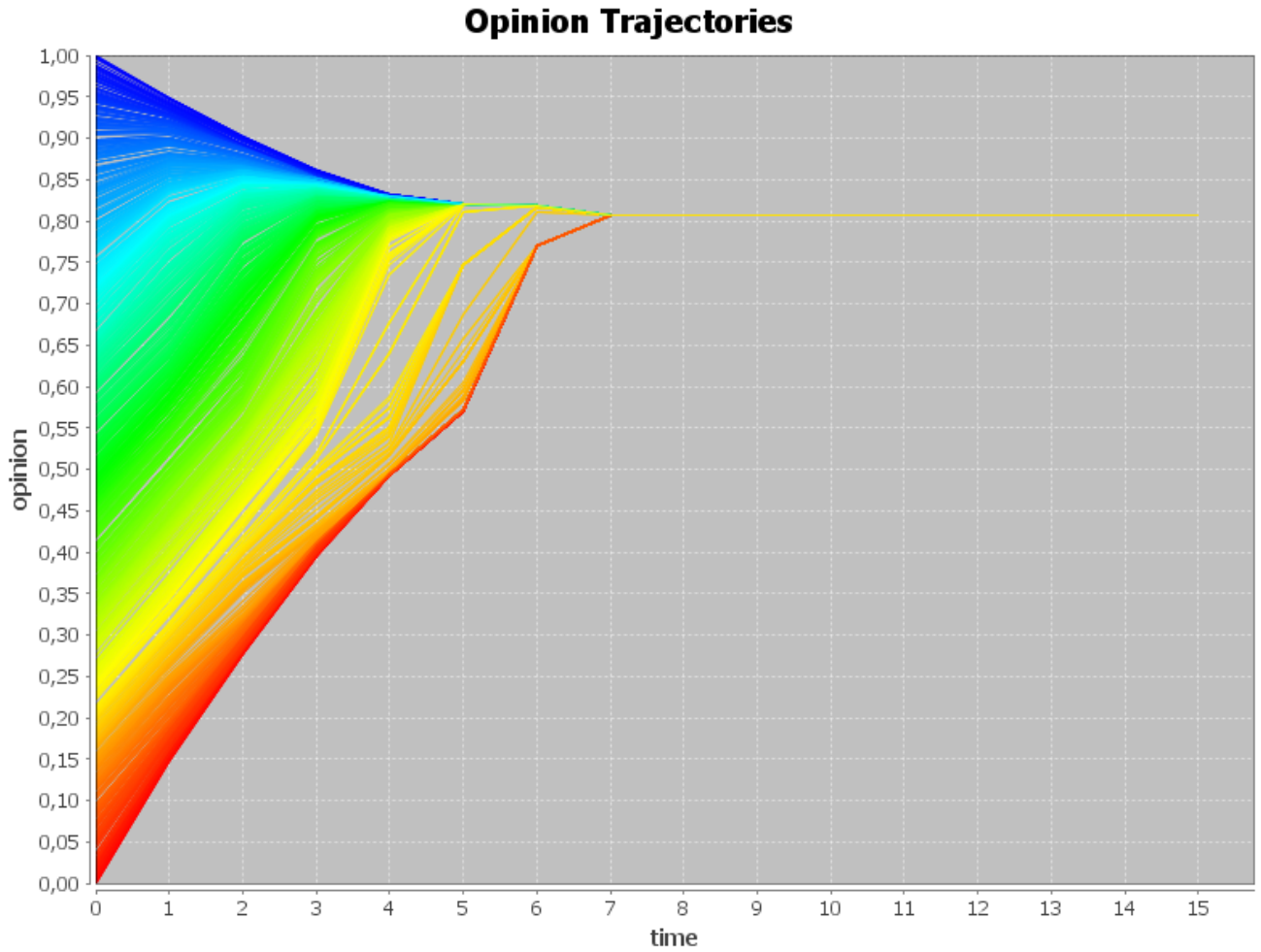
- $\epsilon_l = 0.02, \epsilon_r = 0.04$



- $\epsilon_l = 0.05, \epsilon_r = 0.15$



- $\epsilon_l = 0.10, \epsilon_r = 0.30$



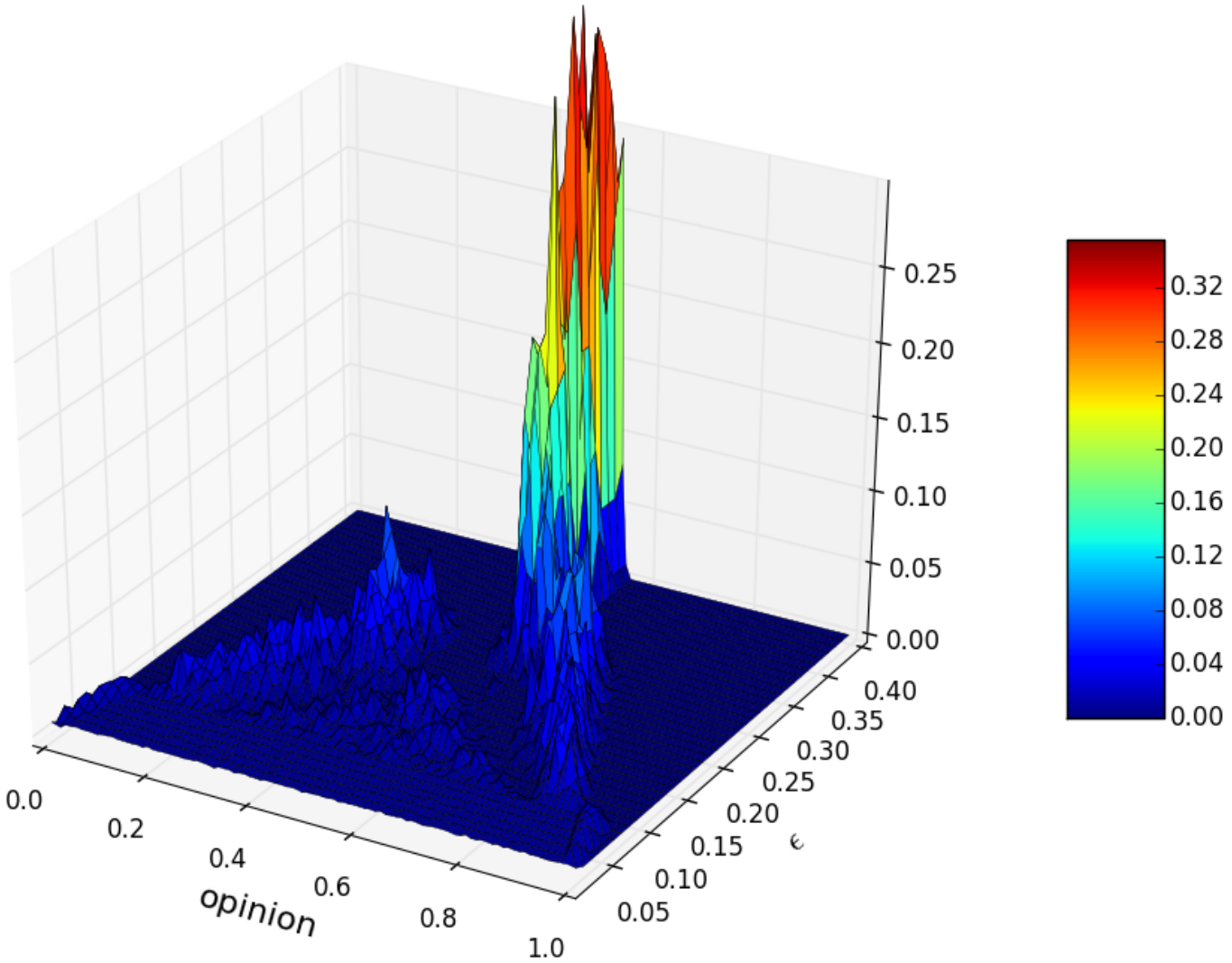
- Similar to previous results
- Dynamics somehow driven into favored direction
- Now systematic walk through parameter space [1]

