Suppose that we have one electron with a spin-state given by

$$|S\rangle = a|\uparrow\rangle + b|\downarrow\rangle$$

According to the linearity of the equations of QM, when performing a measurement we get

$$|S + \text{Detector}\rangle = a|\uparrow\rangle|\Uparrow\rangle + b|\downarrow\rangle|\Downarrow\rangle$$

Detector showing spin-up

Detector showing spin-down

The state $|S + \text{detector}\rangle$ represents a superposition of 2 macroscopically distinct states: 'detector up' + 'detector down'.

This is analogous to the Schrödinger cat. This is a so-called "cat state".

According to Everett's interpretation:

- NO COLLAPSE
- The full state $|S + \text{detector}\rangle$ is the superposition of two distinct "pieces" which we call worlds → MANY WORLDS (BUT ONE UNIVERSE).

Note, this is 'rational' and actually a simple consequence of the linearity of the Schrödinger equation.

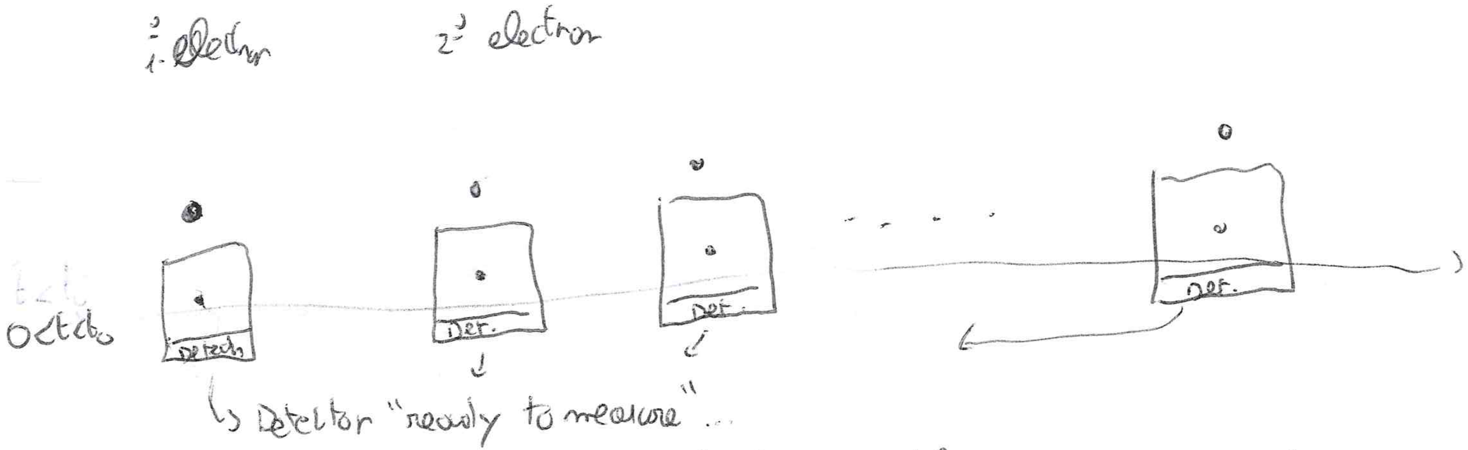
It is however important to show that in a 'certain' world the probabilities of QM emerge.

We can do it in a special case:

At $t=0$ prepare N electrons all in the state $a|\uparrow\rangle + b|\downarrow\rangle$:

$$|S\rangle_i = a|\uparrow\rangle_i + b|\downarrow\rangle_i \quad , \quad i = 1, \dots N$$

On each electron we perform a measurement at - say - $t = t_0 >$



The full quantum state before the measurement is given by

$$|\text{system}\rangle = \underbrace{(a|\uparrow\rangle_1 + b|\downarrow\rangle_1)}_{|S\rangle_1}\underbrace{(a|\uparrow\rangle_2 + b|\downarrow\rangle_2)}_{|S\rangle_2}\cdots\underbrace{(a|\uparrow\rangle_N + b|\downarrow\rangle_N)}_{|S\rangle_N}$$

