Pion DA and $\gamma\gamma^* \longrightarrow \pi$ *FF in QCD*

A. Bakulev, S. Mikhailov, N. Stefanis

Bogolyubov Lab. Theor. Phys., JINR (Dubna, Russia)

Contents

- **Pion Distribution Amplitude** in QCD

Contents

- **Pion Distribution Amplitude** in QCD
- **QCD SRs** with **Nonlocal Condensates** for Pion DA

Contents

- **Pion Distribution Amplitude** in QCD
- **QCD SRs** with **Nonlocal Condensates** for Pion DA
- **Comparison** with **CLEO** Data on $F_{\gamma^*\gamma\to\pi}(Q^2)$

Contents

- **Pion Distribution Amplitude** in QCD
- **QCD SRs** with **Nonlocal Condensates** for Pion DA
- **Comparison** with **CLEO** Data on $F_{\gamma^*\gamma\to\pi}(Q^2)$
- **Comparison** with **Lattice** Data on Pion DA

Contents

- **Pion Distribution Amplitude** in QCD
- **QCD SRs** with **Nonlocal Condensates** for Pion DA
- **Comparison** with **CLEO** Data on $F_{\gamma^*\gamma\to\pi}(Q^2)$
- **Comparison** with **Lattice** Data on Pion DA
- **Comparison** with **JLab** Data on $F_\pi(Q^2)$

Contents

- **Pion Distribution Amplitude** in QCD
- **QCD SRs** with **Nonlocal Condensates** for Pion DA
- **Comparison** with **CLEO** Data on $F_{\gamma^*\gamma\to\pi}(Q^2)$
- **Comparison** with **Lattice** Data on Pion DA
- **Comparison** with **JLab** Data on $F_{\pi}(Q^2)$
- **Pre-Conclusions**

Contents

Contents

- **Pion Distribution Amplitude** in QCD
- **QCD SRs** with **Nonlocal Condensates** for Pion DA
- **Comparison** with **CLEO** Data on $F_{\gamma^*\gamma\to\pi}(Q^2)$
- **Comparison** with **Lattice** Data on Pion DA
- **Comparison** with **JLab** Data on $F_\pi(Q^2)$
- **Pre-Conclusions**
- **BaBar** Data on $F_{\gamma^*\gamma\to\pi}(Q^2)$: **Challenge** for QCD
- **Preliminary conclusions about BaBar data**

Publications

A. B., S. M., N. S.	PLB	508	(2001)	279
A. B., S. M., N. S.	PRD	67	(2003)	074012
A. B., S. M., N. S.	PLB	578	(2004)	91
A. B., N. S. *et al.*	PRD	70	(2004)	033014
A. B., N. S.	NPB	721	(2005)	50
A. B., S. M., N. S.	PRD	73	(2006)	056002
A. B., A. P.	APPB	37	(2006)	3627
A. B., O. T., N. S.	PRD	76	(2007)	074032
A. B., A. P., N. S.	PRD	79	(2009)	093010
S. M., N. S.	NPB	821	(2009)	291
S. M., N. S.	MPLA	24	(2009)	2858

QCD SRs for π Distribution Amplitude

Pion distribution amplitude (DA)

- Matrix element of nonlocal axial current on light cone

$$\langle 0 \mid \bar{d}(z)\gamma_\mu\gamma_5 E(z,0) u(0) \mid \pi(P)\rangle\Big|_{z^2=0} =$$

$$i f_\pi P_\mu \int_0^1 dx\, e^{ix(zP)}\, \varphi_\pi^{\text{Tw-2}}(x,\mu^2)$$

- **gauge-invariance** due to Fock–Schwinger string:

$$E(z,0) = \mathcal{P} e^{ig\int_0^z A_\mu(\tau) d\tau^\mu}$$

- Physical meaning of $\varphi_\pi(x;\mu^2)$ — amplitude for transition $\pi \longrightarrow u + d$

Representation of Pion DA

- It is convenient to represent the pion DA:
$\varphi_\pi(x;\mu^2) = \varphi^{\text{As}}(x)\times$
$\times\left[1 + a_2(\mu^2)C_2^{3/2}(2x-1) + a_4(\mu^2)C_4^{3/2}(2x-1) + \ldots\right]$
where $C_n^{3/2}(2x-1)$ are the **Gegenbauer** polynomials (1-loop eigenfunctions of ER-BL kernel)

Representation of Pion DA

- It is convenient to represent the pion DA:
  $\varphi_\pi(x;\mu^2) = \varphi^{\text{As}}(x)\times$
  $\times\left[1 + a_2(\mu^2)C_2^{3/2}(2x-1) + a_4(\mu^2)C_4^{3/2}(2x-1) + \ldots\right]$
  where $C_n^{3/2}(2x-1)$ are the **Gegenbauer** polynomials (1-loop eigenfunctions of ER-BL kernel)
- That means

$$\left\{a_2(\mu^2), a_4(\mu^2), \ldots\right\} \Leftrightarrow \varphi_\pi(x;\mu^2)$$

Representation of Pion DA

- It is convenient to represent the pion DA:
$$\varphi_\pi(x;\mu^2) = \varphi^{\text{As}}(x)\times$$
$$\times\left[1+a_2(\mu^2)C_2^{3/2}(2x-1)+a_4(\mu^2)C_4^{3/2}(2x-1)+\ldots\right]$$
where $C_n^{3/2}(2x-1)$ are the **Gegenbauer** polynomials (1-loop eigenfunctions of ER-BL kernel)
- That means
$$\left\{a_2(\mu^2), a_4(\mu^2), \ldots\right\} \Leftrightarrow \varphi_\pi(x;\mu^2)$$
- ER-BL solution at 2-loop level $\left[\begin{array}{l}\textbf{Mikhailov\&Radyushkin; 1986}\\ \textbf{Müller; 1994–95}\\ \textbf{A.B.\&Stefanis; 2005}\end{array}\right]$

Non-Local Condensates in QCD SR

- Illustration of
  NLC-model: $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$

Non-Local Condensates in QCD SR

- Illustration of
  NLC-model: $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$
- A **single scale** parameter $\lambda_q^2 = \langle k^2 \rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1\ \textbf{GeV}^2 & [\ \textbf{QCD SRs, 1987}\ ] \\ 0.5 \pm 0.05\ \textbf{GeV}^2 & [\ \textbf{QCD SRs, 1991}\ ] \\ 0.4 - 0.5\ \textbf{GeV}^2 & [\ \textbf{Lattice, 1998-2002}\ ] \end{cases}$$

Non-Local Condensates in QCD SR

- Illustration of
  NLC-model: $\langle \bar{q}(0) q(z) \rangle = \langle \bar{q}(0) q(0) \rangle e^{-|z^2|\lambda_q^2/8}$
- A **single scale** parameter $\lambda_q^2 = \langle k^2 \rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1\ \textbf{GeV}^2 & [\ \textbf{QCD SRs, 1987}\ ] \\ 0.5 \pm 0.05\ \textbf{GeV}^2 & [\ \textbf{QCD SRs, 1991}\ ] \\ 0.4 - 0.5\ \textbf{GeV}^2 & [\ \textbf{Lattice, 1998-2002}\ ] \end{cases}$$

- Correlation length $\lambda_q^{-1} \sim \rho$-meson size

Non-Local Condensates in QCD SR

- Illustration of
  NLC-model: $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$
- A **single scale** parameter $\lambda_q^2 = \langle k^2\rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1\ \textbf{GeV}^2 & [\ \textbf{QCD SRs, 1987}\ ] \\ 0.5 \pm 0.05\ \textbf{GeV}^2 & [\ \textbf{QCD SRs, 1991}\ ] \\ 0.4 - 0.5\ \textbf{GeV}^2 & [\ \textbf{Lattice, 1998-2002}\ ] \end{cases}$$

- Correlation length $\lambda_q^{-1} \sim \rho$-meson size
- Possible to include second ($\Lambda \simeq 450$ **MeV**) scale with $\langle\bar{q}(0)q(z)\rangle\Big|_{|z|\gg 1\ \textbf{Fm}} \sim \langle\bar{q}q\rangle e^{-|z|\Lambda}$ (not included here)

Axial-axial correlator

We study correlator:

$$\Pi^N_{\mu\nu} = i \int d^4x \, e^{iqx} \langle 0 | T \left[J^N_{\mu 5}(0) J^+_{\nu 5}(x) \right] | 0 \rangle$$

of two axial currents

$$J^N_{\mu 5}(0) = \bar{d}(0) \gamma_\mu \gamma_5 \left[-in\nabla \right]^N u(0) \, ; \;\; J^+_{\nu 5}(x) = \bar{u}(x) \gamma_\nu \gamma_5 d(x)$$

corresponding to charged π-meson. Current $J^N_{\mu 5}(0)$ produces

$$\langle 0 \mid J^N_{\mu 5}(0) \mid \pi(P) \rangle = i f_\pi P_\mu \, (nP)^N \int_0^1 dx \, x^N \, \varphi_\pi(x)$$

Axial-axial correlator

We study correlator:

$$\Pi^N_{\mu\nu} = i \int d^4x\, e^{iqx} \langle 0 | T \left[J^N_{\mu 5}(0) J^+_{\nu 5}(x) \right] | 0 \rangle$$

of two axial currents

$$J^N_{\mu 5}(0) = \bar{d}(0)\gamma_\mu \gamma_5 \left[-in\nabla \right]^N u(0)\,;\;\; J^+_{\nu 5}(x) = \bar{u}(x)\gamma_\nu \gamma_5 d(x)$$

corresponding to charged π-meson. Current $J^N_{\mu 5}(0)$ produces

$$\langle 0 \mid J^N_{\mu 5}(0) \mid \pi(P) \rangle = i f_\pi P_\mu \, (nP)^N \, \langle x^N \rangle_\pi$$

