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Genesis einer hadronischen Theorie „von Quarks und Gluonen zu Mesonen und Baryonen“

Genesis of a hadronic theory
„from quarks and gluons to mesons and baryons“

Antrittsvorlesung

von

Francesco Giacosa

15/5/2013

Outline

The mystery of hadron masses

The Lagrangian of QCD and its symmetries

Development and results of a hadronic theory: general considerations

Development and results of a hadronic theory: mesons

Development and results of a hadronic theory: baryons

Nonzero density

Summary

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The mystery of hadron masses

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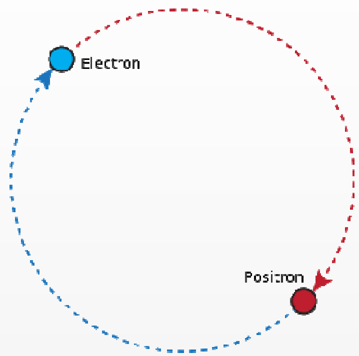
Positronium mass



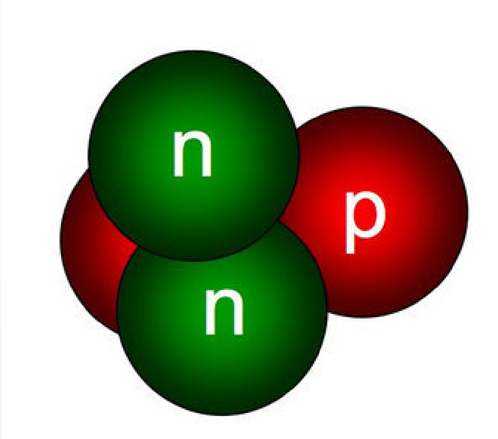
$$m_e = 0.511 \ \text{MeV}$$

$$m_{\text{Positronium}} = 2m_e - 6.8 \cdot 10^{-6} \ \text{MeV}$$

$$m_{\text{Positronium}} \approx 2m_e$$

Mass of the α particle

Nucleus of a Helium-atom



$$m_\alpha = 3.727379240 \ \text{GeV}$$

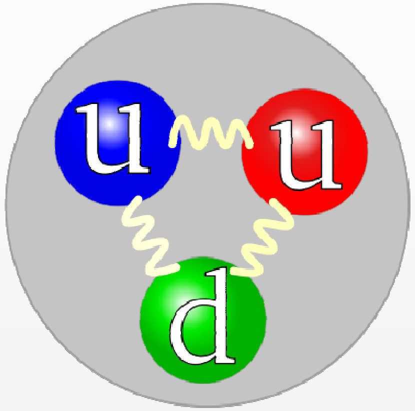
$$m_p = 0.93827 \ \text{GeV} \ ; \ m_n = 0.93956 \ \text{GeV}$$

$$m_\alpha \approx 2m_p + 2m_n$$

$$m_\alpha = (2m_p + 2m_n) - 28.2956 \ \text{MeV}$$

Proton

$$m_p = 938.27\,\mathrm{MeV}$$

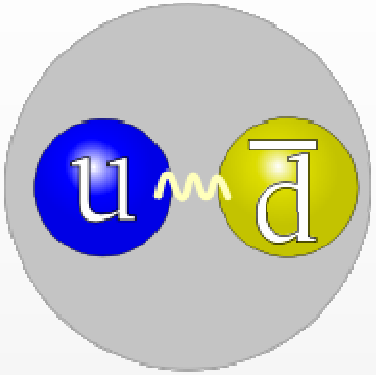


$$m_u = 2.3^{+0.7}_{-0.5}\ \mathrm{MeV}$$

$$m_d = 4.8^{+0.7}_{-0.5}\ \mathrm{MeV}$$

$$m_p >> 2m_u + m_d \approx 10\ \mathrm{MeV}$$

The ρ and the π mesons



Rho-meson $$m_{\rho^+} = 775 \ \text{MeV}$$

Pion $$m_{\pi^+} = 139 \ \text{MeV}$$

$$m_u + m_d \approx 7 \ \text{MeV}$$

The Lagrangian of QCD and its symmetries

Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

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Fields of the QCD Lagrangian

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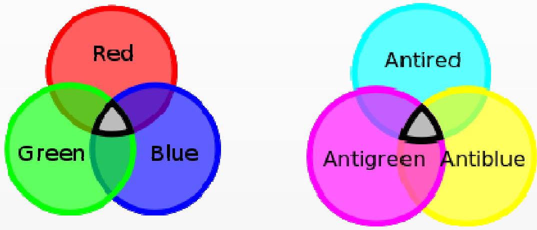
Quark: u,d,s R,G,B



$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \quad i = u,d,s$$

8 type of gluons ($R\overline{G}$,$B\overline{G}$,…)

$$A_\mu^a \; ; \quad a = 1,\ldots,8$$

SU(3)color: local gauge group

$$U(x) \in SU(3) \to U^{+}U = 1,\ \det U = 1. \qquad U = e^{it^a\theta^a}$$

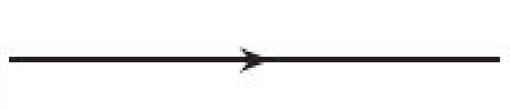
$$q_i(x) \to U(x)q_i(x) \qquad A_\mu = A_\mu^a t^a \to U(x)A_\mu U^{+}(x) - \frac{i}{g_0}U(x)\partial_\mu U^{+}(x)$$

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i(i\gamma^\mu D_\mu - m_i)q_i - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu}$$

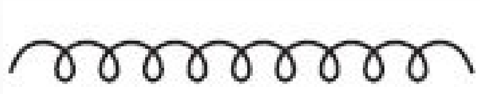
$$D_\mu = \partial_\mu - ig_0 A_\mu^a t^a$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_0 f^{abc} A^b_\mu A^c_\nu,\ \ a,b,c = 1,...,8$$

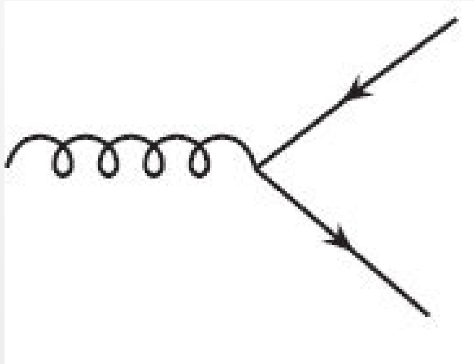
Feynman diagrams of QCD



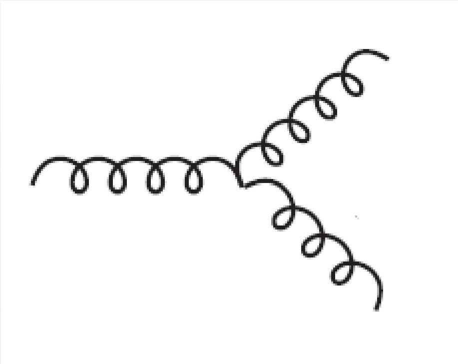
Quark



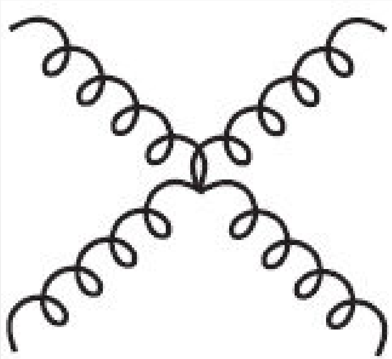
Gluon



Gluon-quark-antiquark vertex



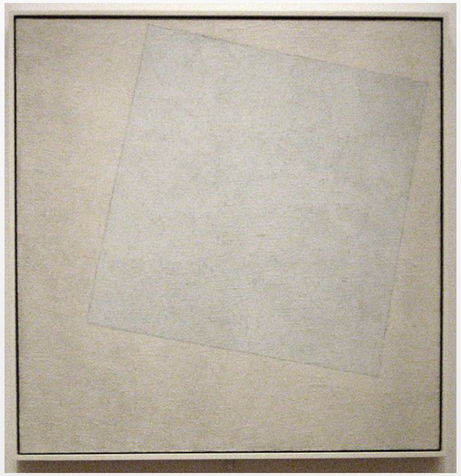
3-gluon vertex



4-gluon vertex

No ‚colored‘ state has been seen.

Confinement: physical states (hadrons) are white.



Painting of K. Malevich: ‚white on white‘.

Immediate question: bound state of gluons?