Just after the measurement (for $t = t_0^+$) we have the full state

$$|\text{system} + \text{detectors}\rangle = (a|\uparrow\rangle_1|\Uparrow\rangle_1 + b|\downarrow\rangle_1|\Downarrow\rangle_1)\cdot$$

$$\cdots(a|\uparrow\rangle_N|\Uparrow\rangle_N + b|\downarrow\rangle_N|\Downarrow\rangle_N)$$

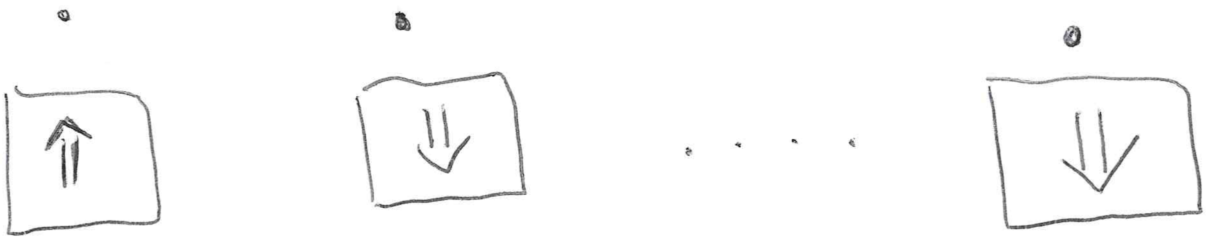
↳ First detector showing spin "up"

" " " " "down"

... and so on and so forth...

$t = t_0$

1 electron　　　2° electron　　　N electron



"Possible outcome of the measurement"

$$|\text{System + Detectors}\rangle = \overbrace{a^N |\uparrow\rangle_1 |\Uparrow\rangle_1 |\uparrow\rangle_2 |\Uparrow\rangle_2 \dots |\uparrow\rangle_N |\Uparrow\rangle_N}^{\text{very atypical world, in which the spin is always up!}} + a^{N-1} b\, |\uparrow\rangle_1 |\Uparrow\rangle_1 \dots |\downarrow\rangle_{N-1} |\Downarrow\rangle_{N-1} |\uparrow\rangle_N |\Uparrow\rangle_N$$

$$+ a^{N-1} b\, |\uparrow\rangle_1 |\Uparrow\rangle_1 \dots |\uparrow\rangle_{N-1} |\Uparrow\rangle_{N-1} |\downarrow\rangle_N |\Downarrow\rangle_N + \dots$$

$$\dots + a^{N/2} b^{N/2} |\uparrow\rangle_1 |\Uparrow\rangle_1 \dots |\uparrow\rangle_{\frac{N}{2}-1} |\Uparrow\rangle_{\frac{N}{2}-1} |\downarrow\rangle_{\frac{N}{2}+1} |\Downarrow\rangle_{\frac{N}{2}+1} \dots |\downarrow\rangle_N |\Downarrow\rangle_N$$

$$\dots + b^N |\downarrow\rangle_1 |\Downarrow\rangle_1 \dots |\downarrow\rangle_N |\Downarrow\rangle_N .$$

Each possibility corresponds to a world. In total, there are 2^N world.

The first world is

$$a^N |\uparrow\rangle_1 |\Uparrow\rangle_1 \dots |\uparrow\rangle_N |\Uparrow\rangle_N$$

is "atypical"... the spin is always up. It is a "Maverick world"...