NLC QCD SR for Pion DA

Here is example of QCD SR with Non-Local Condensates

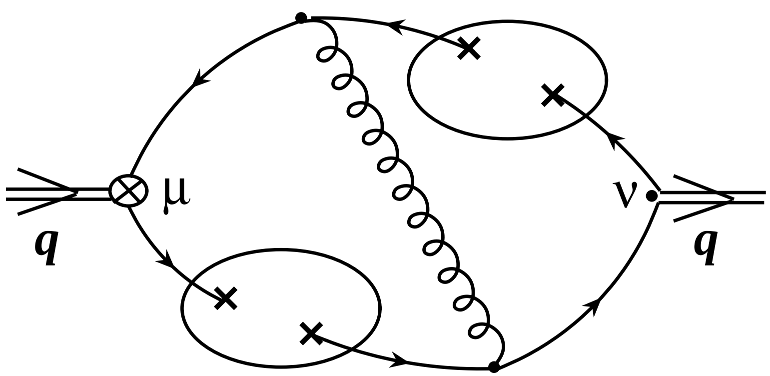
$$f_\pi^2\,\varphi_\pi(x) = \int_0^{s_0} \rho^{\text{pert}}(x;s)\,e^{-s/M^2}ds + \frac{\alpha_s\langle GG\rangle}{24\pi M^2}\varphi_G(x;\Delta) + \frac{16\pi\alpha_s\langle\bar{q}q\rangle^2}{81M^4}\sum_{i=2V,3L,4Q}\varphi_i(x;\Delta)$$

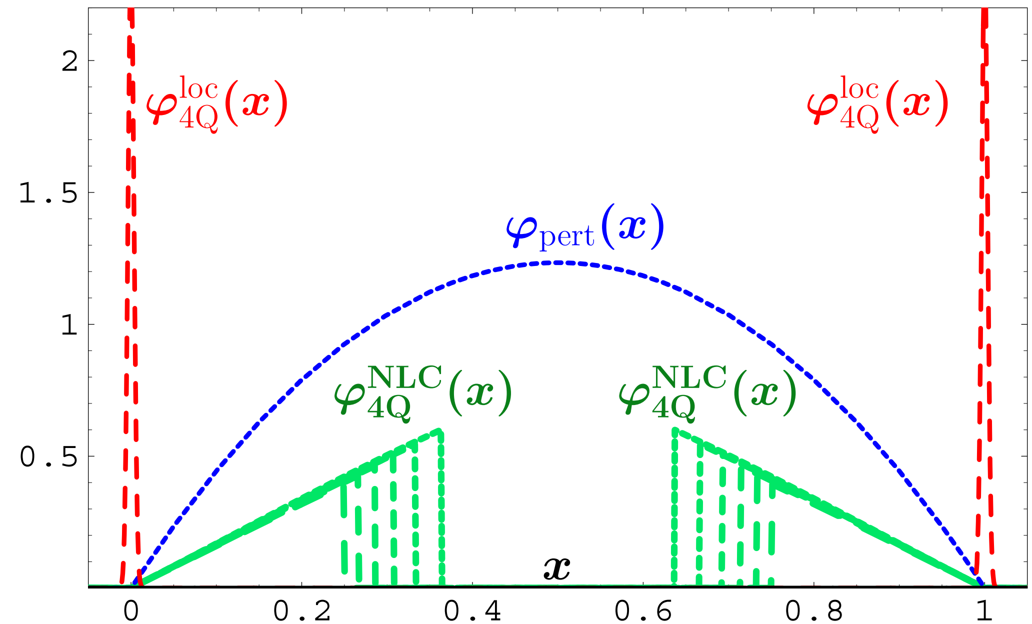
Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\begin{aligned}
\varphi_G(x;\Delta=0) &= [\delta(x)+\delta(1-x)] \\
\varphi_{2V}(x;\Delta=0) &= [x\delta'(1-x)+(1-x)\delta'(x)] \\
\varphi_{4Q}(x;\Delta=0) &= 9[\delta(x)+\delta(1-x)]
\end{aligned}$$

NLC contributions to QCD SR

Examples for Gaussian NLC with a single parameter λ_q^2



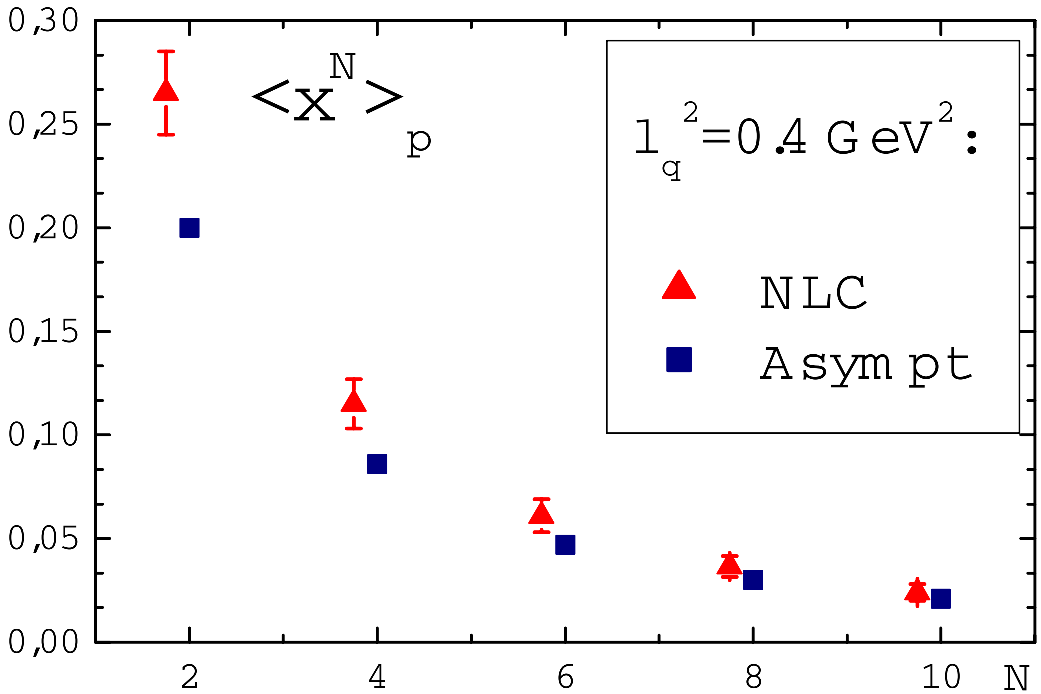


Local limit: $\lambda_q^2/M^2 \equiv \Delta \to 0$,

$$\varphi_{4Q}^{\text{loc}}(x) \equiv \lim_{\Delta\to 0} \varphi_{4Q}^{\text{NLC}}(x;\Delta) = 9[\delta(x) + \delta(1-x)]$$

NLC SRs for pion DA

Moments $\langle\xi^N\rangle_\pi = \int_0^1 \varphi_\pi(x)\,(2x-1)^N\,dx$ at $\mu^2 \approx 1$ **GeV^2**



from NLC SRs

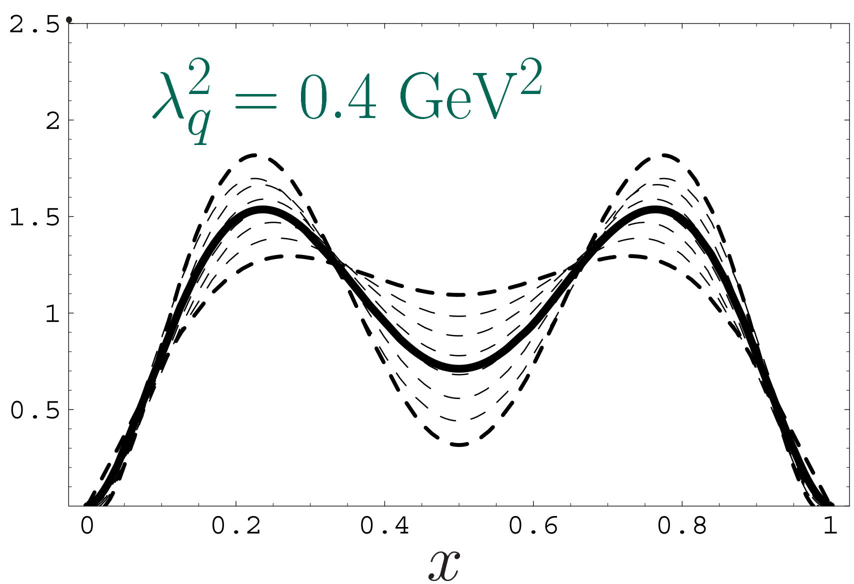
▲ **PLB 508 (2001) 279**

These $\langle\xi^N\rangle_\pi$ values allow one to restore DA $\varphi_\pi(x)$

NLC SRs for Pion DA

produce**bunch** of self-consistent 2-parameter models $\varphi_\pi(x)$ at $\mu^2 \simeq 1$ **GeV**^{2}:

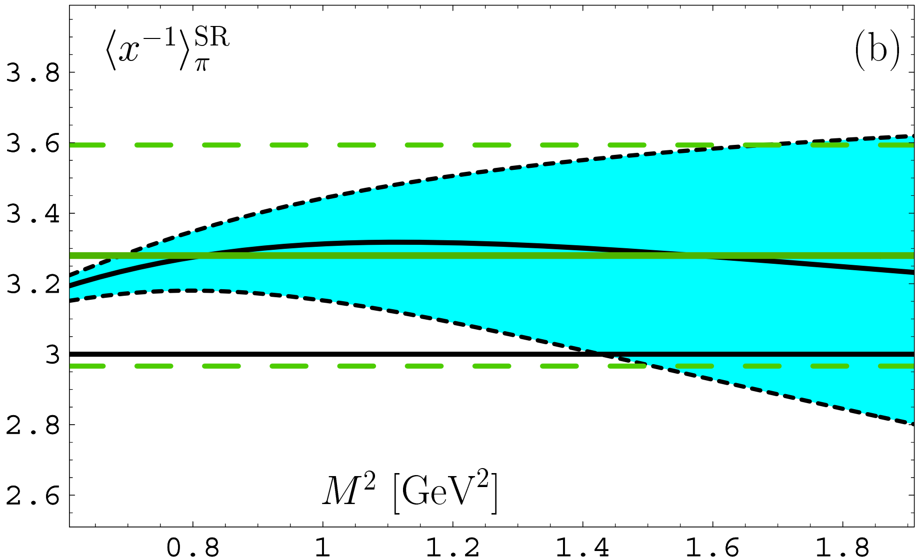
$$\varphi_\pi(x) = \varphi^{\text{As}}(x)\left[1 + a_2\, C_2^{3/2}(2x-1) + a_4\, C_4^{3/2}(2x-1)\right]$$



$$\begin{array}{rcl} a_2^{\text{b.f.}} & = & +0.188 \\ a_4^{\text{b.f.}} & = & -0.130 \\ \chi^2 & \approx & 0.001 \\ \langle x^{-1}\rangle^{\text{SR}} & = & 3.30(30) \end{array}$$