asymmetric_walk.png

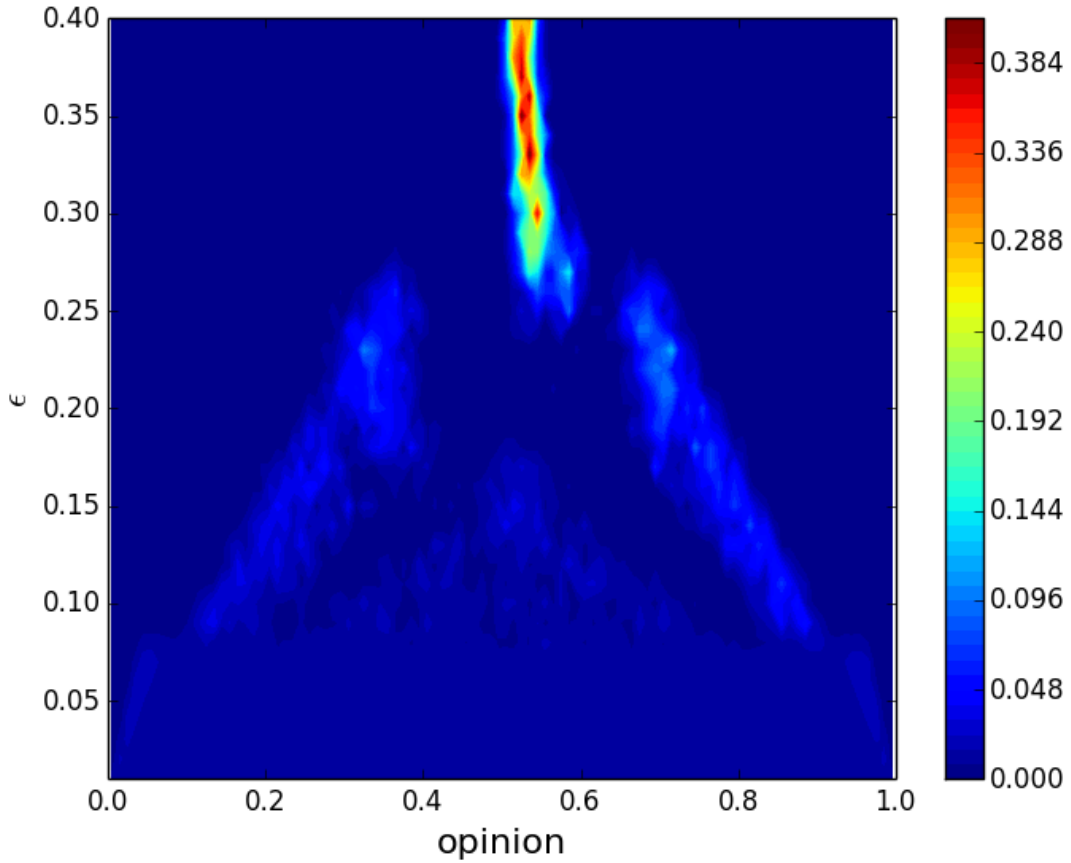
- Again, in each ϵ-step:
  - Generate 1000 opinions at random
  - Simulate until convergence
  - Repeat 100 times

[1] Figure from: Rainer Hegselmann and Ulrich Krause. Opinion Dynamics and Bounded Confidence, Models, Analysis and Simulation. Journal of Artificial Societies and Social Simulation, 5(3):2, 2002.

