a) $\vec{\nabla}\cdot\vec{E} = \frac{\varrho}{\varepsilon_0}$,

$\nabla \times \vec{E} = \vec{0}$.

b) $\varrho(\vec{r}) = q_1\,\delta(x)\,\delta(y)\,\delta(z) + q_2\,\delta(x-1)\,\delta(y-1)\,\delta(z-1)$.

c)

$$\varphi_{S(V)}(\vec{E}) \overset{\text{def}}{=} \int_{S(V)} \vec{E}\cdot d\vec{F} \overset{\text{Gauß}}{=} \int_V \vec{\nabla}\cdot\vec{E}\,dV \overset{\text{Mxw}}{=} \int_V \frac{\varrho}{\varepsilon_0}\,dV$$

$$= \frac{q_1}{\varepsilon_0}\int_V dV\,\delta(x)\,\delta(y)\,\delta(z) + \frac{q_2}{\varepsilon_0}\int dV\,\delta(x-1)\,\delta(y-1)\,\delta(z-1) =$$

$$= \frac{q_1+q_2}{\varepsilon_0}.$$

d)

$$C_{K(S)}(\vec{E}) \overset{\text{def}}{=} \oint_{K(S)} \vec{E}\cdot d\vec{\ell} \overset{\text{Stokes}}{=} \int_S \nabla\times\vec{E}\cdot d\vec{F} \overset{\text{Mxw}}{=} 0.$$

a) $\nabla \times \vec{B} = \mu_0 \vec{J}$

$\vec{\nabla} \cdot \vec{B} = 0$.

b) $\varphi_{S(V)}(\vec{B}) \overset{\text{def}}{=} \int_{S(V)} \vec{B} \cdot d\vec{F} \overset{\text{Gauß}}{=} \int \vec{\nabla} \cdot \vec{B} \, dV \overset{\text{Mxw}}{=} 0$.

c) $\oint_{K(S)} \vec{A} \cdot d\vec{s} \overset{\text{stokes}}{=} \int_S \vec{\nabla} \times \vec{A} \cdot d\vec{F} = \int_S \vec{B} \cdot d\vec{F}$

def. von $\vec{A}$ / $\vec{B} = \vec{\nabla} \times \vec{A}$.

d) $\vec{F} = q(\vec{v} \times \vec{B})$ (Lorentz-Kraft)

Die infinitesimale ~~Kraft~~ Arbeit verschwindet:

$dL = \vec{F} \cdot d\vec{r} = q(\vec{v} \times \vec{B}) \cdot d\vec{r} = q\left(\frac{d\vec{r}}{dt} \times \vec{B}\right) \cdot d\vec{r} = 0$

weil $\frac{d\vec{r}}{dt} \parallel d\vec{r}$.

Daraus folgt: $\underbrace{\oint_K dL = 0 \quad \forall K}_{\text{def of a conservative force.}}$ (geschlossene Kurve).

a) $\mathcal{L}_{ED} = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu$

b) $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$;

Gauge: $A^\mu \mapsto A^\mu + \partial^\mu \eta$

$$F^{\mu\nu} \mapsto \partial^\mu (A^\nu + \partial^\nu \eta) - \partial^\nu (A^\mu + \partial^\mu \eta)$$

$$= \underbrace{\partial^\mu A^\nu - \partial^\nu A^\mu}_{F^{\mu\nu}} + \underbrace{\partial^\mu \partial^\nu \eta - \partial^\nu \partial^\mu \eta}_{=0} = F^{\mu\nu}$$

$F^{\mu\nu}$ ist invariant.

c) $\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$

$F_{\alpha\beta}$ ist eichinvariant (siehe b) $\rightarrow$ $\tilde{F}^{\mu\nu}$ ist auch eichinvariant.

d) $S_{ED} = \int d^4x \, \mathcal{L}_{ED}$

$$\mathcal{L}_{ED} \xmapsto{A_\mu \mapsto A_\mu + \partial_\mu \eta} \mathcal{L}_{ED} - J_\mu \partial^\mu \eta$$

Ergo:

$$\int d^4x \, \mathcal{L}_{ED} = S_{ED} \longmapsto \int d^4x \, \mathcal{L}_{ED} - \int d^4x \, J_\mu \partial^\mu \eta$$

$$= S_{ED} - \int d^4x \, J_\mu \partial^\mu \eta .$$

$$\left[\int d^4x \, J_\mu \partial^\mu \eta = \int d^4x \left[\partial_\mu (J^\mu \eta) - \underbrace{\partial_\mu J^\mu \cdot \eta}_{=0, \text{ weil: } \partial_\mu J^\mu = 0.} \right] \right.$$

~~partielle~~

$$= \int d^4x \, \partial_\mu (J^\mu \eta) \overset{\text{Gaußsatz}}{=} \int_{S \to \infty} d\sigma_\mu \, J^\mu \eta = 0$$

da die Felder für $|\vec{r}| \to \infty$ verschwinden.

Letztendlich:

$S_{ED} \longmapsto S_{ED}$: EICHINVARIANT.

e) $$\mathcal{L}_{mass} = m^2 A_\mu A^\mu \xrightarrow{\text{Eichtransf.}} m^2 A_\mu A^\mu + m^2 \partial_\mu \eta A^\mu + m^2 A_\mu \partial^\mu \eta + m^2 \partial_\mu \eta \partial^\mu \eta .$$

DAS IST <u>NICHT</u> EICHINVARIANT.