NLC SR estimate of $\langle x^{-1}\rangle_{\pi}^{SR}$

BMS [PLB (2001)]: at $\mu^2 \simeq 1$ **GeV**^{2}



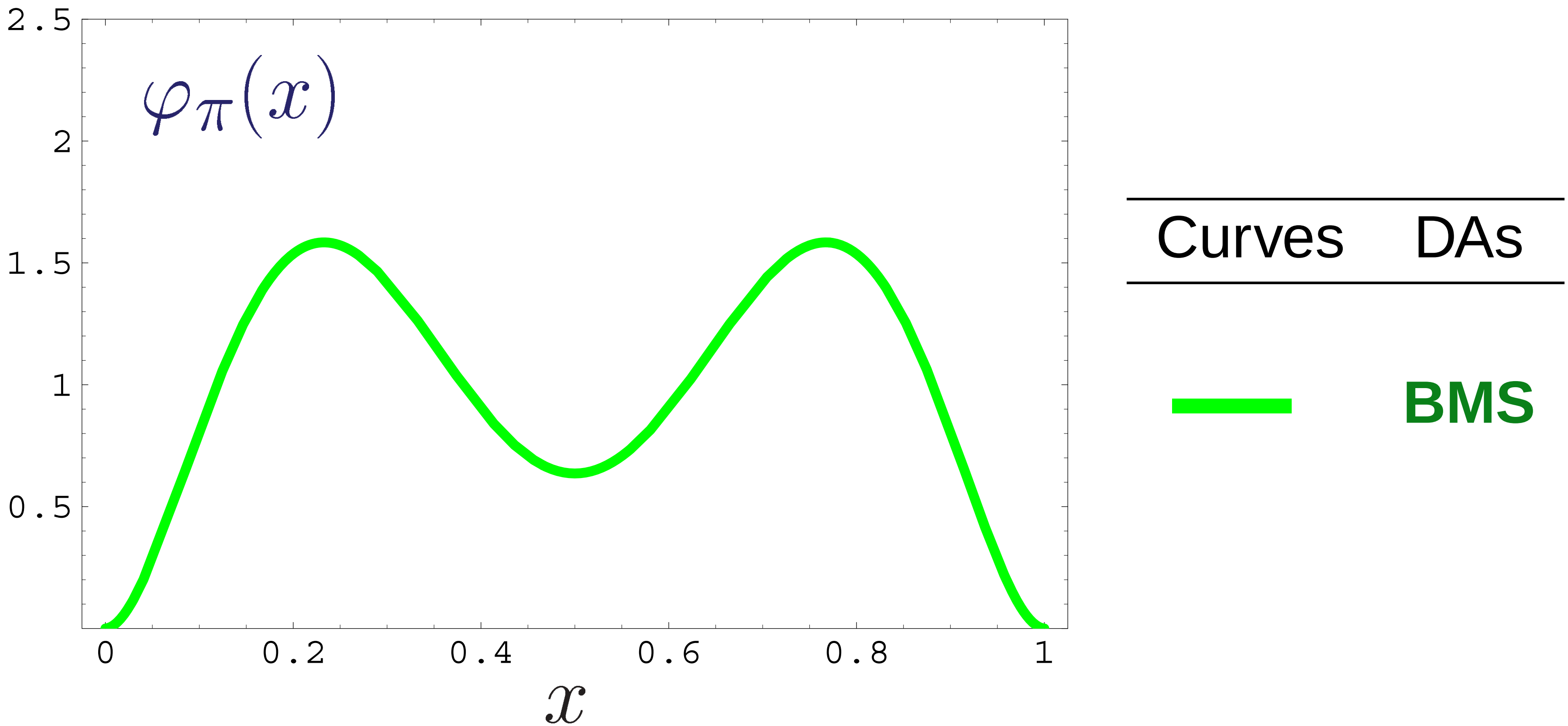
$\lambda_q^2 = 0.4$ **GeV**^{2},

$\langle x^{-1}\rangle_{\pi}^{\textbf{SR}} = 3.3 \pm 0.3$,

$\langle x^{-1}\rangle_{\pi}^{\textbf{b.f.}} = 3.17$

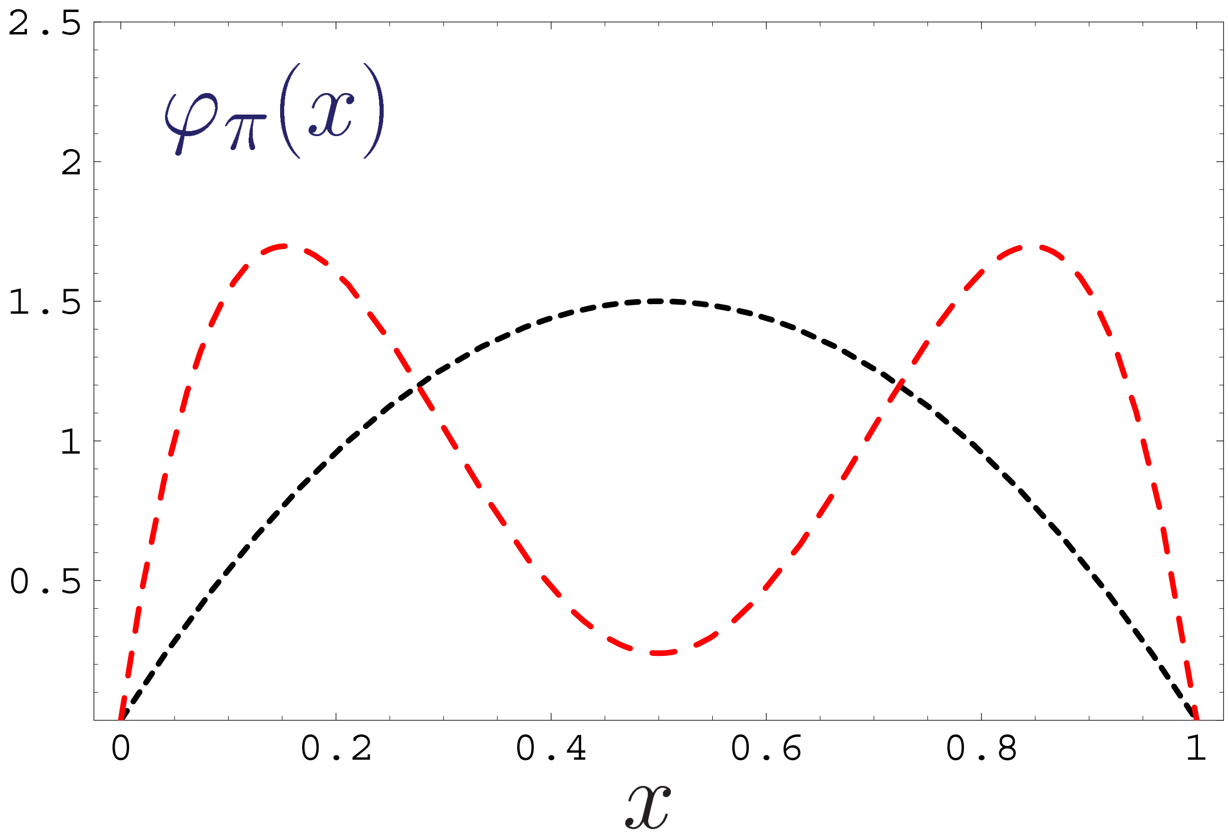
The moment $\langle x^{-1}\rangle_{\pi}^{\textbf{SR}}$ could be determined **only in NLC SRs** because end-point singularities absent ☛

BMS vs CZ distribution amplitude



BMS DA is end-point suppressed!