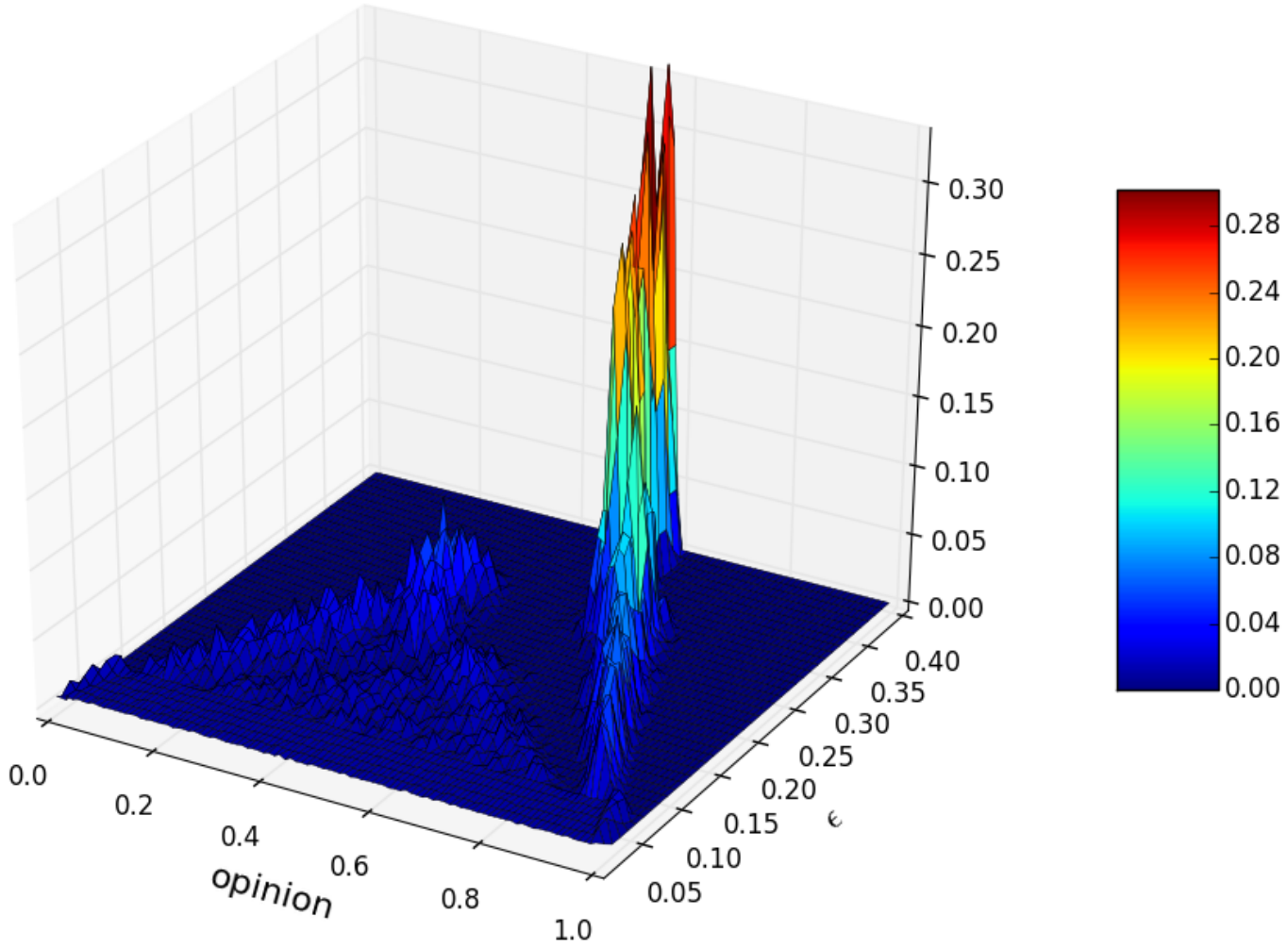
- $\epsilon_l = 0.9\,\epsilon_r$



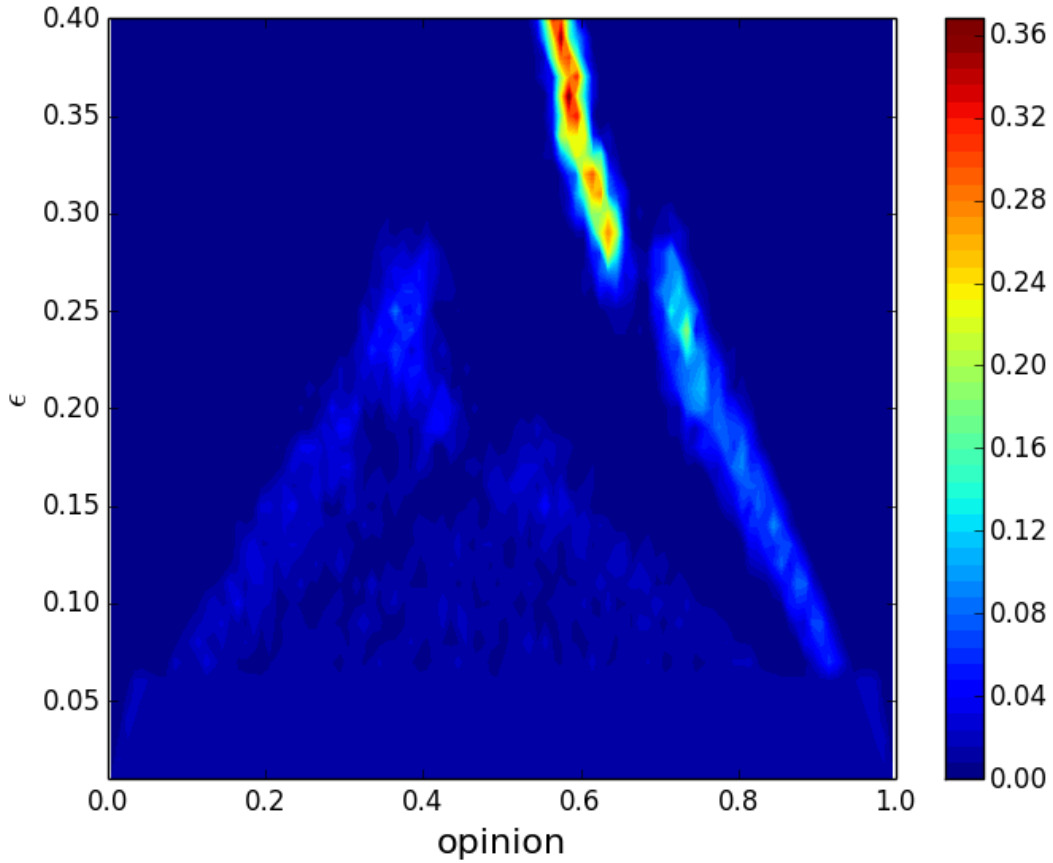
- $\epsilon_l = 0.9\,\epsilon_r$



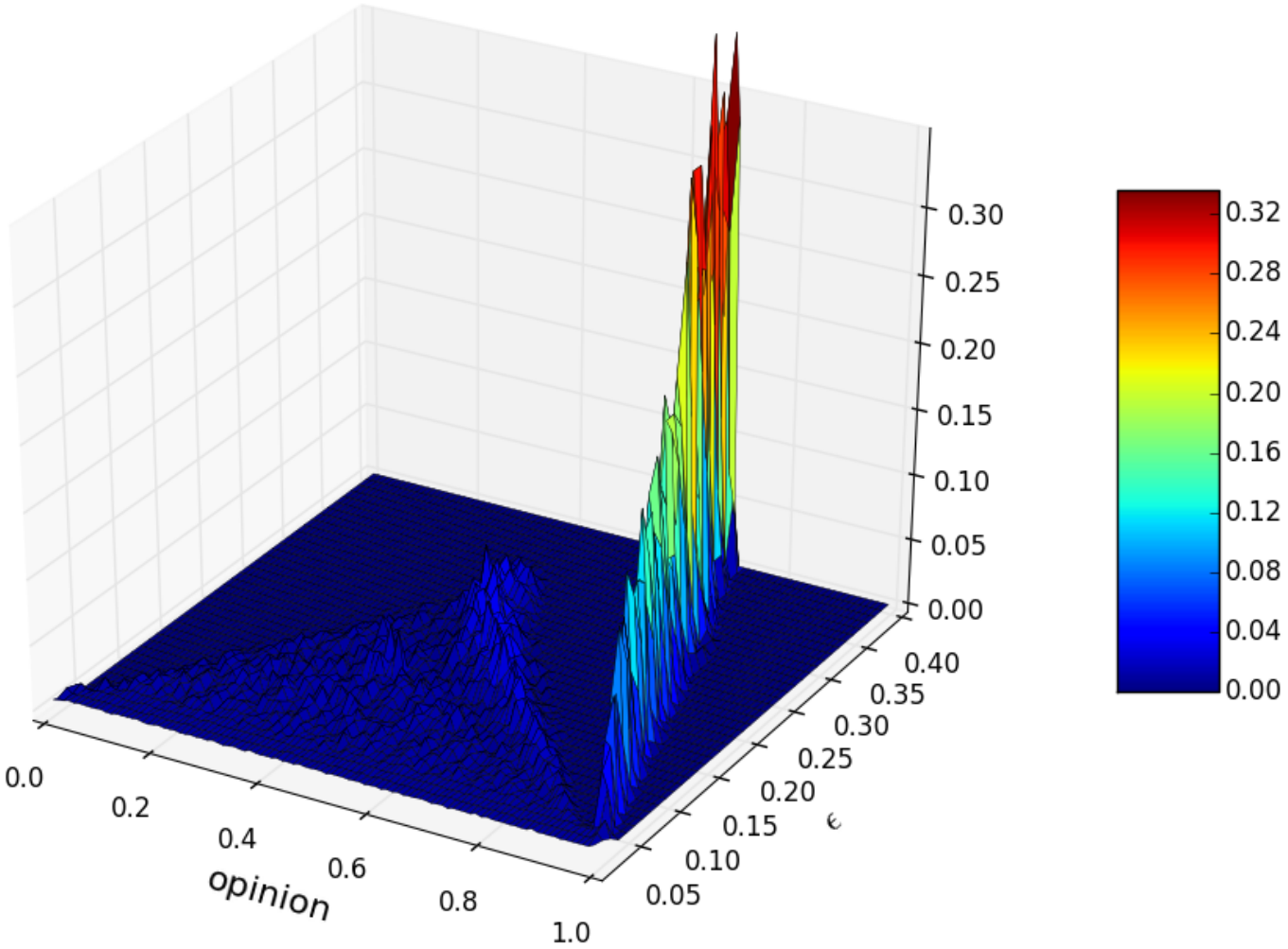
- $\epsilon_l = 0.75\,\epsilon_r$



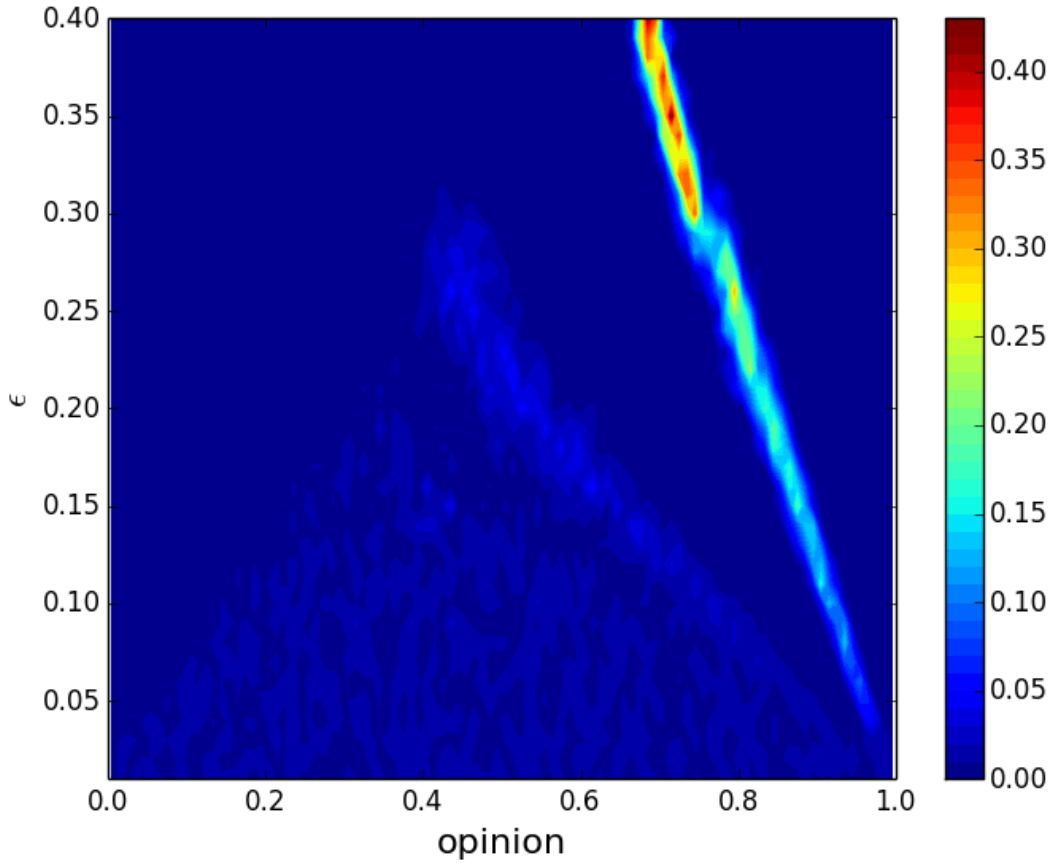
- $\epsilon_l = 0.75\,\epsilon_r$



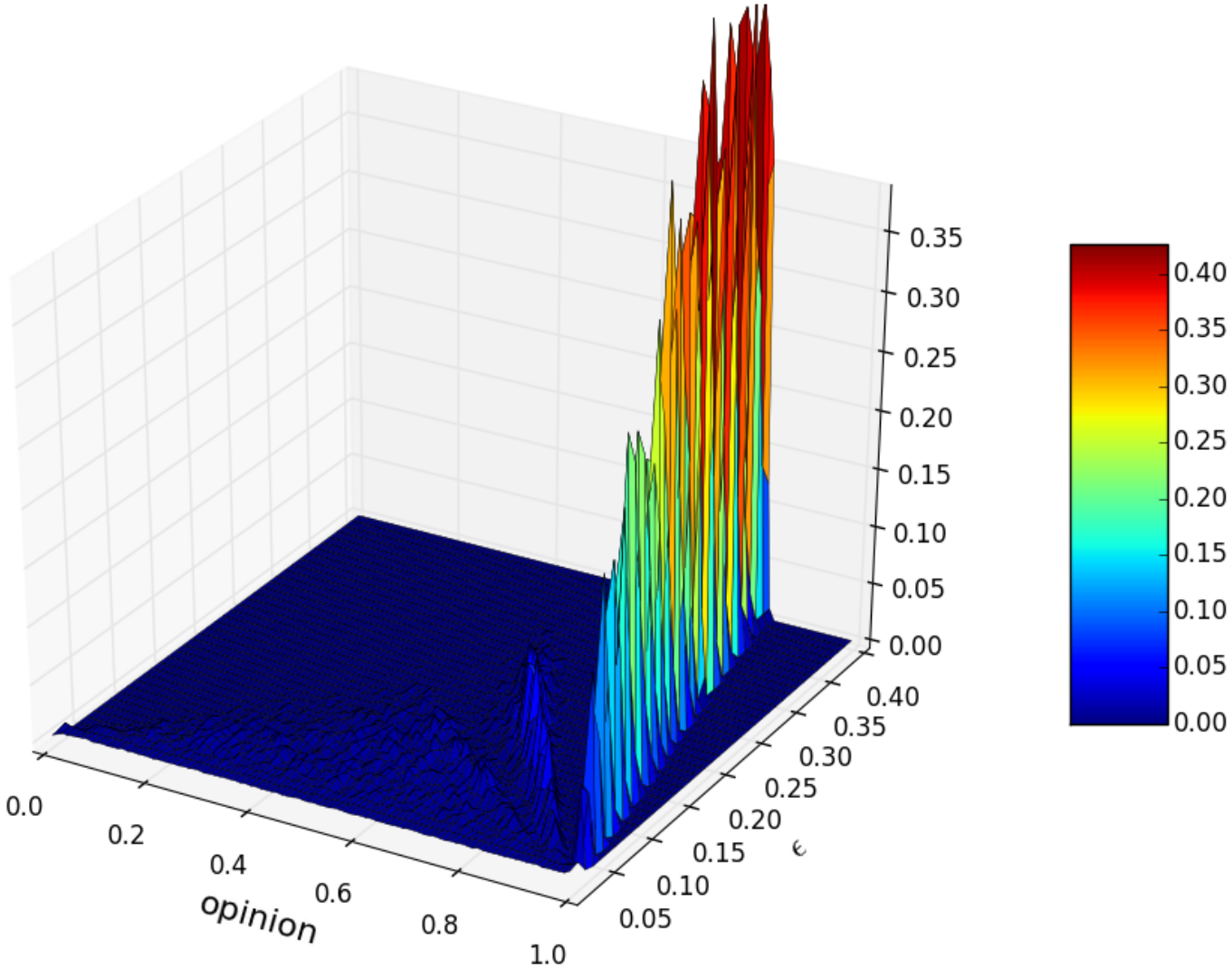
- $\epsilon_l = 0.5\,\epsilon_r$



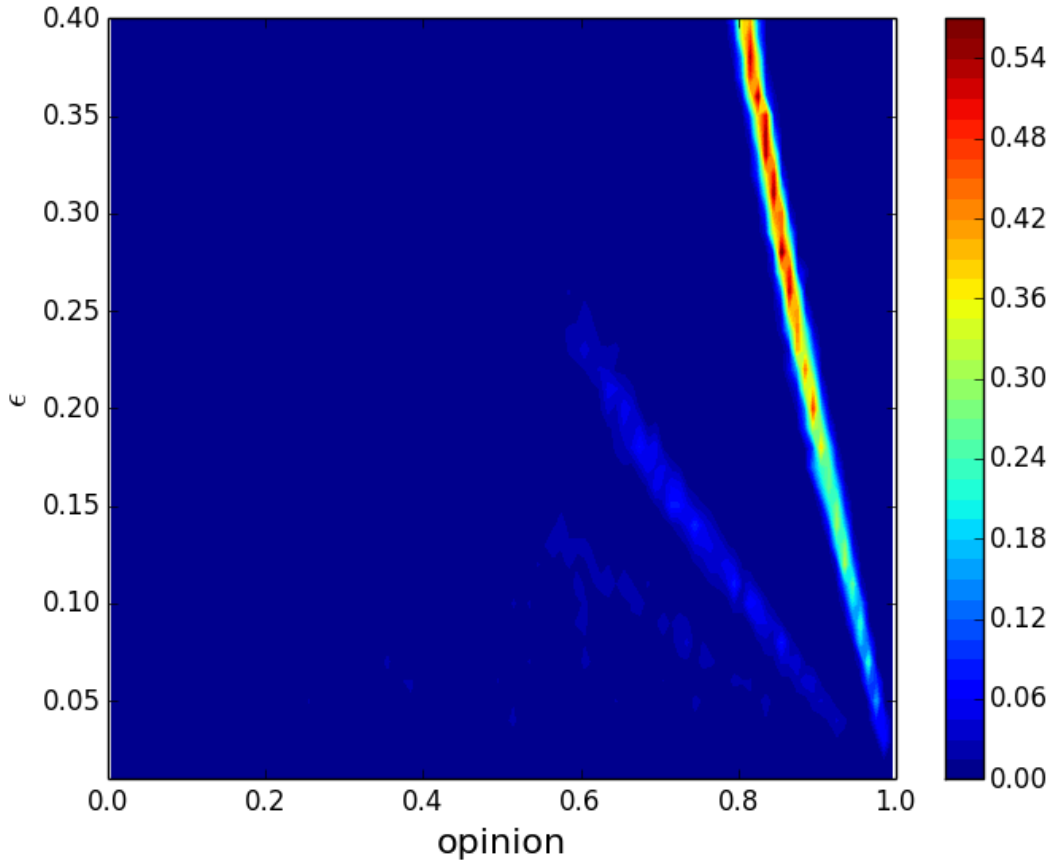
- $\epsilon_l = 0.5\,\epsilon_r$



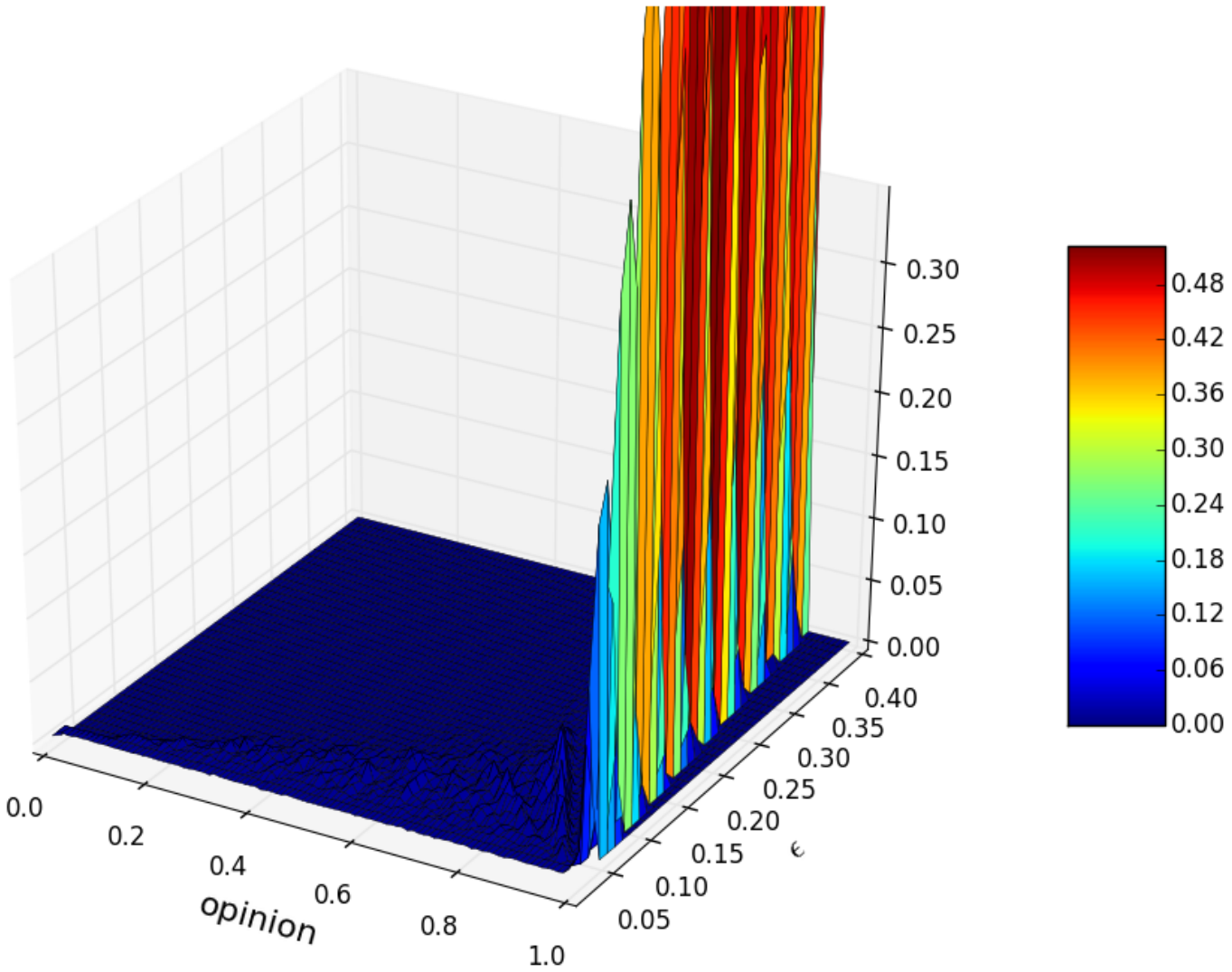
- $\epsilon_l = 0.25\,\epsilon_r$



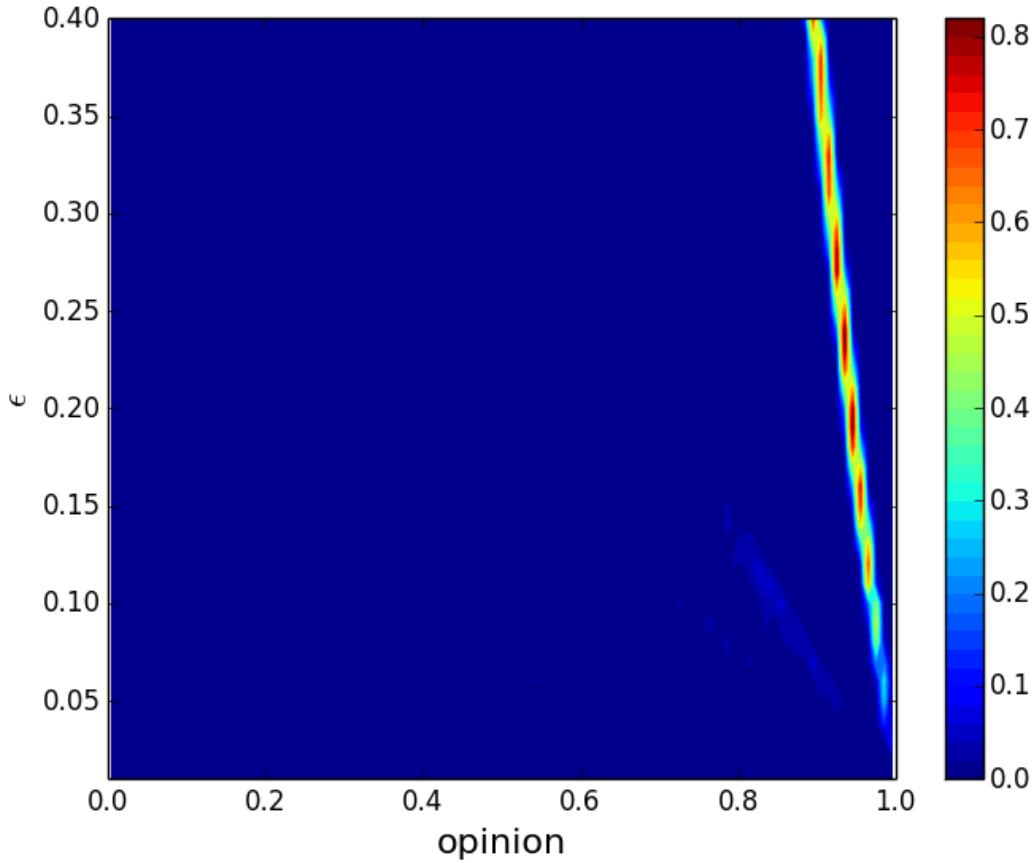