Trace anomaly

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$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \; D_\mu = \partial_\mu - i g_0 A^a_\mu t^a$$

Chiral limit: $m_i = 0$

$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

$$g_0 \overset{\text{Renormierung}}{\longrightarrow} g(\mu)$$

$$\partial_\mu J^\mu = T^\mu_\mu = \frac{\beta(g)}{4g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0$$

$$\beta(g) = \mu \frac{\partial g}{\partial \mu}$$

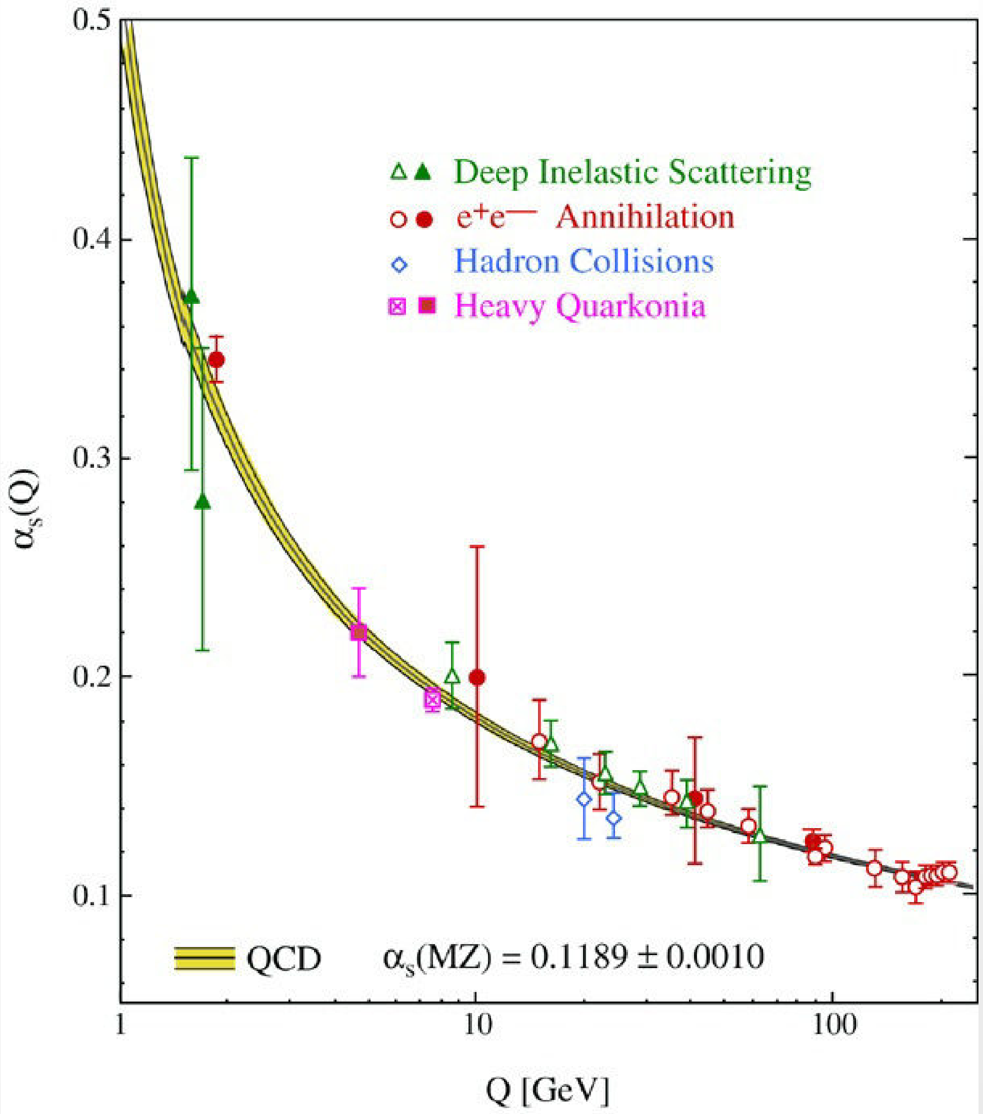
$$g^2(\mu) = \frac{1}{2b \log \frac{\mu}{\Lambda_{YM}}}$$

$$\Lambda_{YM} \approx 250 \quad \text{MeV}$$

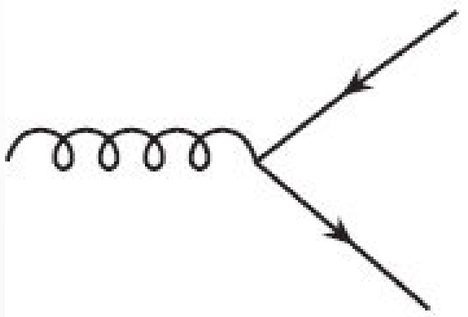
Dimensional transmutation

Trace anomaly and running coupling

$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Flavor symmetry



Gluon-quark-antiquark vertex.

It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$\mathrm{U} \in \mathrm{U}(3)_{\mathrm{V}} \rightarrow \mathrm{U}^{+}\mathrm{U} = 1$$

Chiral symmetry/1

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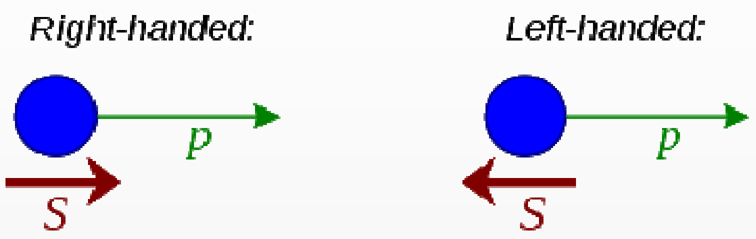


$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

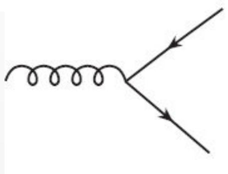
$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow (U_R)_{ij}\, q_{j,R} + (U_L)_{ij}\, q_{j,L}$$

$$U_R \subset U(3)_R \; ; \; U_L \subset U(3)_L$$

Chiral symmetry/2

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \; D_\mu = \partial_\mu - i g_0 A^a_\mu t^a$$



$$\overline{q}_i i\gamma^\mu D_\mu q_i = \overline{q}_{i,R} i\gamma^\mu D_\mu q_{i,R} + \overline{q}_{i,L} i\gamma^\mu D_\mu q_{i,L}$$ is chirally invariant

$$m_i \overline{q}_i q_i = m_i \overline{q}_{i,R} q_{i,L} + m_i \overline{q}_{i,L} q_{i,R}$$ is **not** chirally invariant

In the chiral limit (m_i=0) chiral symmetry is exact

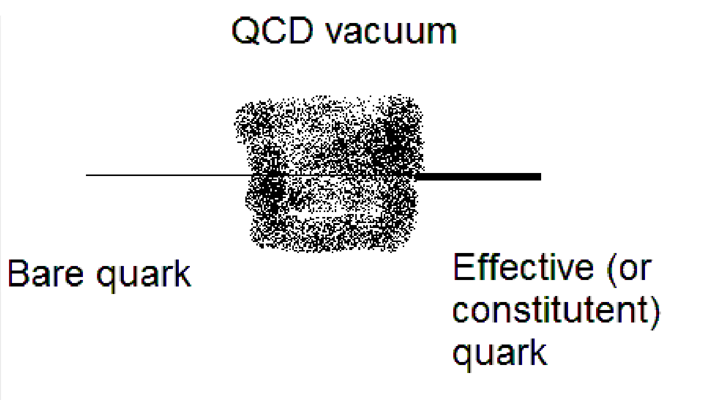
Spontaneous breaking of chiral symmetry

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

$$SU(3)_R \times SU(3)_L \rightarrow SU(3)_{V=R+L} \quad \Rightarrow \quad q_i \rightarrow U_{ij} q_j$$

$$\left\langle \bar{q}_i q_i \right\rangle = \left\langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \right\rangle \neq 0$$

$$m \simeq 5 \text{ MeV} \rightarrow m^* \simeq 300 \text{ MeV} \gg m$$



Masses revisited

$$m^* \approx 300\,\mathrm{MeV}$$

$$m_p \approx 3m^*$$

$$m_\rho \approx 2m^*$$

$$m_\pi << 2m^*$$

Pion: (quasi) Goldstone boson. $m_\pi^2 \propto (m_u + m_d)\langle \bar{q}q \rangle$

Symmetries of QCD: summary

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$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (trace anomaly) and by small quark masses

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is spontaneously broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (chiral anomaly)

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Development of a hadronic model: general considerations

Francesco Giacosa

Objectives

• Development of a chirally symmetric model for mesons and baryons including (axial-)vector d.o.f.

'Extended Linear Sigma Model (eLSM)'

• Study of the model for $T = \mu = 0$ (spectroscopy in vacuum)

(Masses, decay, scattering lengths,…)

Interrelation between these two aspects!

• Second goal: properties at nonzero T and μ

(condensates and masses in thermal/matter medium,…)

Fields of the model

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• Quark-antiquark mesons: scalar, pseudoscalar, vector and axial-vector quarkonia.

• Additional mesons: The scalar and the pseudoscalar glueballs

• Baryons: nucleon doublet and its partner

(in the so-called mirror assignment)

How to construct the eLSM

- Confinement: only hadrons.
- Dilatation invariance and its anomalous breaking
- Chiral symmetry $SU_R(N_f) \times SU_L(N_f) \times U_V(1)$ and its spontaneous as well as explicit breaking.
- Chiral anomaly

Development of a hadronic model: mesons

Francesco Giacosa

Confinement: from gluons to glueballs

$$m_{gluon} = 0 \rightarrow m^{*}_{gluon} \approx 500 - 800\ \mathrm{MeV}$$

$$\left\langle G^{a}_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$$

Confinement implies glueballs.

Where are they? We are still looking for them.