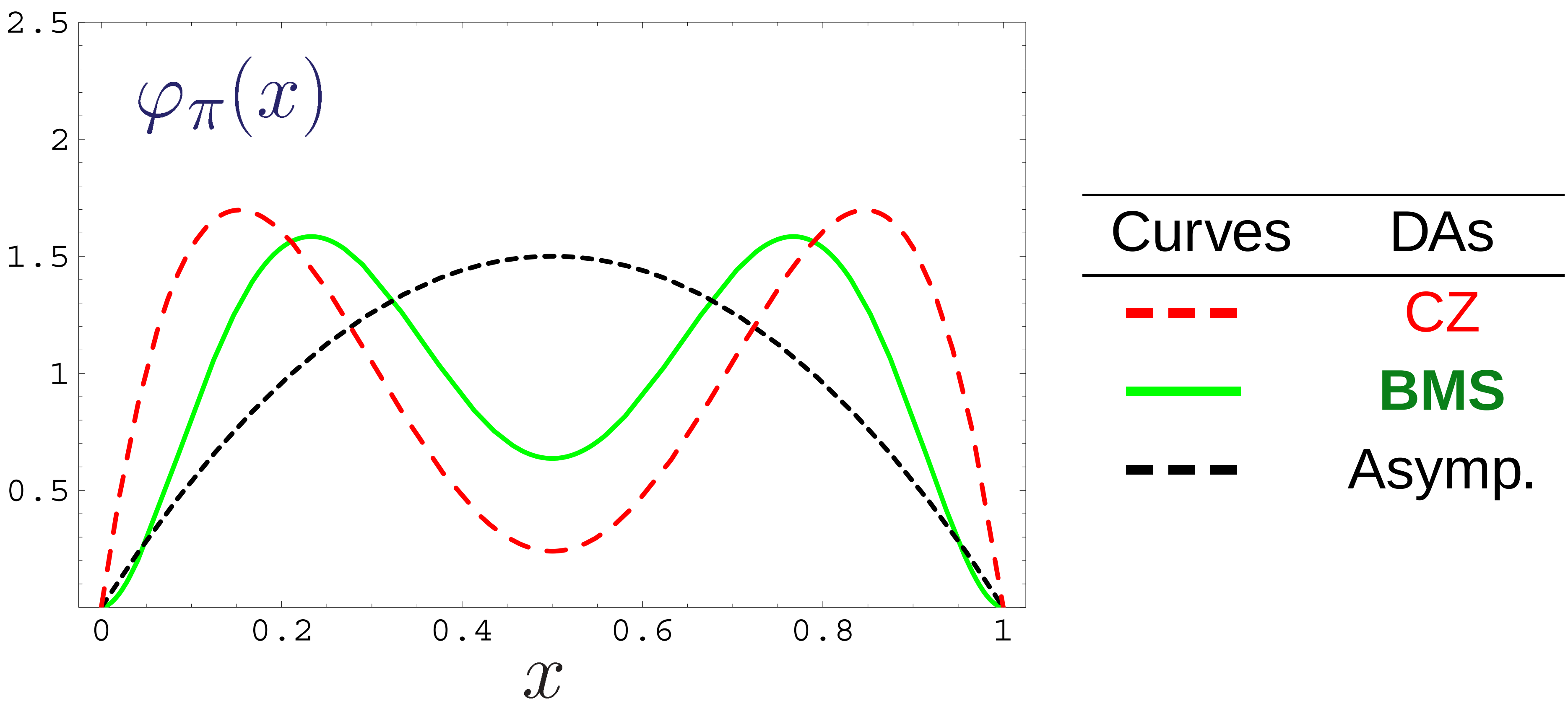
BMS vs CZ distribution amplitude



Curves	DAs
red dashed	CZ
black dashed	Asymp.

CZ DA: end-point enhancement

BMS vs CZ distribution amplitude

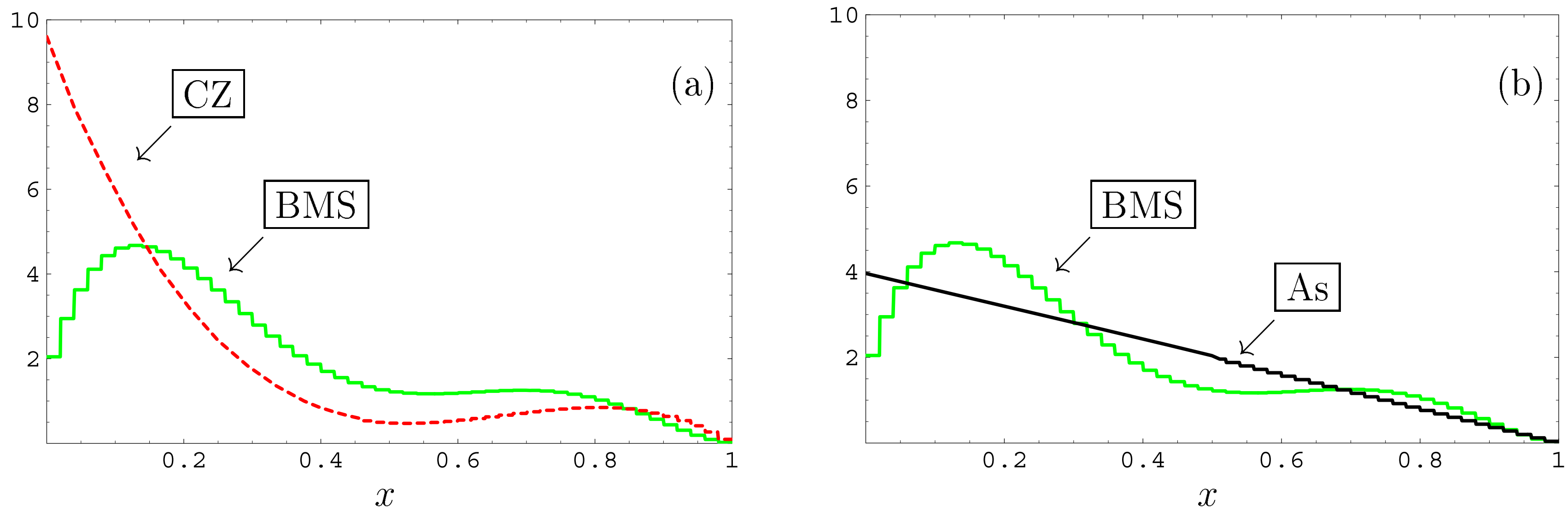


BMS bunch is 2-humped, but end-point suppressed!

Histograms for inverse moment $\langle x^{-1} \rangle_\pi$

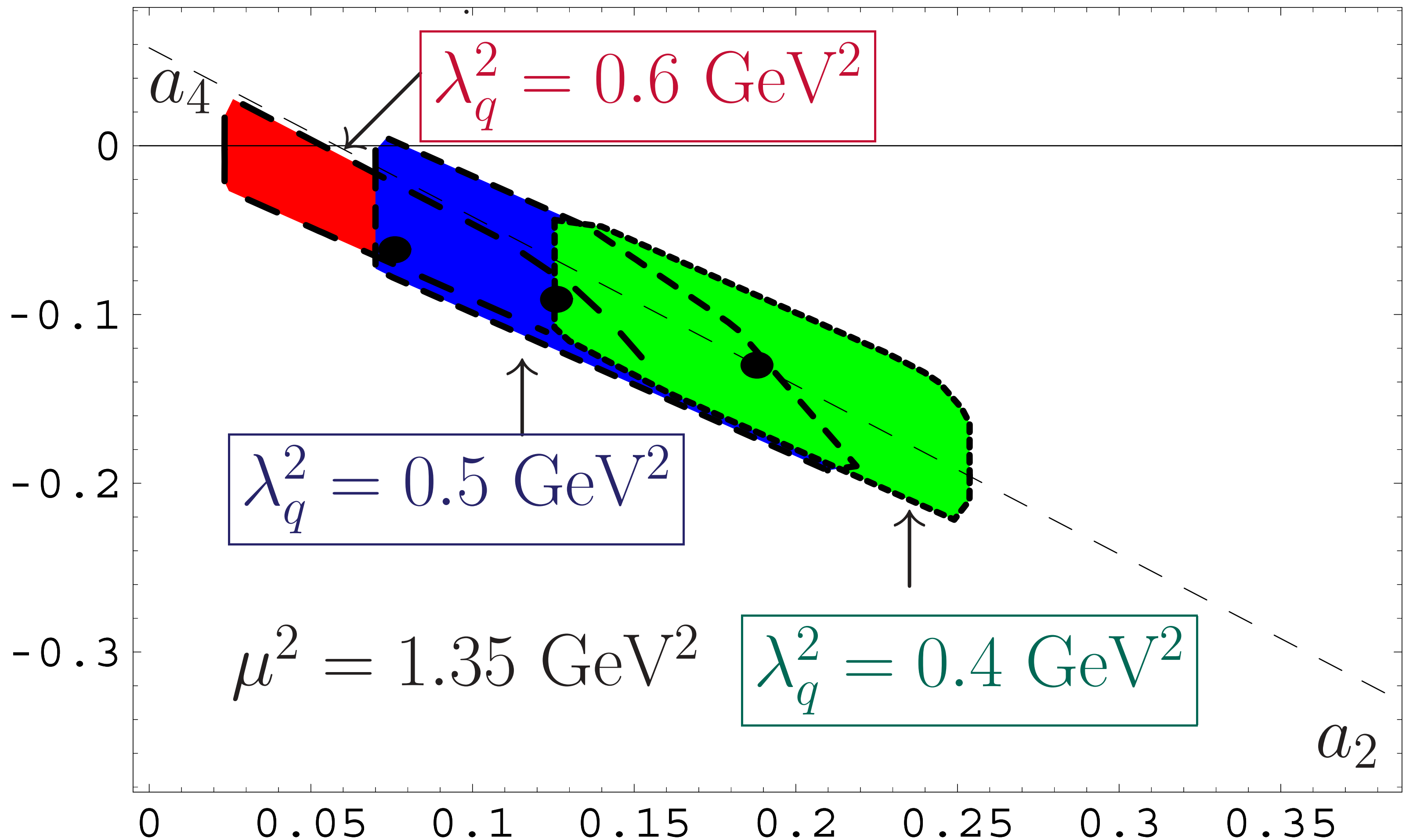
Contributions of different DAs to inverse moment $\langle x^{-1} \rangle_\pi$, calculated as $\int_x^{x+0.02} \phi(x)dx$ and normalized to 100%, for:

(a) **CZ** and**BMS** DAs; (b) Asympt. and**BMS** DAs.



In**BMS** case region $x \leq 0.1$ contributes even less than in Asymptotic DA case.

NLC SR Constraints on a_2, a_4 of Pion DA



Estimated **bunches of pion DAs** for different values of λ_q^2.

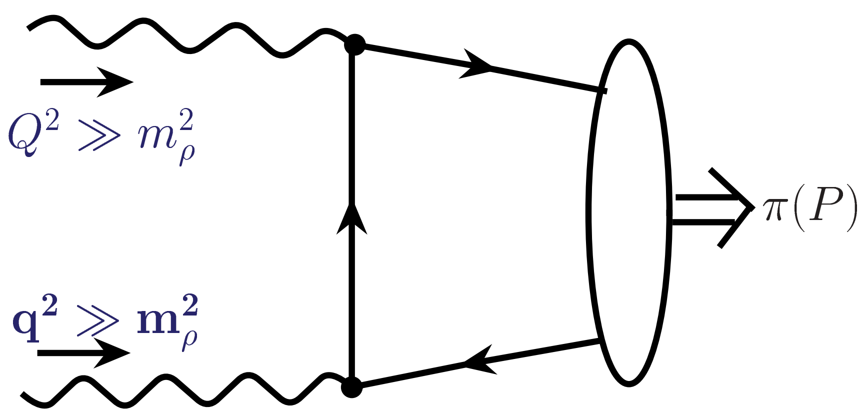
NLO Light-Cone SRs $\Rightarrow$

CLEO data on $F_{\gamma\gamma^*\pi}(Q^2) \Rightarrow$

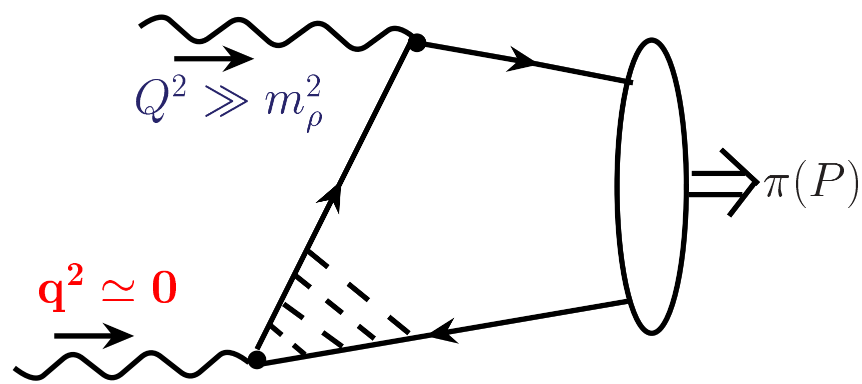
Constraints on Pion DA

$\gamma^*\gamma \to \pi$: *Why Light-Cone Sum Rules?*

For $Q^2 \gg m_\rho^2, \;\; q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are of importance**[Radyushkin–Ruskov, NPB (1996)]**.
Reason: if $q^2 \to 0$ one needs to take into account interaction of real photon at long distances $\sim O(1/\sqrt{q^2})$