- $\epsilon_l = 0.25\,\epsilon_r$



- $\epsilon_l = 0.1\,\epsilon_r$

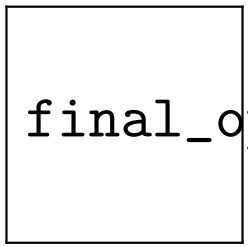


- $\epsilon_l = 0.1\,\epsilon_r$



- As ϵ_r increases all walks lead to region where consensus is achieved
- Consensus moves into favored direction
- ϵ_r and ϵ_l close: 'symmetric' polarization, nearly same size and same distance of the two camps from center of opinion space
- ϵ_r significantly greater than ϵ_l: left camp vanishes; new camp emerges at right border, but left from the main camp

- How many opinions survive?



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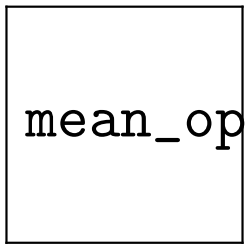
[2] Figures from: Rainer Hegselmann and Ulrich Krause. Opinion Dynamics and Bounded Confidence, Models, Analysis and Simulation. Journal of Artificial Societies and Social Simulation, 5(3):2, 2002.

- What is the final average opinion?



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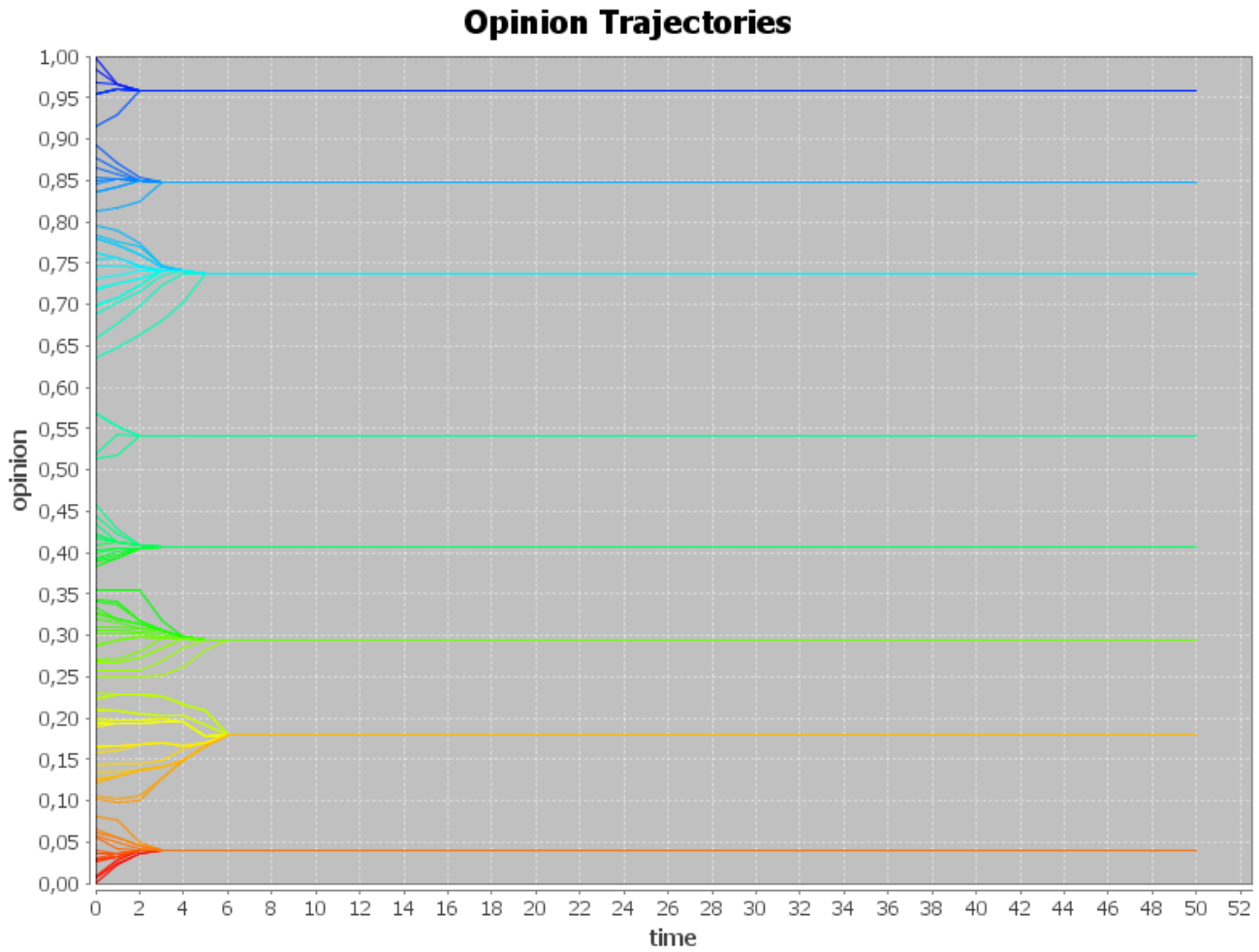
[3] Figures from: Rainer Hegselmann and Ulrich Krause. Opinion Dynamics and Bounded Confidence, Models, Analysis and Simulation. Journal of Artificial Societies and Social Simulation, 5(3):2, 2002.