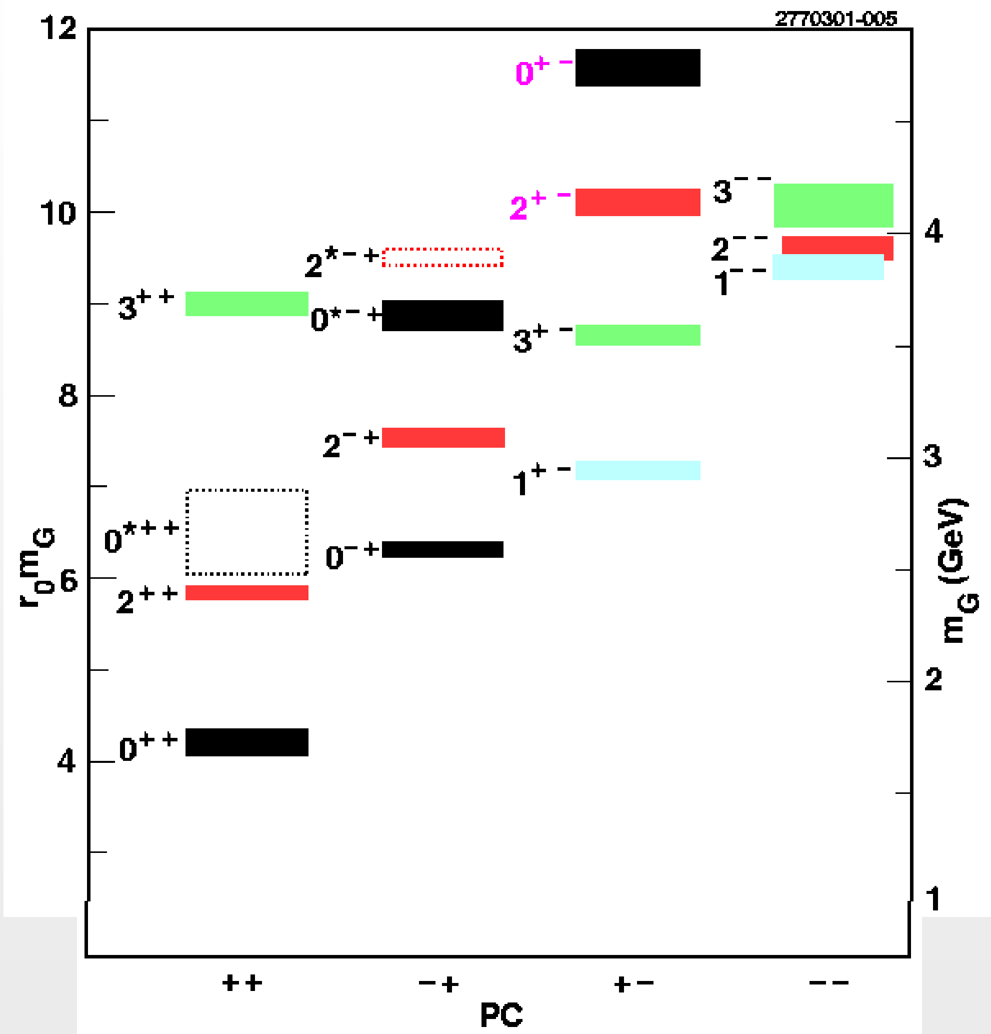
Glueballs from Lattice QCD

$$M_G = 1.6 - 1.8 \text{ GeV}$$

$$J^{PC} = 0^{++}$$

$$I = 0$$

lightest predicted glueball



Morningstar (1999)

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The lightest glueball is part of an effective Lagrangian which reproduces at a composite level the breaking of dilatation invariance.

Development of a dilaton potential.

Dilaton / Scalar glueball

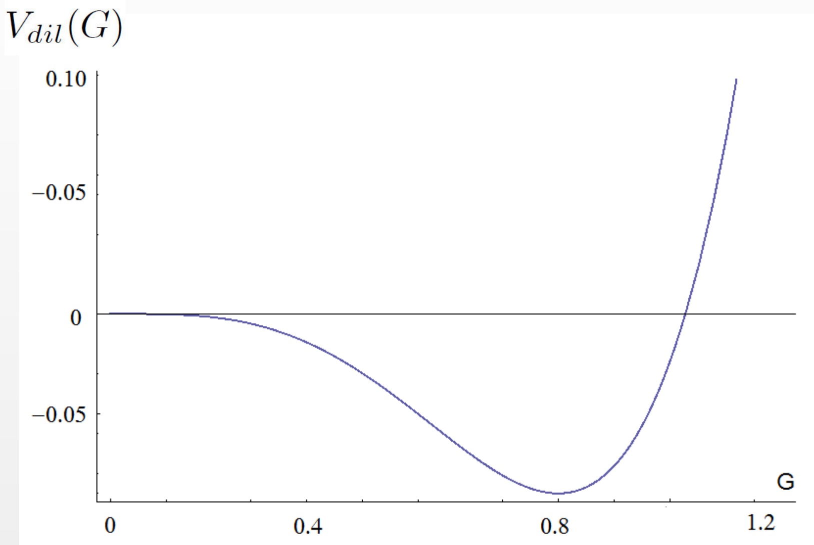
At the hadronic level, we describe these properties as:

$$G^4 \sim G^a_{\mu\nu} G^{a,\mu\nu}$$

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V_{dil}(G)$$

$$V_{dil}(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left[G^4 \ln\left(\frac{G}{\Lambda_G}\right) - \frac{G^4}{4}\right]$$

Λ_G dimensionful param that breaks dilatation inv!



$$\partial_\mu J^\mu = T^\mu_\mu = -\frac{1}{4}\frac{m_G^2}{\Lambda_G^2} G^4$$

In QCD it is:

$$\partial_\mu J^\mu = T^\mu_\mu = \frac{\beta(g)}{4g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0$$

Dilaton / Scalar glueball (2)

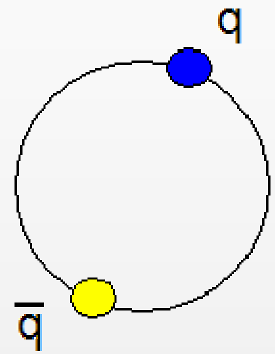
Where is the glueball G in the PDG ???

(Too many) candidates, most notably: $f_0(1500)$ and $f_0(1710)$

Quark-Antiquark mesons

Quark: u,d,s R,G,B

Quark-antiquark bound states: conventional mesons



$$\left| color \right\rangle = \sqrt{1/3}\left(\overline{R}R + \overline{B}B + \overline{G}G\right)$$

$\vec{L}, \vec{S}$ ⟹ $P = -(-1)^{L}$ $C = (-1)^{L+S}$

$\vec{L}, \vec{S}$ ⟹ $\vec{J} = \vec{L} + \vec{S}$ J^{PC}

(Pseudo)scalar sector

9 pseudoscalar fields: $L = S = 0 \rightarrow J^{PC} = 0^{-+}$

$$P = P_a \lambda^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & K^0 \\ K^- & \overline{K}^0 & \eta_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \qquad \Gamma = i\gamma^5$$

$$\pi^+ \equiv u\bar{d}$$

$$K^+ \equiv u\bar{s}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_\eta & \sin\theta_\eta \\ -\sin\theta_\eta & \cos\theta_\eta \end{pmatrix} \begin{pmatrix} \eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S \equiv \bar{s}s \end{pmatrix}$$

$$-36^\circ < \theta_\eta < -45^\circ$$

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…and 9 scalar fields: $L = S = 1 \rightarrow J^{PC} = 0^{++}$

$$S = S_a \lambda^a = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \qquad \Gamma = 1$$

$a_0^+ = a_0(1450) \equiv u\bar{d}$ and not $a_0(980)$!!!

$K_S^+ = K_0^{*+}(1430) \equiv u\bar{s}$ and not k(700)!!!

$\sigma_N \equiv \sqrt{1/2}(u\bar{u} + d\bar{d}) \approx f_0(1370)$

and not $f_0(500)$!!!

$\sigma_S \equiv u\bar{s} \approx f_0(1500)$ or $f_0(1710)$

and not $f_0(980)$!!!

Chiral transformation of (pseudo)scalar mesons

$$q_i = q_{i,R} + q_{i,L} \rightarrow (U_R)_{ij}\, q_{j,R} + (U_L)_{ij}\, q_{j,L} \qquad U_R,\ U_L \subset U(3)$$

$$\Phi = S + iP$$

$$\Phi_{ij} = \bar{q}_j q_i + i\bar{q}_j i\gamma^5 q_i = \sqrt{2}\,\bar{q}_{R,j} q_{L,i}$$

$$\Phi \rightarrow U_L \Phi U_R^+$$

Example of an invariant term

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$$\Phi \rightarrow U_L \Phi U_R^+ \qquad U_R,\ U_L \subset SU(3)$$

$$\lambda_2 Tr\left[\Phi^+ \Phi \Phi^+ \Phi\right] \rightarrow$$

$$\lambda_2 Tr\left[U_R \Phi^+ U_L^+ U_L \Phi U_R^+ U_R \Phi^+ U_L^+ U_L \Phi U_R^+\right] = \lambda_2 Tr\left[\Phi^+ \Phi \Phi^+ \Phi\right]$$

$$U_L^+ U_L = 1\ ,\ U_R^+ U_R = 1$$

(Axial-)Vector sector

9 vector fields… $L = 0, S = 1 \rightarrow J^{PC} = 1^{--}$

$$V^{\mu} = V^{\mu}{}_{a}\lambda^{a} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & \rho^{+} & K_{*}(892)^{+} \\ \rho^{-} & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & K_{*}(892)^{0} \\ K_{*}(892)^{-} & \overline{K}_{*}(892)^{0} & \phi_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \qquad \Gamma = \gamma^{\mu}$$

$$\rho^{+} \equiv u\bar{d}$$

$$K_{*}^{+}(892) \equiv u\bar{s}$$

$$\omega \approx \omega_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

$$\phi \approx \phi_S \equiv \bar{s}s$$

…and 9 axial-vector fields… $L = S = 1 \rightarrow J^{PC} = 1^{++}$

$$A^{\mu} = A^{\mu}{}_{a}\lambda^{a} = \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_{1,N}}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & K_1^0 \\ K_1^- & \overline{K}_1^0 & f_{1,S} \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix}$$

$$\Gamma = \gamma^{\mu}\gamma^{5}$$

$$a_1^+ = a_1^+(1260) \equiv u\bar{d}$$

$$K_1^+ = K_1^+(1270) \equiv u\bar{s}$$

$$f_1\,(1285) \approx f_{1,N} \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

$$f_1\,(1510) \approx f_{1,S} \equiv \bar{s}s$$

…where A and V are coupled in the following way:

$$L^{\mu} = V^{\mu} + A^{\mu}$$

$$R^{\mu} = V^{\mu} - A^{\mu}$$

which under chiral transformations transform as

$$R^{\mu} \rightarrow U_R R^{\mu} U_R^{+}$$

$$L^{\mu} \rightarrow U_L L^{\mu} U_L^{+}$$

Examples of further invariant objects:

$$G^2 Tr\left[L_\mu^+ L_\mu + R_\mu^+ R_\mu\right]$$

$$Tr\left[\Phi R_\mu \Phi^+ L_\mu\right]$$

$$\ldots$$

Meson sector: how many fields do we have?