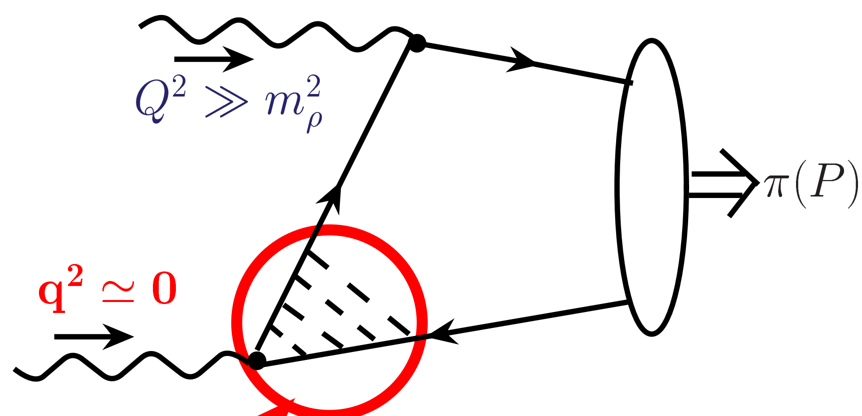
pQCD is OK



LCSR should be applied

$\gamma^*\gamma \to \pi$: Why Light-Cone Sum Rules?

For $Q^2 \gg m_\rho^2,\ q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are of importance**[Radyushkin–Ruskov, NPB (1996)]**.
Reason: if $q^2 \to 0$ one needs to take into account interaction of real photon at long distances $\sim O(1/\sqrt{q^2})$



To account for long-distance effects in pQCD one needs to introduce light-cone DA of real photon

$\gamma^*\gamma \to \pi$: Light-Cone Sum Rules!

Khodjamirian [EJPC (1999)]: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) = \frac{1}{\pi}\int_0^{s_0} \frac{\mathrm{Im}F^{\mathrm{PT}}_{\gamma^*\gamma^*\pi}(Q^2, s)}{m_\rho^2 + q^2} e^{(m_\rho^2 - s)/M^2} ds + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\mathrm{Im}F^{\mathrm{PT}}_{\gamma^*\gamma^*\pi}(Q^2, s)}{s + q^2} ds$$

$s_0 \simeq 1.5$ GeV^2 – effective threshold in vector channel,
M^2 – Borel parameter ($0.5 - 0.9$ GeV^2).

Real-photon limit $q^2 \to 0$ can be easily done ...

$\gamma^*\gamma \to \pi$: *Light-Cone Sum Rules!*

Khodjamirian **[EJPC (1999)]**: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2

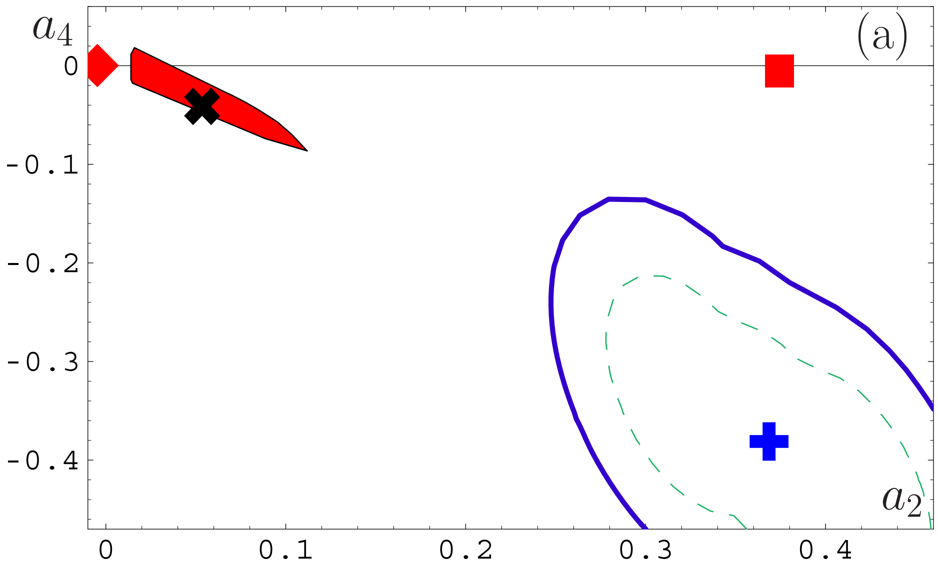
$$F_{\gamma\gamma^*\pi}(Q^2,0) = \frac{1}{\pi}\int_0^{s_0} \frac{\text{Im}F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2,s)}{m_\rho^2} e^{(m_\rho^2-s)/M^2} ds + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\text{Im}F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2,s)}{s} ds$$

$s_0 \simeq 1.5$ **GeV^2** – effective threshold in vector channel,
M^2 – Borel parameter ($0.5 - 0.9$ GeV^2).

... as demonstrated here.

NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]



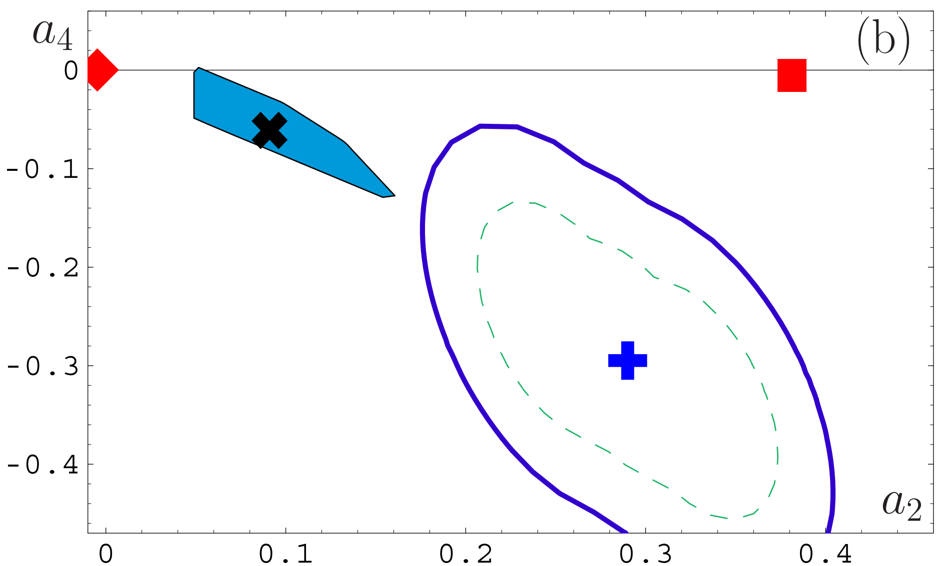
$\Leftrightarrow \lambda_q^2 = 0.6$ **GeV**^{2},

$\delta^2_{\text{Tw-4}} = 0.28(3)$ **GeV**^{2}

No agreement with CLEO data for $\lambda_q^2 = 0.6$ GeV^2

NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]

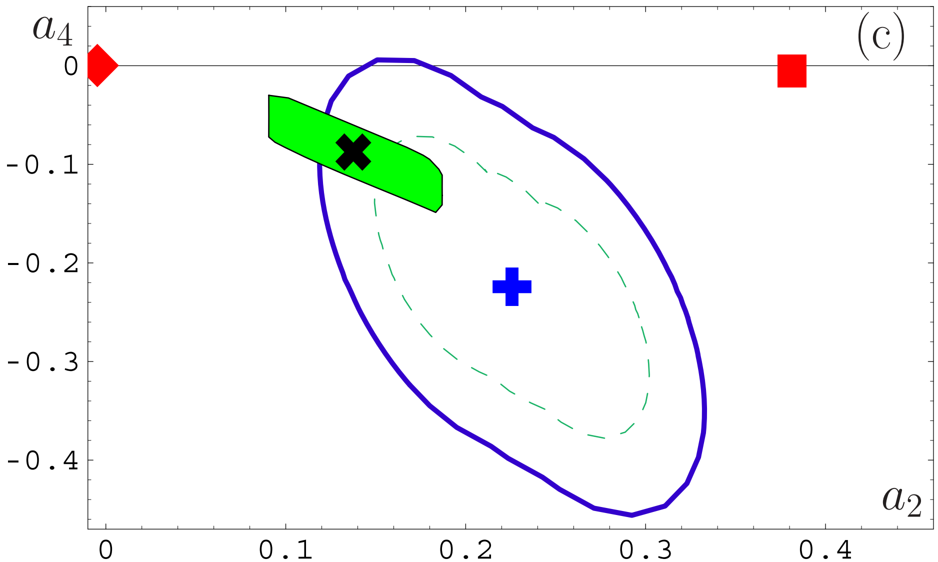


$\Leftrightarrow \lambda_q^2 = 0.5$ **GeV**^{2},
$\delta^2_{\text{Tw-4}} = 0.23(2)$ **GeV**^{2}

Bad agreement with CLEO data for $\lambda_q^2 = 0.5$ GeV^2

NLC SR Results vs NLO CLEO Constraints

[BMS, PRD 67 (2003) 074012]