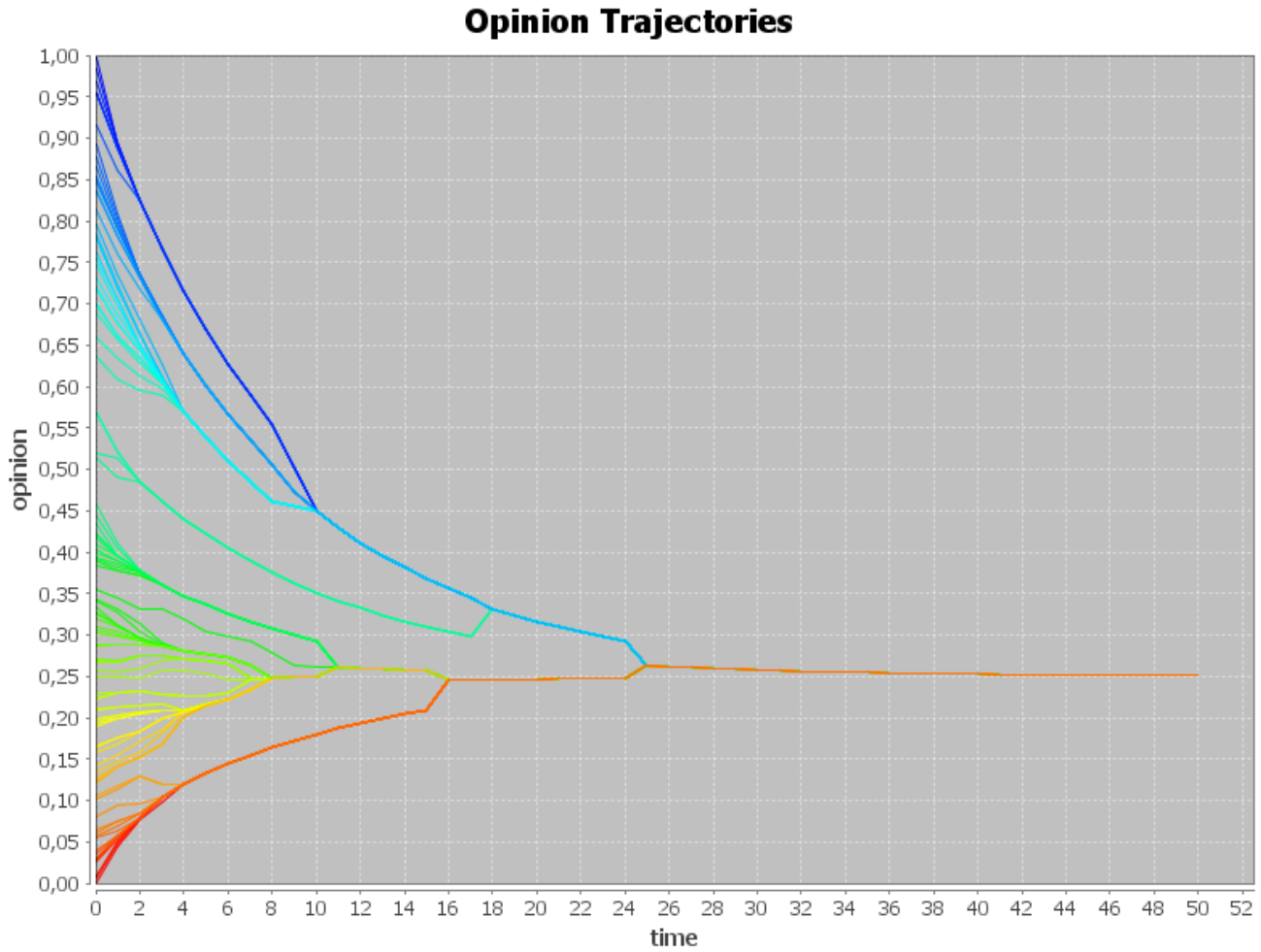
Extension: Truth finding

- Assumption: there is a true value T in our opinion space [0,1]
- T somehow attracts opinions
- Extend HK model to:
  $x_i(t+1) = \alpha_i T + (1-\alpha_i) f_i(x(t)) \qquad 0 \leq \alpha_i \leq 1$
- $\alpha_i T$: objective component, α_i controls strength of attraction
  α_i could be interpreted as the combined effect of education, training, profession, interest
- $(1-\alpha_i) f_i(x(t))$: social component with $f_i(x(t)$ as defined in HK model
- Case studies: $n = 100$, same random start distribution

- $\epsilon = 0.05$, $\alpha = 0.0$
- Original HK model

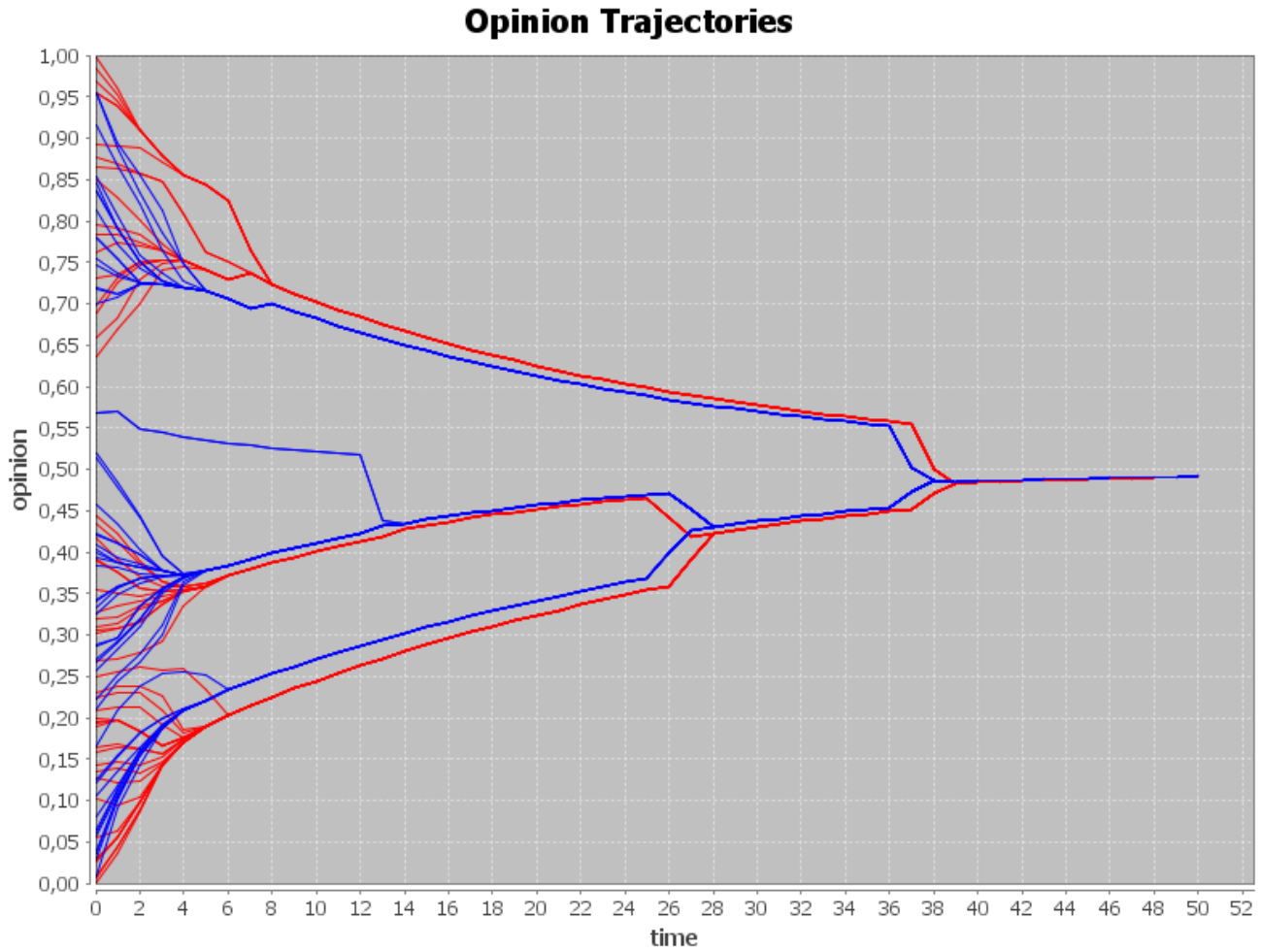


- $\epsilon = 0.05,\ \alpha = 0.1,\ T = 0.25$
- All agents are 'truth seekers'