We can now split the vector $|\text{system + Detectors}\rangle$ as a sum of two pieces:

$|\text{system} + \text{detectors}\rangle = |\bar{\Psi}\rangle + |\Psi_{Rest}\rangle$

where:

$|\bar{\psi}\rangle$ contains only those worlds which are in agreement with QM, that is:

$|a|^2 \cdot N \rightarrow$ the spin is up

$|b|^2 \cdot N \rightarrow$ the spin is down

$$|\bar{\psi}\rangle = a^{|a|^2 N} b^{|b|^2 N} |\uparrow\rangle_1 |\Uparrow\rangle_1 \cdots |\uparrow\rangle_{|a|^2 N} |\Uparrow\rangle_{|a|^2 N} |\downarrow\rangle_{|a|^2 N+1} |\Downarrow\rangle_{|a|^2 N+1} \cdots |\downarrow\rangle_N |\Downarrow\rangle_N$$

$$+ \cdots$$

$$= \sum_{i=1}^{\bar{N}} a^{|a|^2 N} b^{|b|^2 N} |\bar{\psi}_i\rangle$$

$|\Psi_{Rest}\rangle$ contains - on the contrary - those Maverik's worlds which do <u>not</u> respect Quantum Mechanics!

$$|\Psi_{Rest}\rangle = a^N |\uparrow\rangle_1 |\Uparrow\rangle_1 \cdots |\uparrow\rangle_N |\Uparrow\rangle_N + \cdots + b^N |\downarrow\rangle_1 |\Downarrow\rangle_1 \cdots |\Downarrow\rangle_N$$

Now, in order to show that only $|\bar{\psi}\rangle$ survives we calculate

$$1 = \langle system+det \,|\, system+det\rangle = \langle\bar{\psi}|\bar{\psi}\rangle + \langle\psi_{rest}|\psi_{rest}\rangle .$$

We then aim to show that

$\langle\bar{\psi}|\bar{\psi}\rangle \simeq 1$ for N very large.

That is, we can approximate the state as

$$|\, system+det\rangle \simeq |\bar{\psi}\rangle$$

and neglect the Maverick-worlds $|\psi_{rest}\rangle$.

Let us then calculate $\langle\bar{\psi}|\bar{\psi}\rangle$:

$$\langle\bar{\psi}|\bar{\psi}\rangle = \sum_{i=1}^{\bar{N}} |a|^{2|a|^2 N} |b|^{2|b|^2 N} = |a|^{2|a|^2 N} |b|^{2|b|^2 N}$$

but:

$$\bar{N} = \binom{N}{|a|^2 N}$$

: nr of worlds in which $|a|^2 N$ the spin is up and $|b|^2 N$ down.

We then have:

$$\langle\bar\psi|\bar\psi\rangle = |a|^{2|a|^2N}\,|b|^{2|b|^2N}\binom{N}{|a|^2N} :$$

$$= |a|^{2|a|^2N}\,|b|^{2|b|^2N}\,\frac{N!}{(|a|^2N)!\,(N-|a|^2N)!}$$

Stirling: $N! \simeq e^{N\log N}$

$$\langle\bar\psi|\bar\psi\rangle = |a|^{2|a|^2N}\,|b|^{2|b|^2N}\,\frac{e^{N\log N}}{e^{|a|^2N\log(|a|^2N)}\,e^{|b|^2N\log(|b|^2N)}}$$

$$|a|^2 = e^{\log|a|^2}$$

$$\langle\bar\psi|\bar\psi\rangle = \cancel{e^{|a|^2N\log(|a|^2)}}\;\cancel{e^{|b|^2N\log(|b|^2)}}\;\frac{e^{N\cdot\log N}}{\cancel{e^{|a|^2N\log|a|^2}}\,e^{|a|^2N\log N}\,\cancel{e^{|b|^2N\log|b|^2}}\,e^{|b|^2N\log N}}$$

$$= \frac{e^{N\log N}}{e^{|a|^2\log N}\,e^{|b|^2\log N}} = 1 \;!!!$$

Ergo, we have shown:

$$|\text{system} + \text{det}\rangle \simeq |\bar{\psi}\rangle$$

where $|\bar{\psi}\rangle$ represents the worlds which respect QM!

That is, the many world interpretation clearly explains the predictions of QM!!!