$\Leftrightarrow \lambda_q^2 = 0.4$ **GeV^2**,
$\delta^2_{\text{Tw-4}} = 0.19(2)$ **GeV^2**

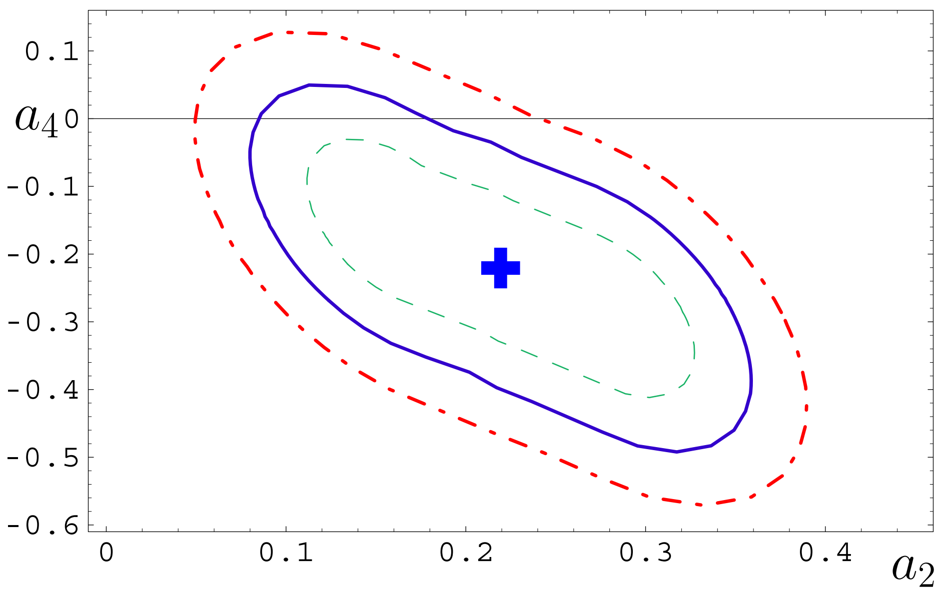
Good agreement with CLEO data for $\lambda_q^2 = 0.4$ GeV^2

NLC SRs vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus(\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV**^{2}

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV**^{2}



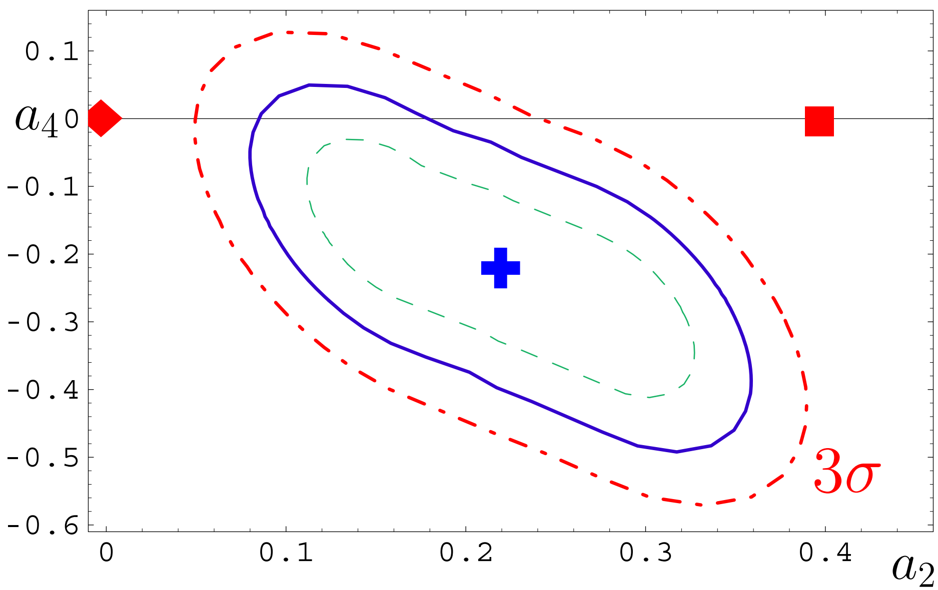
✚ = best-fit point

NLC SRs vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus(\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV**^{2}

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV**^{2}



✚ = best-fit point

◆ = Asymptotic DA

■ = CZ DA

Even with 20% uncertainty in twist-4

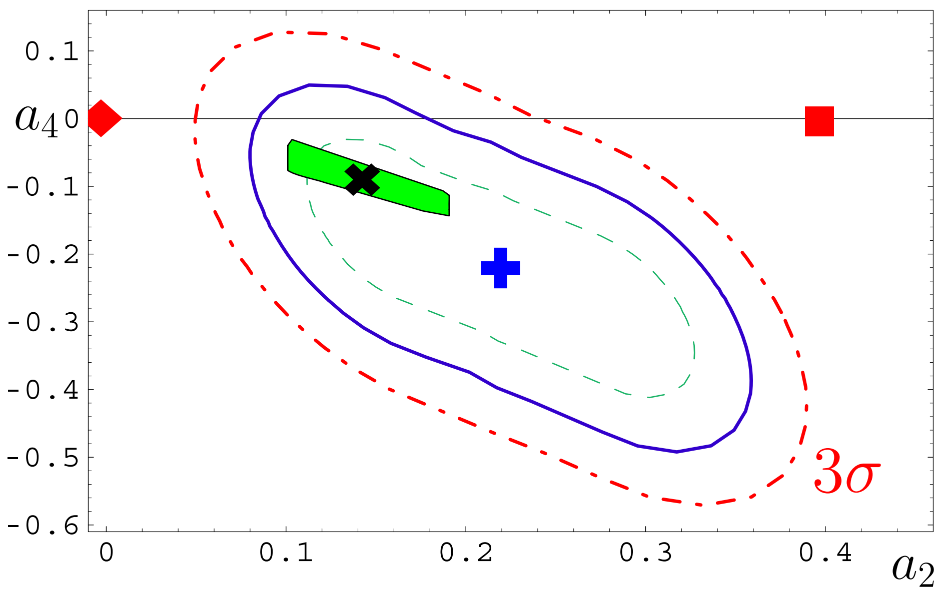
CZ DA excluded **at least** at 4σ-level! **As** DA — at 3σ-level.

NLC SRs vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus(\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV**^{2}

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV**^{2}

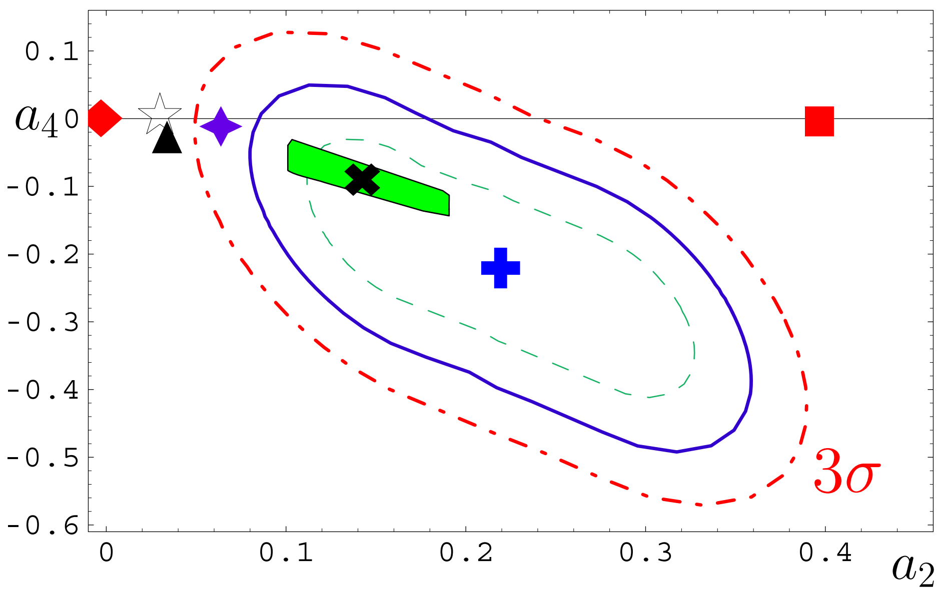


✚ = best-fit point
◆ = Asymptotic DA
■ = CZ DA
✖ = BMS model

CZ DA excluded **at least** at 4σ-level! **As** DA — at 3σ-level.
BMS DA and most of **BMS bunch** — inside 1σ-domain.

NLC SRs vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus(\mu^2 = Q^2)$
with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV^2**
BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV^2**



- ✚ = best-fit point
- ◆ = Asymptotic DA
- ■ = CZ DA
- ✖ = BMS model
- ☆, ▲ and ✦ = instantons

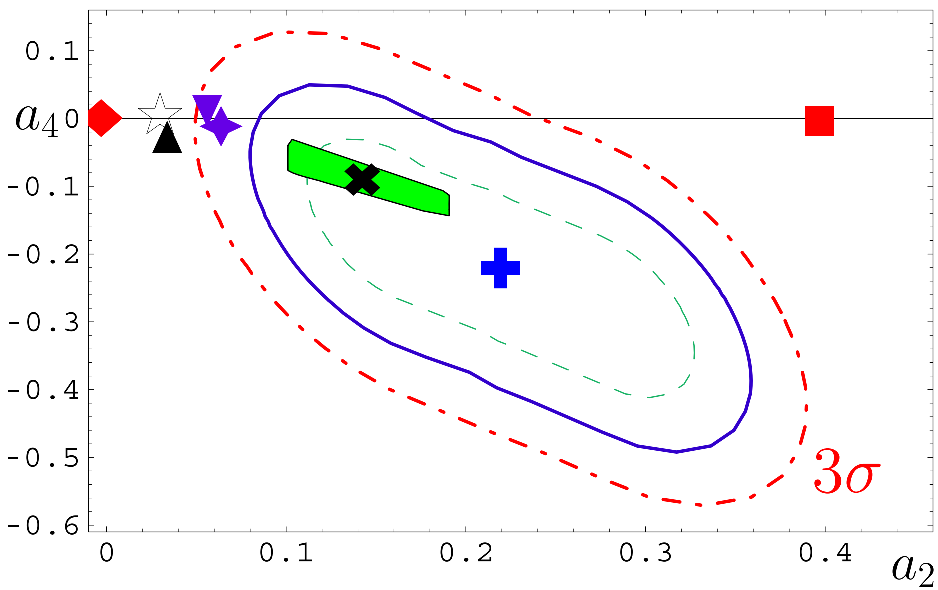
BMS DA and most of **BMS bunch** — inside 1σ-domain.
Instanton-based models — near 3σ-boundary
(**PR**-model is close to 2σ-boundary).