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$4N_f^2 + 2$ fields

For $N_f = 3$ there are 38 mesons

36 quark-antiquark fields + 2 glueballs

Criteria (repetita juvant!)

We construct the Lagrangian of the so-called Extended Linear Sigma Model (ELSM) according to:

dilatation symmetry

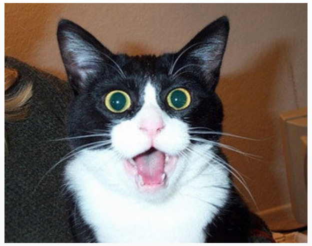
and

chiral invariance.

The breaking of the dilatation symmetry is only included in the „gluonic part"…(scalar glueball and axial anomaly)

Moreover, invariance under C and P is also taken into account.

Model of QCD – eLSM with scalar Glueball



$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right) + \mathrm{Tr}\left[(D^\mu \Phi)^\dagger (D_\mu \Phi)\right] \\
& - m_0^2 \left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\Phi^\dagger \Phi\right] - \lambda_1 (\mathrm{Tr}\left[\Phi^\dagger \Phi\right])^2 - \lambda_2 \mathrm{Tr}\left[(\Phi^\dagger \Phi)^2\right] \\
& + \left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left((L^\mu)^2 + (R^\mu)^2\right)\right] \\
& - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right] \\
& + c_1[\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger \Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\
& + h_2 \mathrm{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \mathrm{Tr}[\Phi R_\mu \Phi^\dagger L^\mu]
\end{aligned}
$$

$$
\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}
\frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\
a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\
K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S
\end{pmatrix}
$$

$$
L^\mu, R^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}
\frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{\star +} \pm K_1^+ \\
\rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{\star 0} \pm K_1^0 \\
K^{\star -} \pm K_1^- & \bar{K}^{\star 0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S}
\end{pmatrix}
$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011**)

D. Parganlija, P. Kovacs, G. Wolf , F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011 arXiv:1208.0585**

Basic features

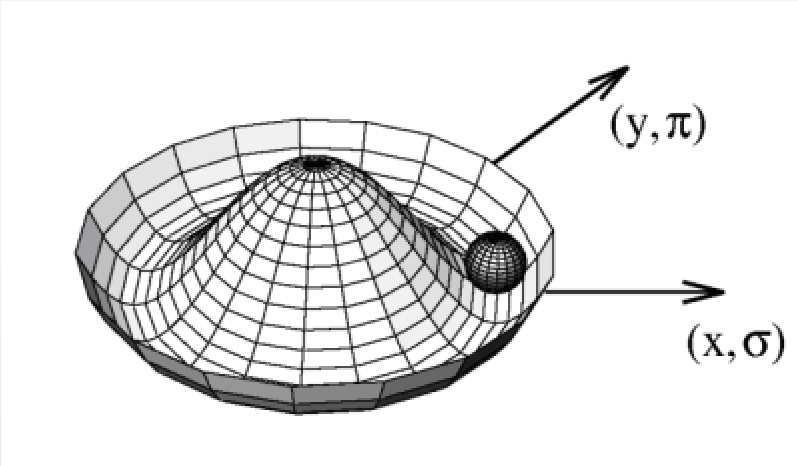
$$V = \frac{m_0^2}{2}\left(\sigma^2 + \pi^2\right) + \frac{\lambda_1 + \lambda_2}{4}\left(\sigma^2 + \pi^2\right)^2$$

$m_0^2 < 0 \;\rightarrow$ Mexican hat

$\pi =$ neutral pion

$$\sigma = \sigma_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \equiv f_0(1370)$$

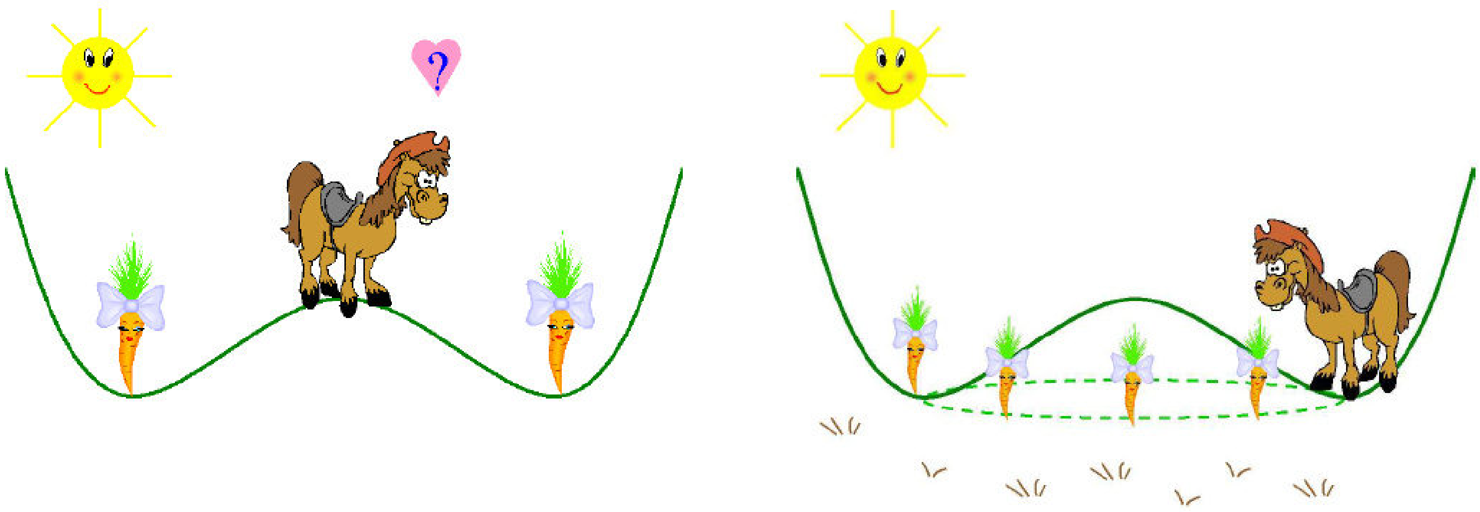
...and not to $f_0(500)$...



The donkey of Buridan

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)

Spontaneous Symmetry Breaking



Although Nicolás likes the symmetric food configuration, he must break the symmetry deciding which carrot is more appealing. In three dimensions, there is a continuous valley where Nicolás can move from one carrot to the next without effort.

Technical remarks

Perform Spontaneous Symmetry Breaking (SSB):

$$\sigma_N \to \sigma_N + \phi_N \ , \ \sigma_S \to \sigma_S + \phi_S$$

Explicit symmetry breaking terms:

$$H = diag\{h_1, h_2, h_3\} \text{ with } h_i \propto m_i \qquad m_\pi^2 \propto (m_u + m_d)\langle \bar{q}q \rangle$$

$$\delta = diag\{\delta_1, \delta_2, \delta_3\} \text{ with } \delta_i \propto m_i^2$$

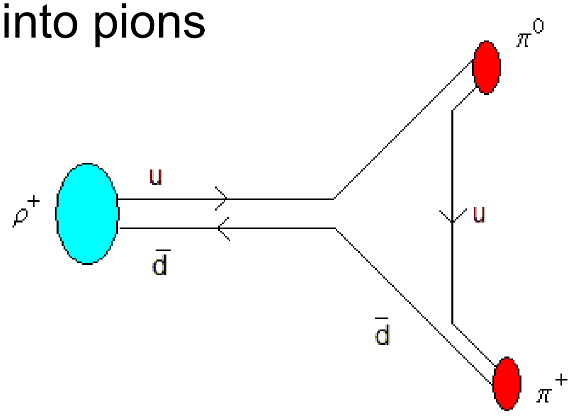
Parameter c: axial anomaly and eta-prime mass

But: only a finite number of terms is allowed!

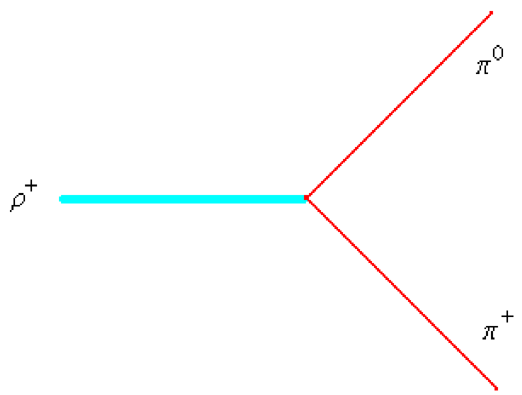
We can calculate: masses, decays, and scattering lengths.