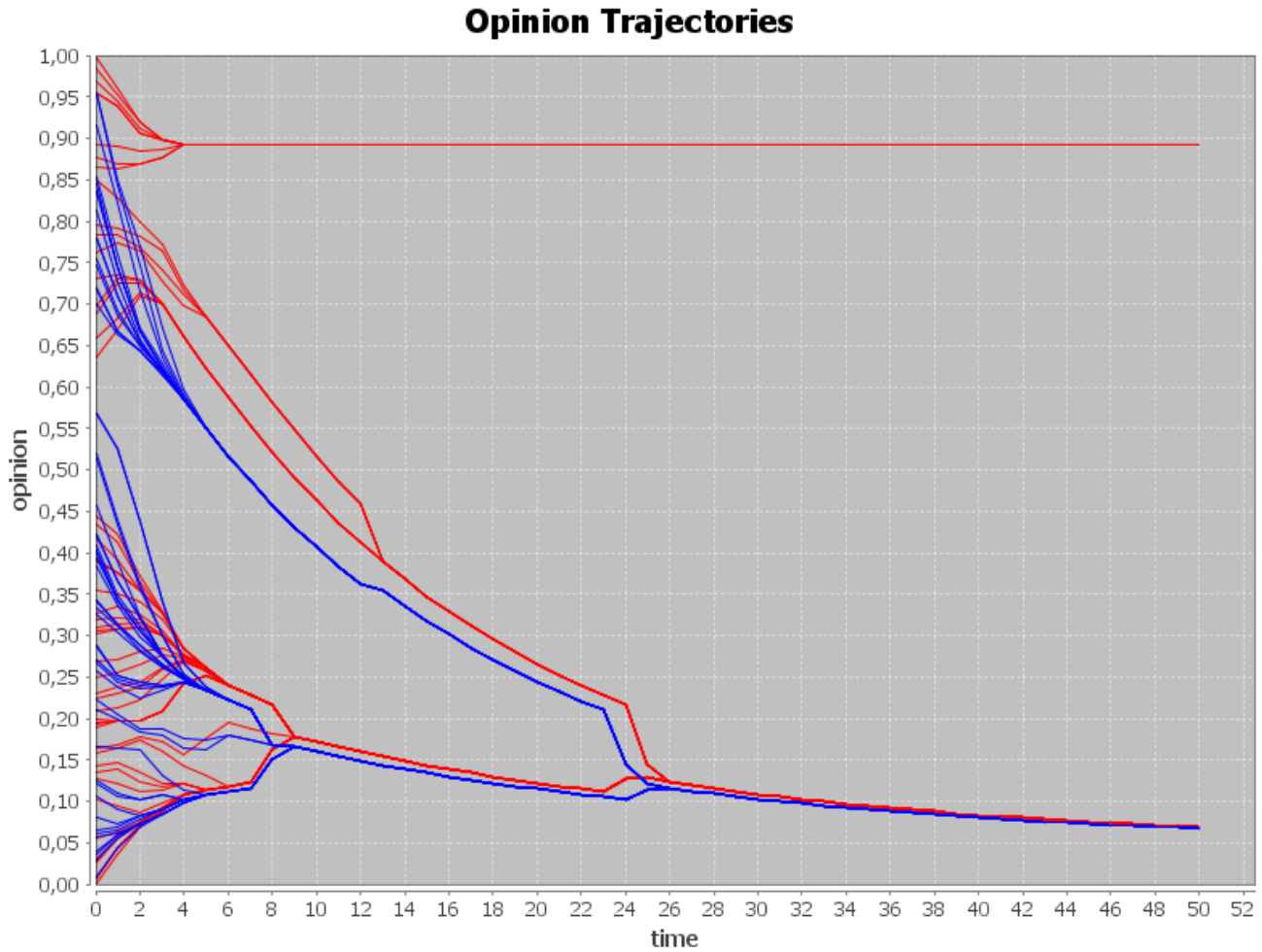


- Has everybody to be a truth seeker to get a consensus on the truth? No!
- $\epsilon = 0.1$, 50% $\alpha = 0.1$ (others $\alpha = 0.0$), $T = 0.25$

- Interplay of seeking for the truth by only some (cognitive division of labor) and social exchange process may lead to consensus

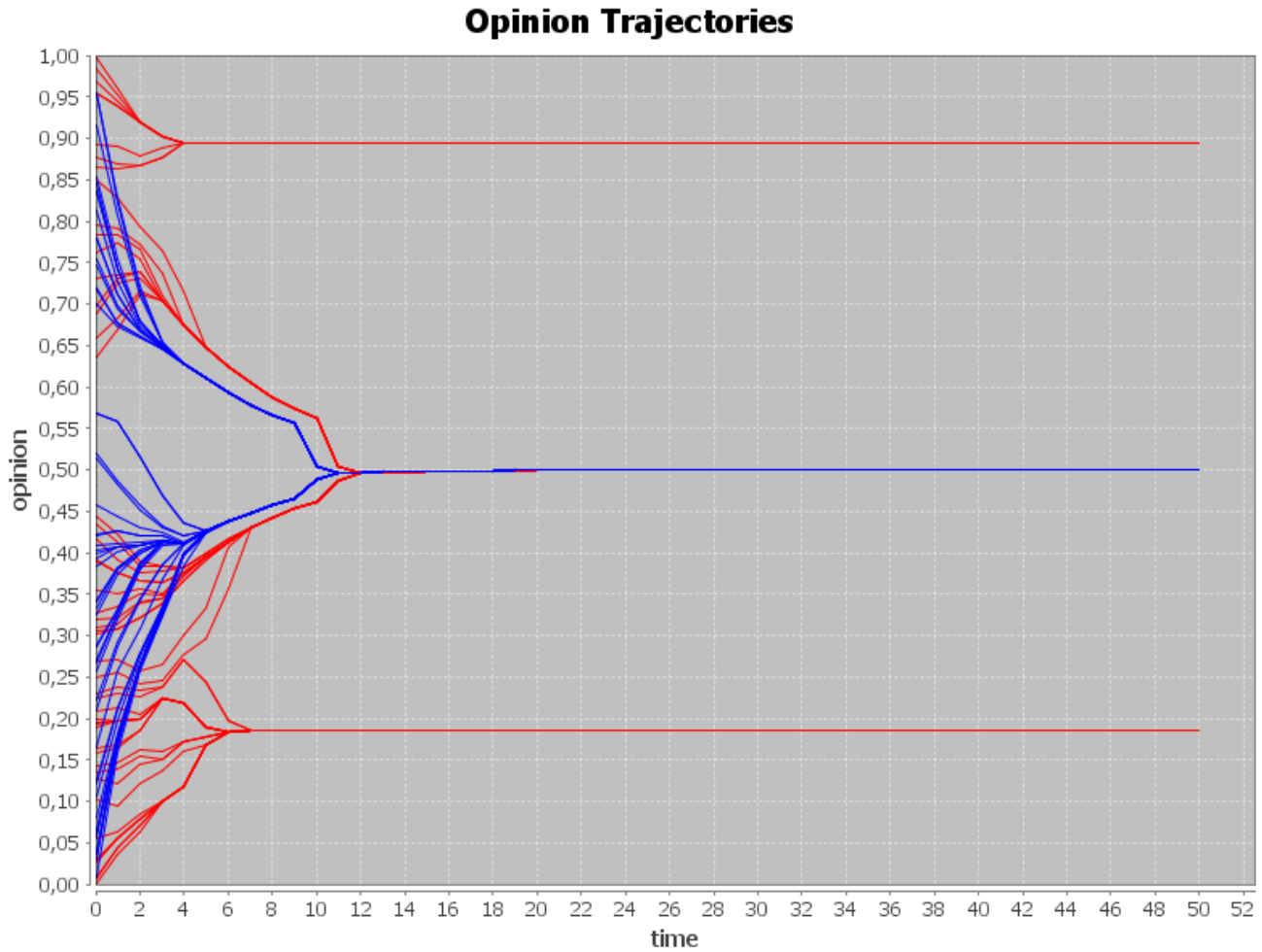


- This does not happen in all cases: consider truth $T = 0.05$ which is extreme, nothing else changes