NLC SRs vs Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus(\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV^2**

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV^2**



- ✚ = best-fit point
- ◆ = Asymptotic DA
- ■ = CZ DA
- ✖ = BMS model
- ☆, ▲ and ✦ = instantons
- ▼ = transverse lattice

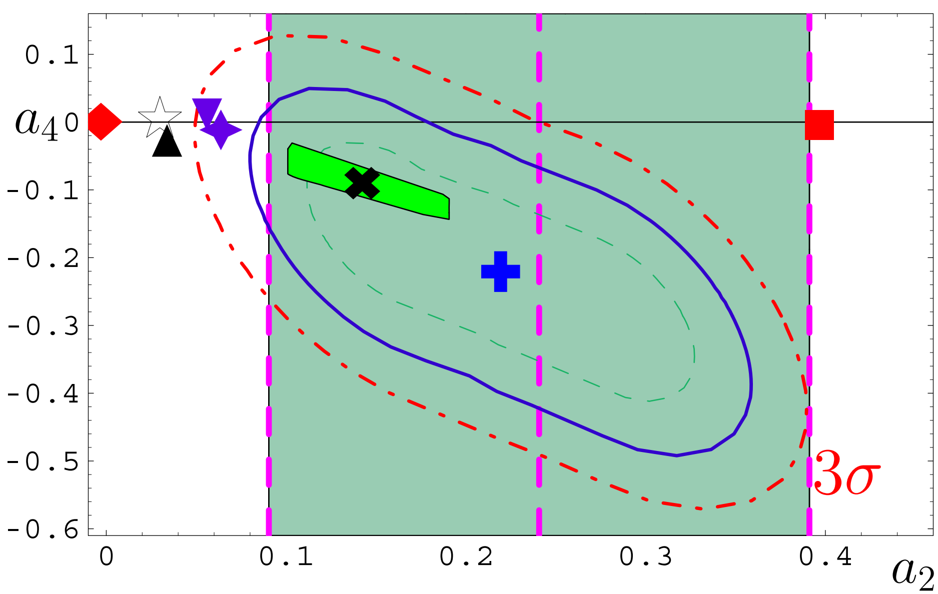
BMS DA and most of **BMS bunch** — inside 1σ-domain.
Transverse lattice model — near 3σ-boundary.

Lattice Data vs. Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus(\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV^2**

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV^2**



- ✚ = best-fit point
- ◆ = Asymptotic DA
- ■ = CZ DA
- ✖ = BMS model
- ☆, ▲ and ✦ = instantons
- ▼ = transverse lattice
- **gray strip** = lattice'04 result

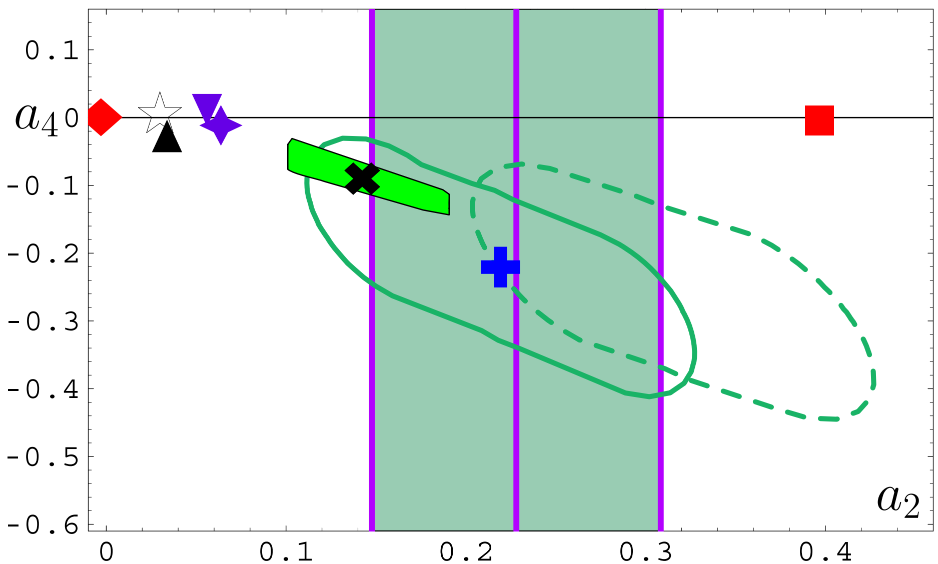
BMS DA and most of **BMS bunch** — inside 1σ-domain and $1/2$ inside **2004 lattice strip [FBSyst. 36 (2005) 77]**.

Lattice Data vs. Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus (\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV**^{2}

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV**^{2}



✚ = best-fit point
◆ = Asymptotic DA
■ = CZ DA
✖ = BMS model
☆, ▲ and ✦ = instantons
▼ = transverse lattice
gray strip = lattice'06 result

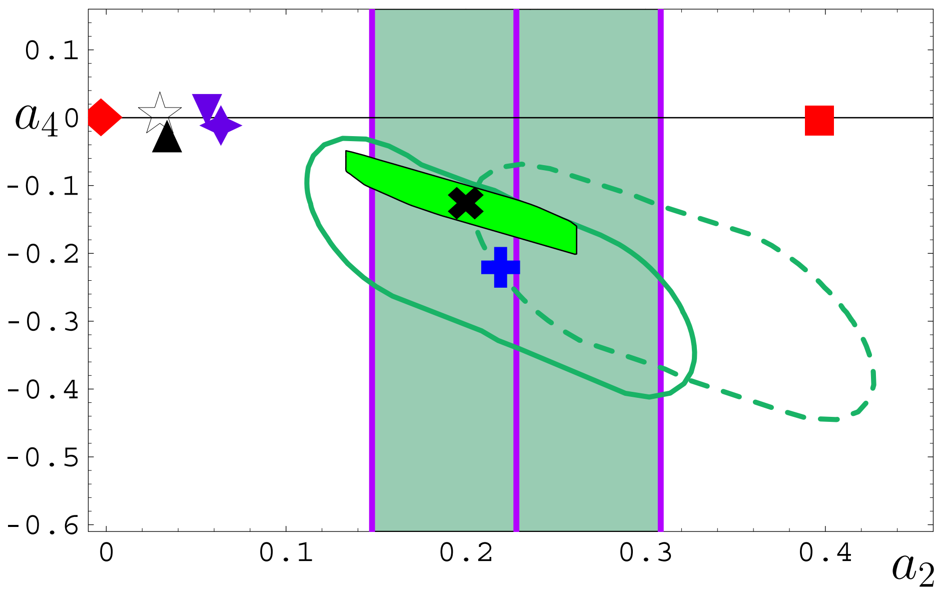
BMS DA and most of **BMS bunch** — inside 1σ-domain and $1/2$ inside **lattice strip**. **Dashed contour** = renormalon model estimation of CLEO data.

Lattice Data vs. Revised CLEO Constraints

NLO Light-Cone SR $\oplus$ **Twist-4** $\oplus(\mu^2 = Q^2)$

with 20% uncertainty of $\delta^2_{\text{Tw-4}}$: $\delta^2_{\text{Tw-4}} = 0.19 \pm 0.04$ **GeV^2**

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = 0.4$ **GeV^2**

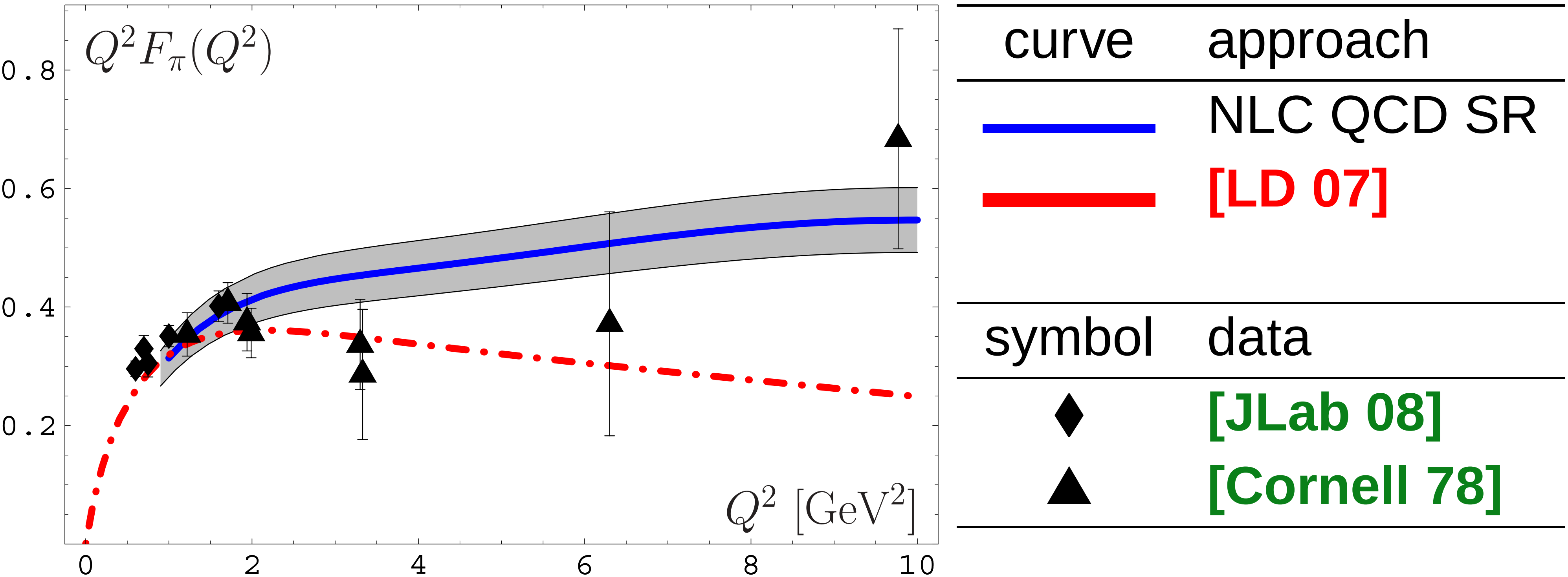


✚ = best-fit point
◆ = Asymptotic DA
■ = CZ DA
✖ = our new model
☆, ▲ and ✦ = instantons
▼ = transverse lattice
gray strip = lattice'06 result

The whole **improved BMS bunch** — inside 1σ-domain and **lattice strip**. **Dashed contour** = renormalon model estimation of CLEO data.