Example: ρ-meson decay into pions

Microscopic



eLSM



Results of the fit (11 parameters, 21 exp. quantities)

Error from PDG or 5%.
Scalar-isoscalar sector not included.

$$\chi^2_{red} = 1.2$$

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

arXiv:1208.0585

Results of the fit: pictorial representation

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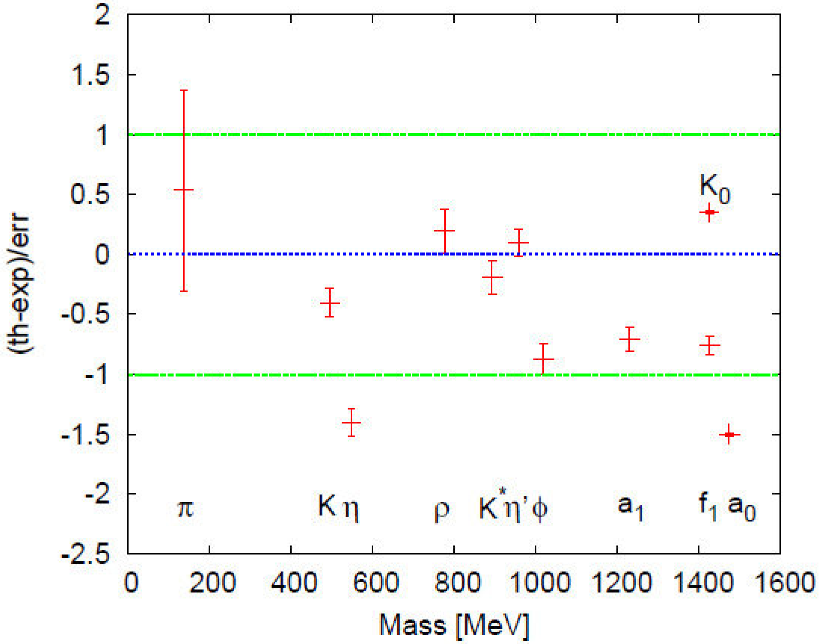

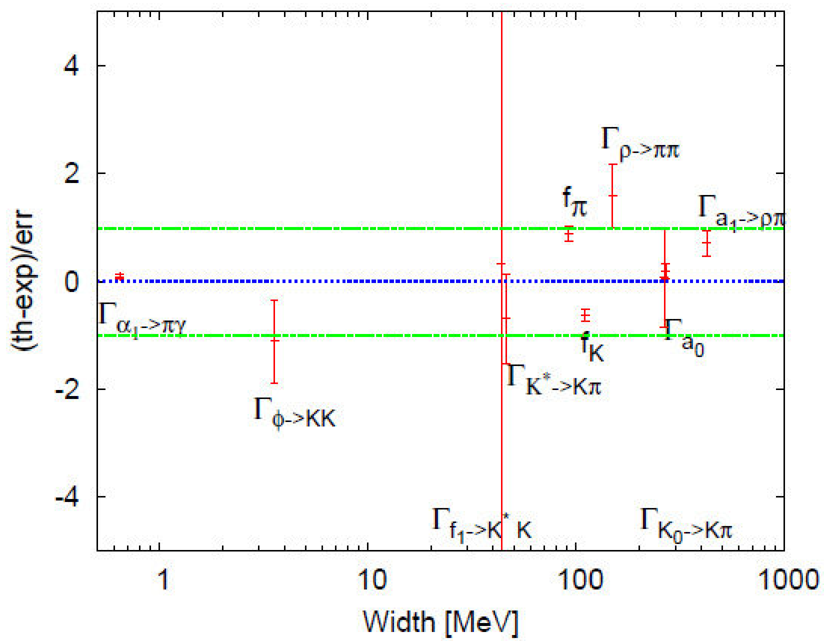

arXiv:1208.0585

Overall phenomenology is good.

Scalar mesons a_0(1450) and K_0(1430) above 1 GeV and are quark-antiquark states.

Importance of the (axial-)vector mesons

Consequences/1: $a_0(1450)$

Theory

$$\frac{\Gamma_{a_0 \to \eta' \pi}}{\Gamma_{a_0 \to \eta \pi}} = 0.19 \pm 0.02\ , \quad \frac{\Gamma_{a_0 \to KK}}{\Gamma_{a_0 \to \eta \pi}} = 1.12 \pm 0.07$$

Exp (PDG)

$$\frac{\Gamma_{a_0(1450) \to \eta' \pi}}{\Gamma_{a_0(1450) \to \eta \pi}} = 0.35 \pm 0.16\ , \quad \frac{\Gamma_{a_0(1450) \to KK}}{\Gamma_{a_0(1450) \to \eta \pi}} = 0.88 \pm 0.23\ .$$

Consequences/2: pseudoscalar mixing angle

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_\eta & \sin\theta_\eta \\ -\sin\theta_\eta & \cos\theta_\eta \end{pmatrix} \begin{pmatrix} \eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}_S$$

Theory $\theta_\eta = -44^\circ$

Exp (KLOE) $\theta_\eta \cong -41^\circ$

Consequences/3: ρ-mass

$$m_\rho^2 = m_1^2 + \frac{1}{2}(h_1 + h_2 + h_3)\phi_N^2 + \frac{h_1}{2}\phi_S^2$$

$$m_1^2 \propto G_0^2 \qquad m_1 = 0.643 \text{ GeV}$$

$$\sqrt{(h_2 + h_3)/2}\phi_N = 0.447 \text{ GeV}$$

Thus, the ρ-mass emerges from the interplay of both the chiral and gluon condensates!

Consequences/4: scalar-isoscalar quarkonia

$$\sigma \equiv \sigma_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \qquad \sigma_S = \bar{s}s$$

Theory $\quad m_{\sigma_N} = 1360 \text{ MeV} \qquad \mathrm{m}_{\sigma_S} = 1530 \text{ MeV}$

Exp (PDG)

$$m_{f_0(1370)} = 1350 \pm 150 \text{ MeV} \quad m_{f_0(1500)} = 1505 \pm 6 \text{ MeV} \quad m_{f_0(1710)} = 1720 \pm 6 \text{ MeV}$$

It then follows: the chiral partner of the pion is (predominantly) $f_0(1370)$ and not $f_0(500)$

The scalars below 1 GeV ($f_0(500)$, k(700), $f_0(980)$, $a_0(980)$ are not quarkonia.
Possibility: tetraquarks.
(F. G. **Phys.Rev. D75 (2007) 054007,** hep-ph/0611388. **)**

An important ongoing work:

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The calculation of the full mixing problem in the I=J=0 sector is ongoing:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = B \begin{pmatrix} \sigma_N \equiv \bar{n}n = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ G \equiv gg \\ \sigma_S \equiv \bar{s}s \end{pmatrix}$$

where B is a 3×3 orthogonal matrix

Our result within the Nf=2 case was:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} \sqrt{0.76} & \sqrt{0.24} & 0 \\ -\sqrt{0.24} & \sqrt{0.76} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_N \equiv \bar{n}n \\ G \equiv gg \\ \sigma_S \equiv \bar{s}s \end{pmatrix}$$

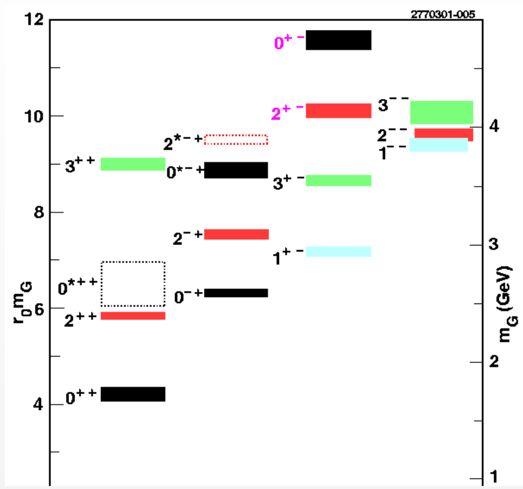
Details in S. Janowski, D. Parganlija, F.G., D. Rischke, **Phys.Rev. D84 (2011) 054007, arXiv:1103.3238**

A new entry: the pseudoscalar glueball

$$\mathcal{L}^{int}_{\tilde{G}\text{-mesons}} = i c_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^\dagger \right)$$

Quantity	Value
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to \eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to \eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to \eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.46
$\Gamma_{\tilde{G}\to \eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to \eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.094

Quantity	Value
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.059
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.083
$\Gamma_{\tilde{G}\to \eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.028
$\Gamma_{\tilde{G}\to \eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012
$\Gamma_{\tilde{G}\to \eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.019



$$\Gamma_{\tilde{G}\to \pi\pi\pi} = 0$$

PANDA/FAIR will be able to scan the energy above 2.5 GeV

Details in:

W. Eshraim, S. Janowski, F.G:, D. Rischke, **Phys.Rev. D87 (2013) 054036. arxiv: 1208.6474** .
W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., **Acta Phys. Pol. B**, Prc. Suppl. 5/4, **arxiv: 1209.3976**

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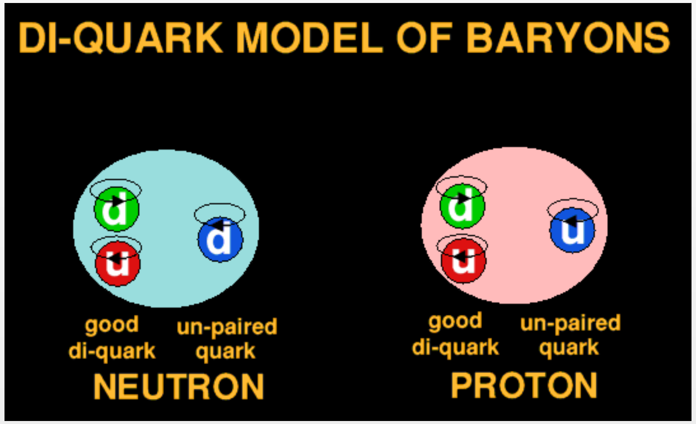
Development of a hadronic model: baryons

Francesco Giacosa

Proton and neutron: white states (as each baryon)

$$\left| baryon - color \right\rangle = \sqrt{\frac{1}{6}}(RGB + BRG + GBR - GRB - BGR - RBG)$$



Nucleon doublet

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$

Chiral transformation: $N_R \to U_R N_R \qquad N_L \to U_L N_L$

A simple mass term $m\,\overline{N}N = m\left(\overline{N}_R N_L + \overline{N}_L N_R\right)$ is forbidden!!!!