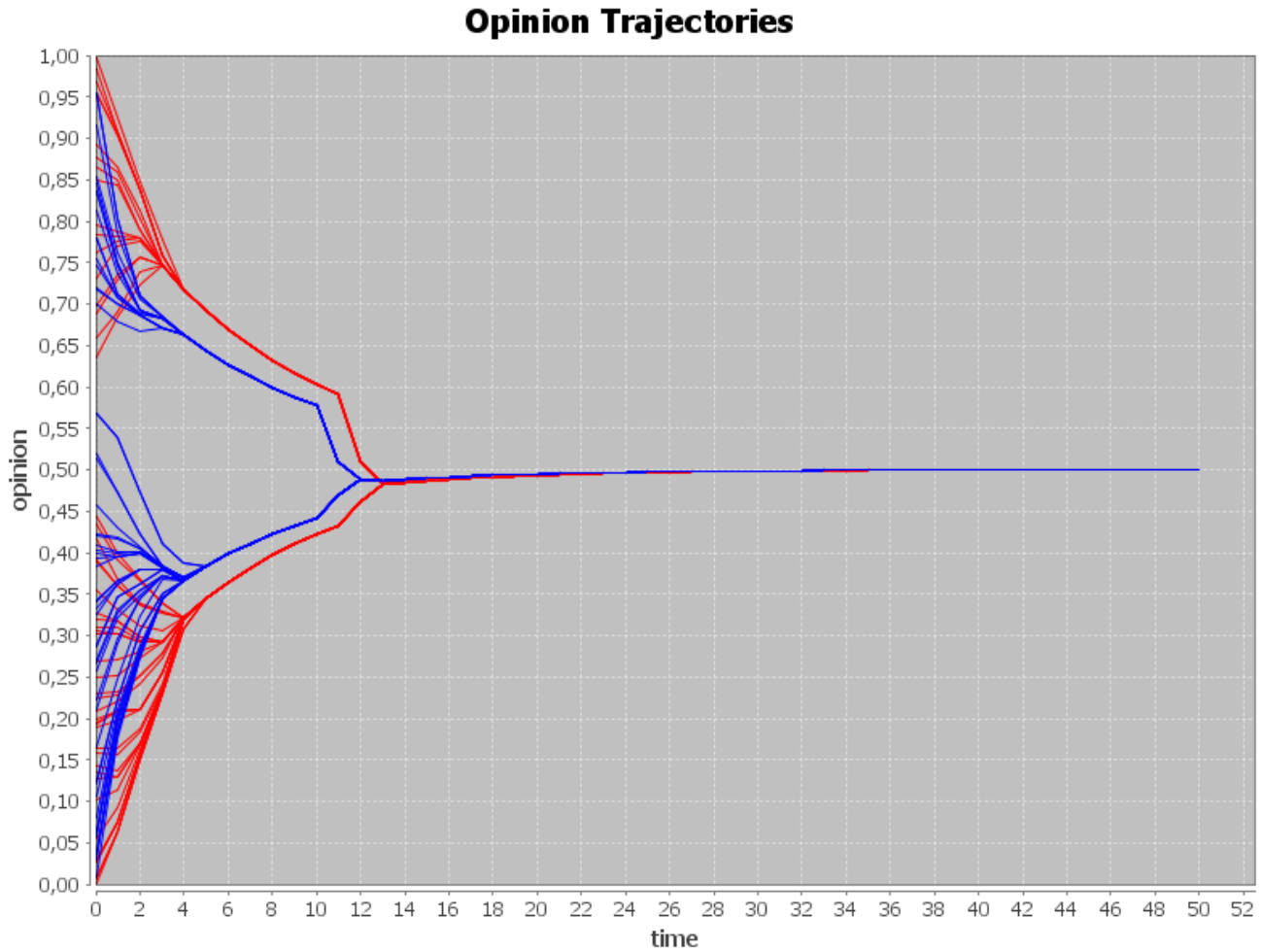
- All truth seekers move direction truth
- Even almost all with $\alpha = 0$, too
- Some non truth seekers are left behind far distant from the truth
- Position of truth matters in respect of finding consensus

- What happens if the attraction to the truth gets stronger? Again $T = 0.5$, but $\alpha = 0.25$ for truth seekers

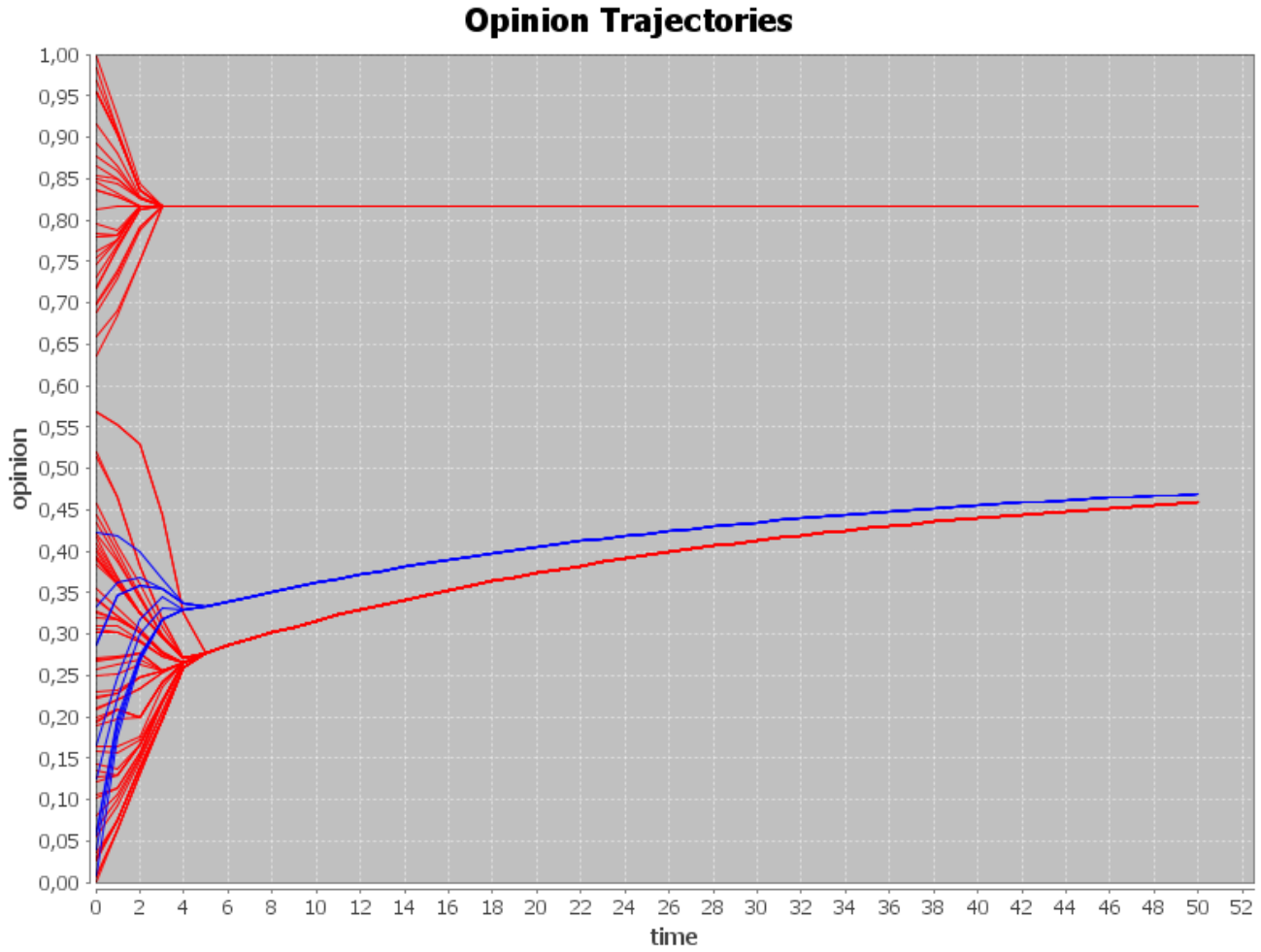


- Truth seekers approach truth much faster than before
- Non truth seekers with start positions more to the extremes of the opinion space are left behind and finally stick to opinions far distant from the truth
- An all including consensus on truth may become impossible if the truth seekers are especially fast and good in getting closer to the truth

- Raise ϵ from 0.10 to 0.15 $\rightarrow$ consensus on truth is possible again



- Keep $\epsilon = 0.15$ but lower percentage of α-positives from 50% to 10% $\rightarrow$ consensus vanishes again



- Rainer Hegselmann and Ulrich Krause. Opinion Dynamics and Bounded Confidence, Models, Analysis and Simulation. Journal of Artificial Societies and Social Simulation, 5(3):2, 2002.
- Jan Lorenz. Continuous opinion dynamics under bounded confidence: A survey. International Journal of Modern Physics C, 2007.
- Guillaume Deffuant, David Neau, Frederic Amblard, and Gerard Weisbuch. Mixing Beliefs among Interacting Agents. Advances in Complex Systems, 3:87-98, 2000.
- Rainer Hegselmann and Ulrich Krause. Truth and Cognitive Division of Labour. Journal of Artificial Societies and Social Simulation, vol. 9, no. 3. 2006.