One has: $g\sigma\overline{N}N + g\vec{\pi}\overline{N}\vec{\tau}N \to g\phi\overline{N}N + \ldots$

$$m_{nucleon} \approx g\phi \propto \left\langle \overline{q}q \right\rangle$$

Baryon sector in the ElSM
(Nf = 2 only)

Nucleon and its chiral partner; chiral symmetry and dilatation invariance

(Axial-)vector mesons are included

Mirror assignment: (C. De Tar and T. Kunihiro, **PRD 39 (1989) 2805**)

$$\Psi_{1,R} \rightarrow U_R \Psi_{1,R} \qquad \Psi_{1,L} \rightarrow U_L \Psi_{1,L}$$

$$\Psi_{2,R} \rightarrow U_L \Psi_{2,R} \qquad \Psi_{2,L} \rightarrow U_R \Psi_{2,L}$$

A chirally invariant mass-term is possible!

$$m_0 \left(\overline{\Psi}_{1,L} \Psi_{2,R} - \overline{\Psi}_{1,R} \Psi_{2,L} - \overline{\Psi}_{2,L} \Psi_{1,R} + \overline{\Psi}_{2,R} \Psi_{1,L} \right)$$

Lagrangian in the baryon sector

Interaction of baryons with (pseudo)scalar and (axial-)vector mesons

$$\mathcal{L}_{mirror} = \overline{\Psi}_{1L} i\gamma_\mu D^\mu_{1L}\Psi_{1L} + \overline{\Psi}_{1R} i\gamma_\mu D^\mu_{1R}\Psi_{1R} + \overline{\Psi}_{2L} i\gamma_\mu D^\mu_{2R}\Psi_{2L} + \overline{\Psi}_{2R} i\gamma_\mu D^\mu_{2L}\Psi_{2R}$$
$$- \widehat{g}_1 \left(\overline{\Psi}_{1L}\Phi\Psi_{1R} + \overline{\Psi}_{1R}\Phi^\dagger\Psi_{1L}\right) - \widehat{g}_2 \left(\overline{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \overline{\Psi}_{2R}\Phi\Psi_{2L}\right) + \mathcal{L}_{\text{mass}}$$

$$D^\mu_{1R} = \partial^\mu - ic_1 R^\mu, D^\mu_{1L} = \partial^\mu - ic_1 L^\mu$$

$$D^\mu_{2R} = \partial^\mu - ic_2 R^\mu, D^\mu_{2L} = \partial^\mu - ic_2 L^\mu$$

$$\mathcal{L}_{\text{mass}} = -m_0(\overline{\Psi}_{1L}\Psi_{2R} - \overline{\Psi}_{1R}\Psi_{2L} - \overline{\Psi}_{2L}\Psi_{1R} + \overline{\Psi}_{2R}\Psi_{1L})$$

Mass of the nucleon

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$\delta = ar\cosh\left[\frac{M_N + M_{N^*}}{2m_0}\right]$$

$N = N(940)$

$N^* = N^*(1535)$

$$m_{N,N^*} = \sqrt{m_0^2 + \left(\frac{\widehat{g}_1 + \widehat{g}_2}{4}\right)^2 \phi^2} \pm \frac{(\widehat{g}_1 - \widehat{g}_2)\phi}{4}$$

If $m_0 = 0 \rightarrow$ only the quark condensate generates the masses. $m_N \sim \phi$

m_0 parameterizes the contribution which does not stem from the quark condensate

Crucial also at nonzero temperature and density

also in the so-called quarkyonic phase: L. McLerran, R. Pisarski **Nucl.Phys.A796:83-100,2007**

Result for m_0

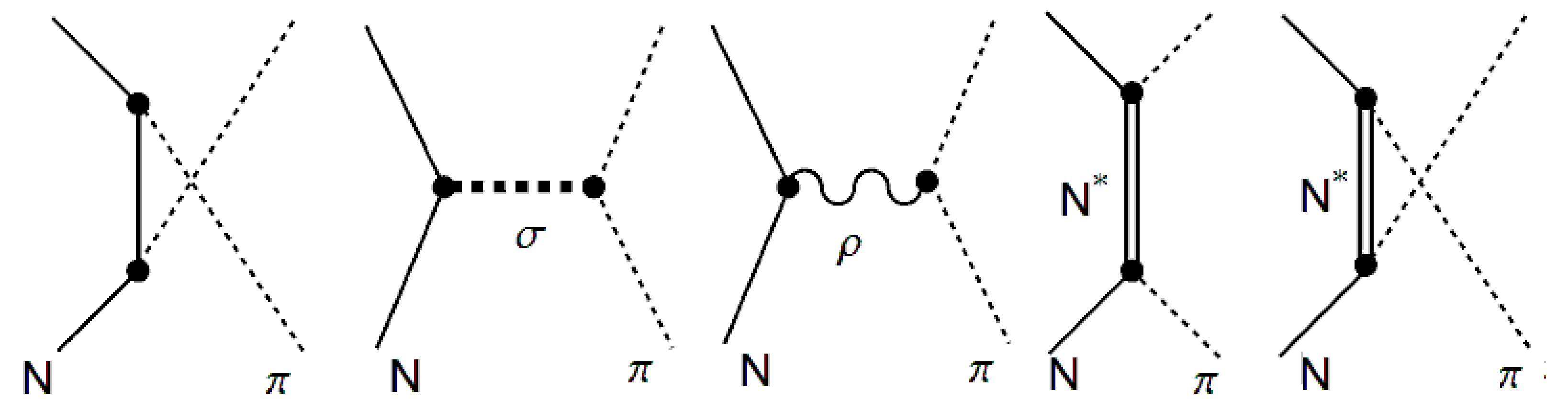
$$m_{N,N^*} = \sqrt{m_0^2 + \left(\frac{\widehat{g}_1 + \widehat{g}_2}{4}\right)^2 \phi^2} \pm \frac{(\widehat{g}_1 - \widehat{g}_2)\phi}{4}$$

$$m_0 = 460 \pm 136 \text{ MeV}$$

Using $g_A^N = 1.26$ (exp), $g_A^{N^*} \approx 0.2$ (latt) and $\Gamma_{N^* \to N\pi} \approx 67$ MeV

The nucleon mass emerges from the interplay of the chiral condensate and the newly introduced mass term, which in turn depends on further condensates: the tetraquark and the gluon condensates.

Test: pion-nucleon scattering lengths



$$a_0^- = (6.04 \pm 0.63) \cdot 10^{-4}\ \mathrm{MeV}^{-1} \qquad a_0^{-(\exp)} = (6.4 \pm 0.1) \cdot 10^{-4}\ \mathrm{MeV}^{-1}$$

$$a_0^+ \approx (\text{from} - 20 \text{ to} + 20 \cdot 10^{-4})\ \mathrm{MeV}^{-1} \qquad a_0^{+(\exp)} = (-8.8 \pm 7.2) \cdot 10^{-4}\ \mathrm{MeV}^{-1}$$

Large theoretical uncertainty due to the scalar-isosocalar sector

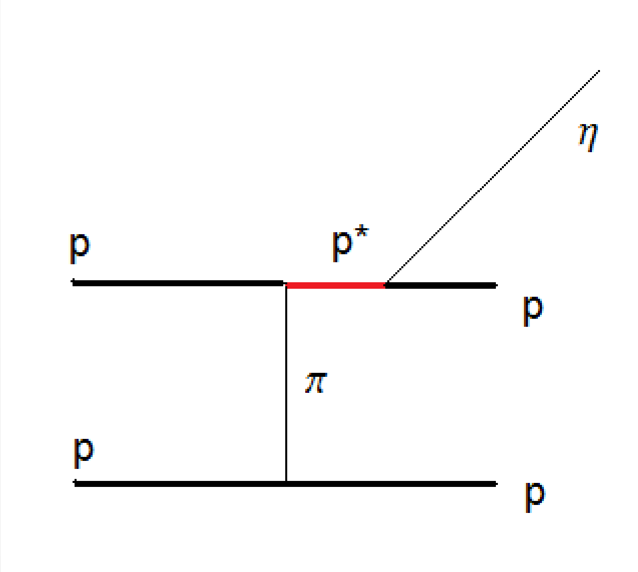
Importance of both vector mesons and mirror assignment in order to get these results

What we are studying rigth now…

$$p + p \rightarrow p + p + X$$

$$X = \eta, \omega, \text{lepton pair}, ...$$

Many diagrams to calculate; the advantage is : chiral symmetry (and also g.i.) built in.



Other processes with nucleons are possible:

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Example: pseudoscalar glueball

$$p + \overline{p} \rightarrow \widetilde{G} \rightarrow \ldots$$

$$\mathcal{L}^{int}_{\tilde{G}\text{-baryons}} = ic_{\tilde{G}\Psi}\tilde{G}\left(\overline{\Psi}_2\Psi_1 - \overline{\Psi}_1\Psi_2\right)$$

$$\frac{\Gamma_{\tilde{G}\rightarrow\overline{N}N}}{\Gamma_{\tilde{G}\rightarrow\overline{N}^*N+h.c.}} = 1.94$$

W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, and F.G., **Acta Phys. Pol. B**, Prc. Suppl. 5/4, **arxiv: 1209.3976**

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Results at nonzero density

Basic considerations for nonzero density

The σ-field of our model is associated with the resonance $f_0(1370)$
…and not with the lightest scalar resonance $f_0(500)$.

The question is: what is $f_0(500)$ and, more in general, what are the scalar states below 1 GeV?

A good phenomenology (masses and decays) is achieved when interpreting the light scalar states as tetraquarks: $f_0(500) \approx [\bar{u}, \bar{d}][u, d]$
(bound states of a diquark and an anti-diquark)

Details in: F.G, Phys.Rev. D **75** (2007) 054007

Back to nucleons: where does m_0 comes from?

$$m_0\left(\overline{\Psi}_{1,L}\Psi_{2,R}-\overline{\Psi}_{1,R}\Psi_{2,L}-\overline{\Psi}_{2,L}\Psi_{1,R}+\overline{\Psi}_{2,R}\Psi_{1,L}\right)$$

By requiring dilatation invariance one should modify the mass-term as:

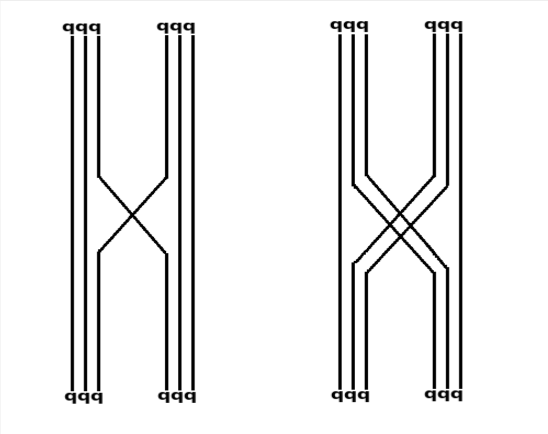
$$(a\chi+bG)\left(\overline{\Psi}_{1,L}\Psi_{2,R}-\overline{\Psi}_{1,R}\Psi_{2,L}-\overline{\Psi}_{2,L}\Psi_{1,R}+\overline{\Psi}_{2,R}\Psi_{1,L}\right)$$

Tetraquark
New field

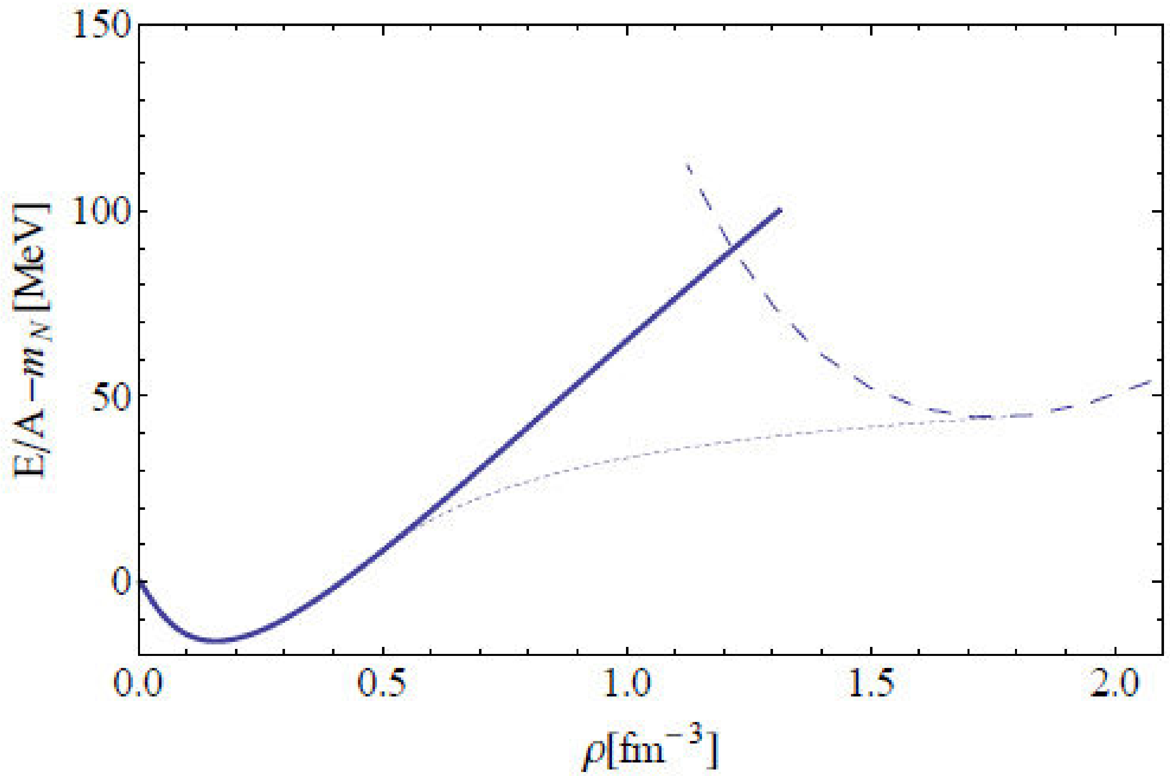
dilaton

By shifting : $\chi\rightarrow\chi_0+\chi,\quad G\rightarrow G_0+G$ one has : $m_0=a\chi_0+bG_0$

m_0 originates form the tetraquark and the gluon condensates.
Note, also, a tetraquark exchange naturally arises in nucleon-nucleon interactions



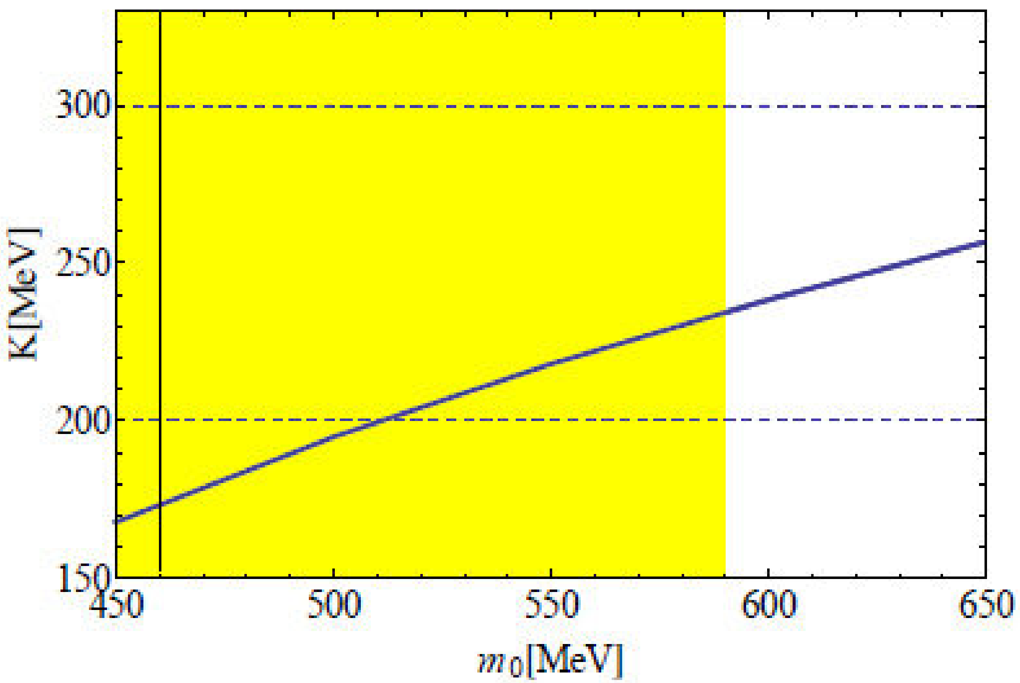
Nuclear matter saturation



Details in: S. Gallas, F. G:, G. Pagliara, **Nucl.Phys. A872 (2011) 13-24 arXiv:1105.5003**

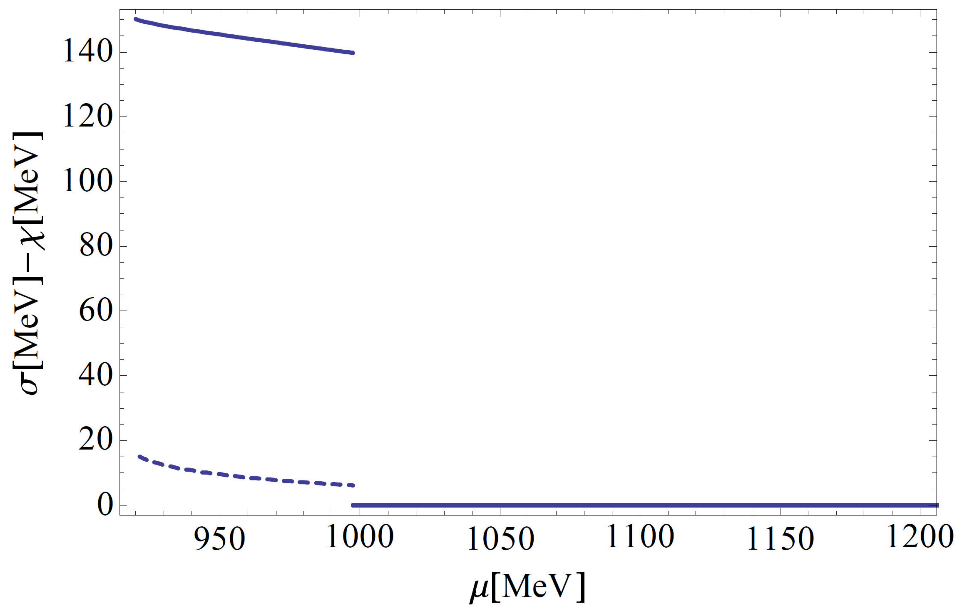
An important test: Compressibility

Compressibility K is in agreement with experiment



arXiv:1105.5003

Chiral phase transition



arXiv:1105.5003

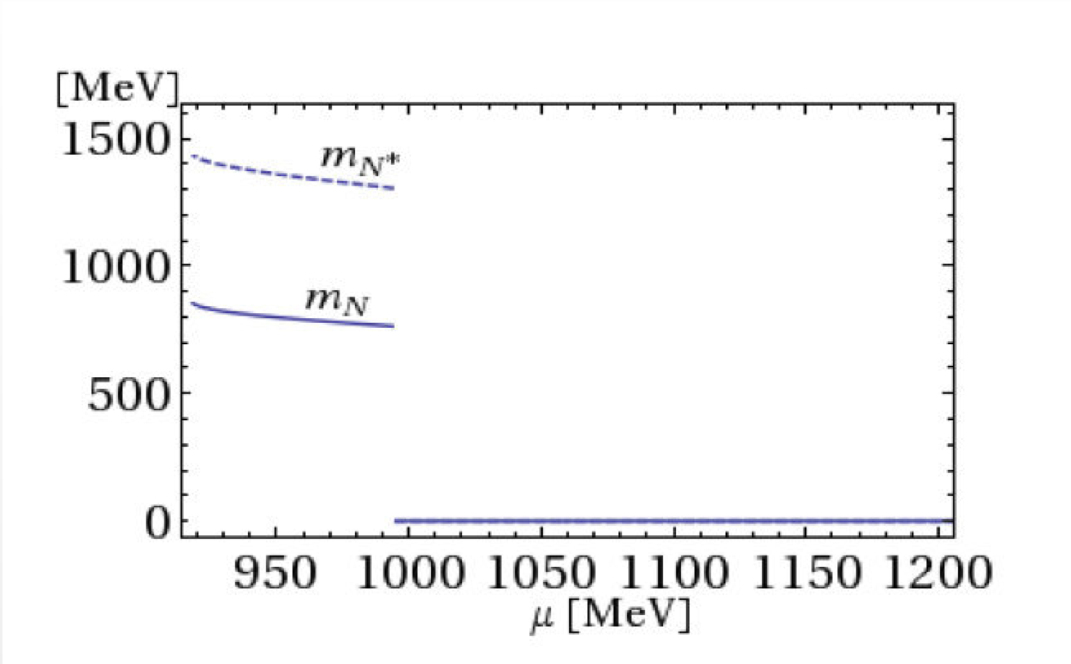
Critical density at the onset of chiral restoration (first order):

$$\rho_{crit} / \rho_0 \approx 2.5$$

(slightly dependent on m_0)

Chiral phase transition/2

Masses



The masses drop almost to zero above the critical value of the chemical potential.

arXiv:1105.5003

Nuclear matter: why does it bind?

The resonance $f_0(500)$, here interpreted as a tetraquark, plays an important role for the stability of nuclear matter.

Related 'amusing' question: does nuclear matter binds at large N_c?

As soon as the lightest scalar $f_0(500)$ is not a quarkonium, nuclear matter ceases to exist already for $N_c=4$.

Of course, for another value of N_c I would not exist and I would not be speaking about it here.

L. Bonanno and F.G., **Nucl.Phys.A859:49-62,2011 arXiv:1102.3367** [hep-ph]

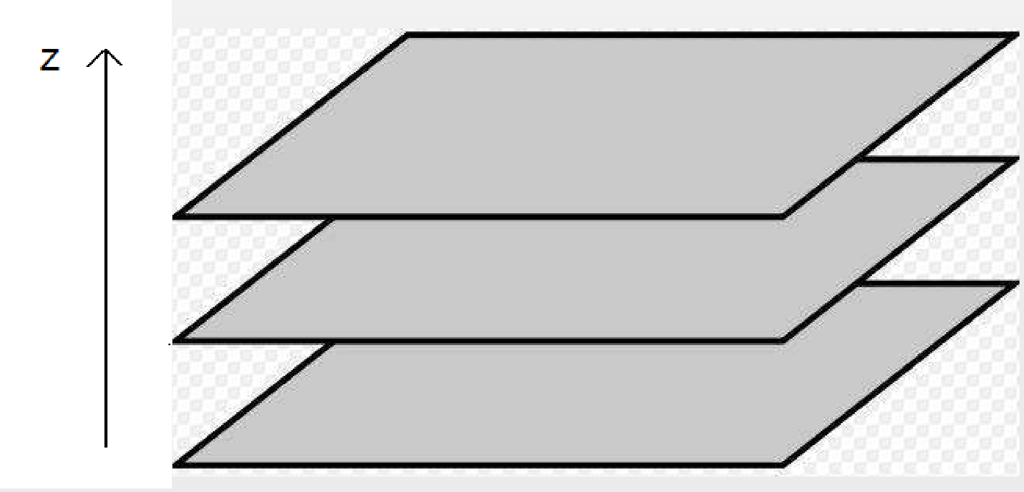
Inhomogeneous condensation at nonzero density

$$\text{Up to now}: \phi = const$$

…but one can have a Chiral Density Wave:

$$\phi(z) = \varphi \cos(2fz)$$

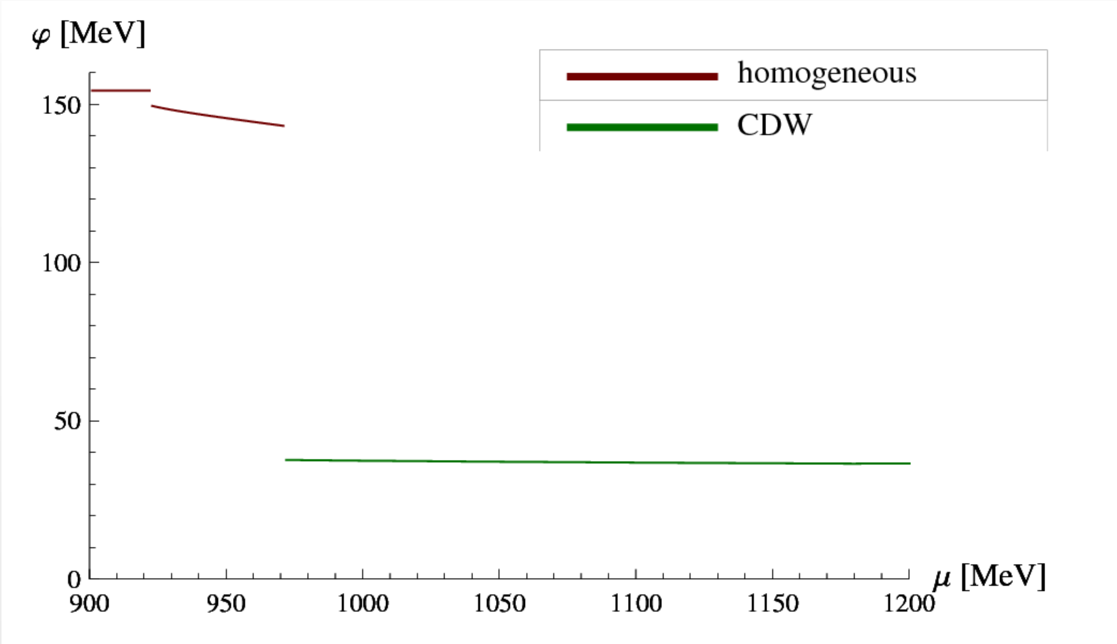
$$\langle \pi^0 \rangle = \varphi \sin(2fz)/Z$$



Inhomogeneous condensation/2

$$\phi(z) = \varphi \cos(2fz)$$

$$\langle \pi^0 \rangle = \varphi \sin(2fz) / Z$$



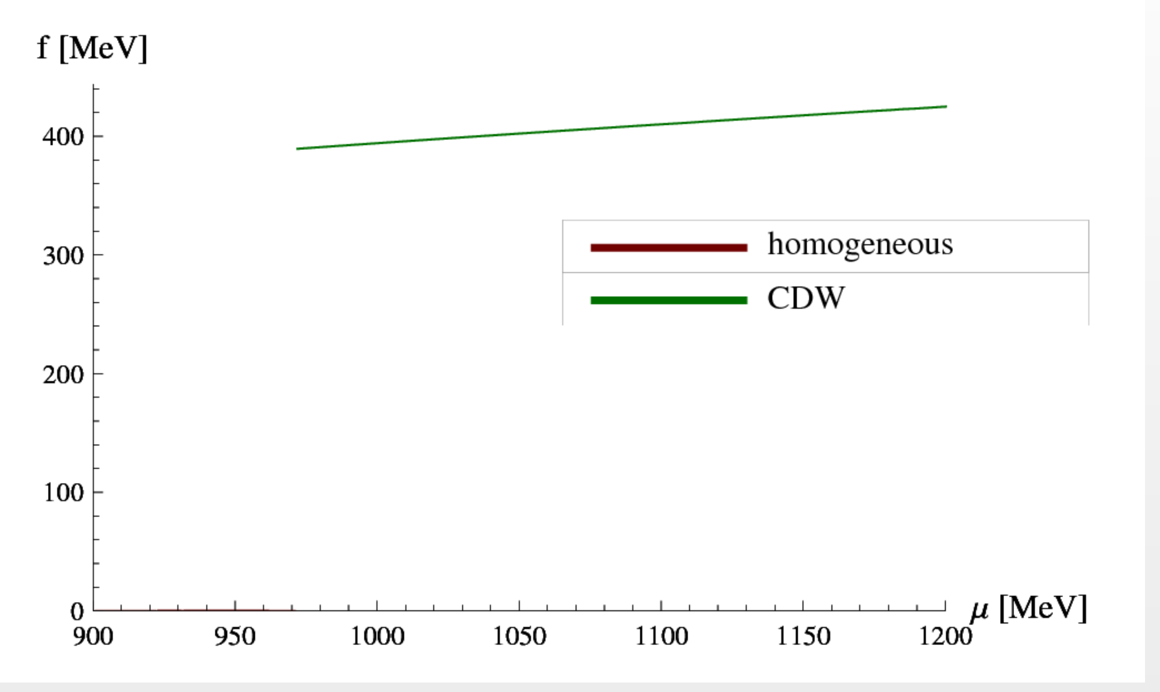
$$m_0 = 500 \text{ MeV}$$

$$\rho_{CDW} / \rho_0 = 2.4$$

Inhomogeneous condensation/3

$$\phi(z) = \varphi \cos(2fz)$$

$$\langle \pi^0 \rangle = \varphi \sin(2fz)/Z$$



$$\rho_{CDW}/\rho_0 = 2.4$$

A. Heinz, F.G., D. H. Rischke, in preparation.

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Summary

Summary

Hadronic Theory (eLSM) for hadrons based on chiral symmetry and dilatation invariance

Important role of (axial-)vector mesons in all phenomenology

Scalar quarkonia and glueball above 1 GeV (effects in the medium)

Nucleon mass contribution which does not stem from the chiral condensate
(but from the tetraquark and glueball condensates)

Ongoing works: N_f =4, additional tetraquark states, weak decays, …

Planned: phase diagram of QCD

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