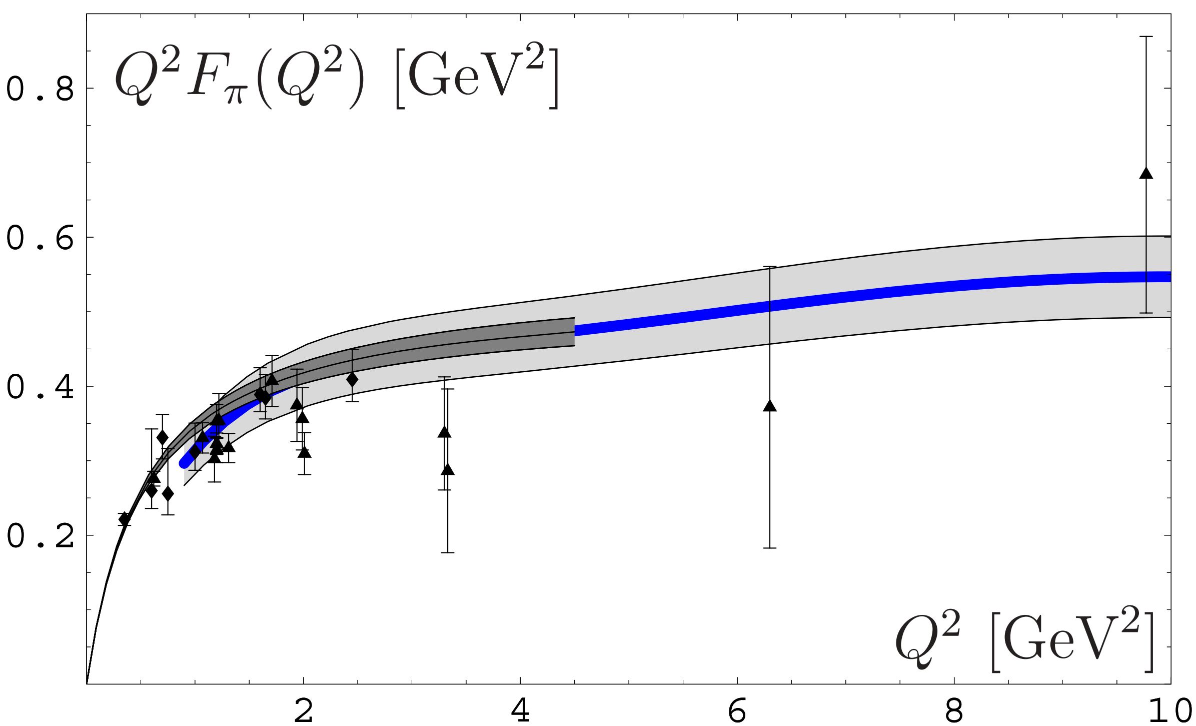
NLC NLO QCD SRs and JLab data for $F_\pi(Q^2)$

NLC QCD SR vs. Lattice QCD results



Pion FF from QCD SR with NLC **(blue solid line)**, in comparison with LD SR results by **Braguta–Lucha–Melikhov [Phys. Lett., B661 (2008) 354]**.

NLC QCD SR vs. Lattice QCD results

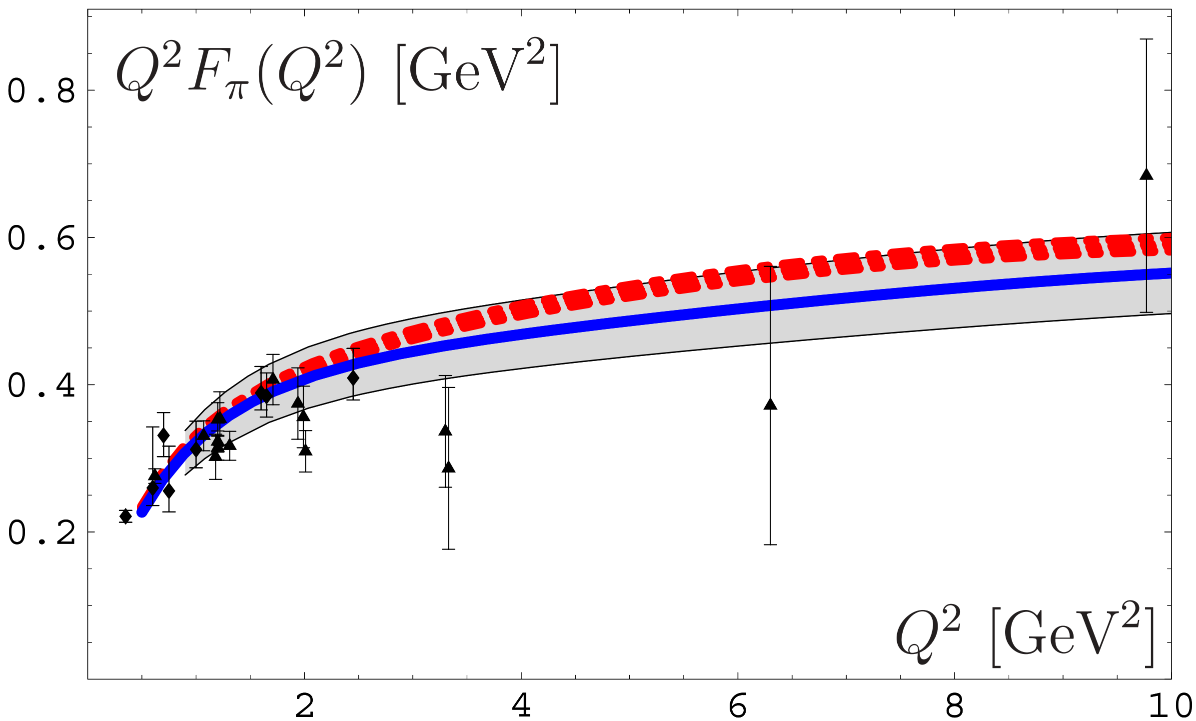


curve	approach
blue line	NLC QCD SR
grey line	**[Lattice 07]**

symbol	data
◆	**[JLab 08]**
▲	**[Cornell 78]**

Pion FF from QCD SR with NLC **(blue solid line)**, in comparison with recent lattice results by **D. Brommel et al. [Eur. Phys. J., C51 (2007) 335]**.

NNLO Correction Estimation using Pion DA



curve	approach
	NLC QCD SR
	$O(\alpha_s^2)$-corr. $\oplus$ BMS DA$_\pi$
symbol	**data**
♦	**[JLab 08]**
▲	**[Cornell 78]**

Pion FF from SRs with NLC **(blue solid line)**
and with **NNLO** correction using **BMS pion DA bunch**:
NNLO correction is of the order of $3-10\%$.

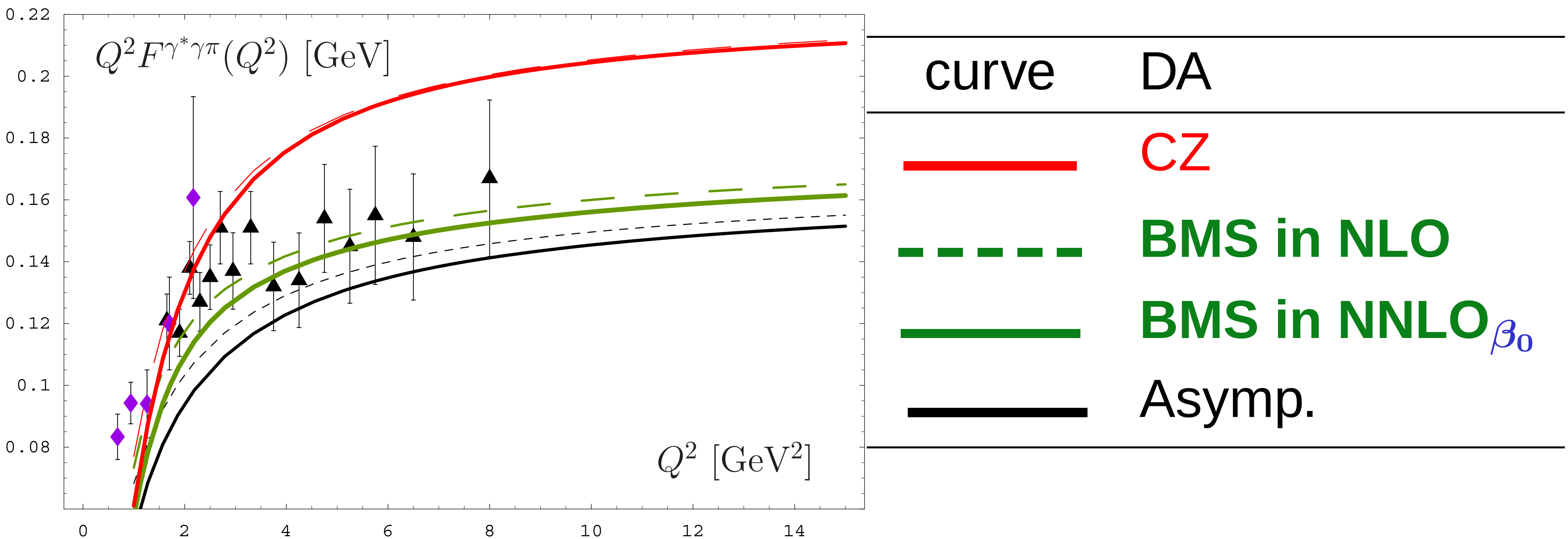
NNLO$_{\beta_0}$ LCSR

vs

NLO LCSR

$NNLO_{\beta_0}$ LCSR vs. NLO LCSR

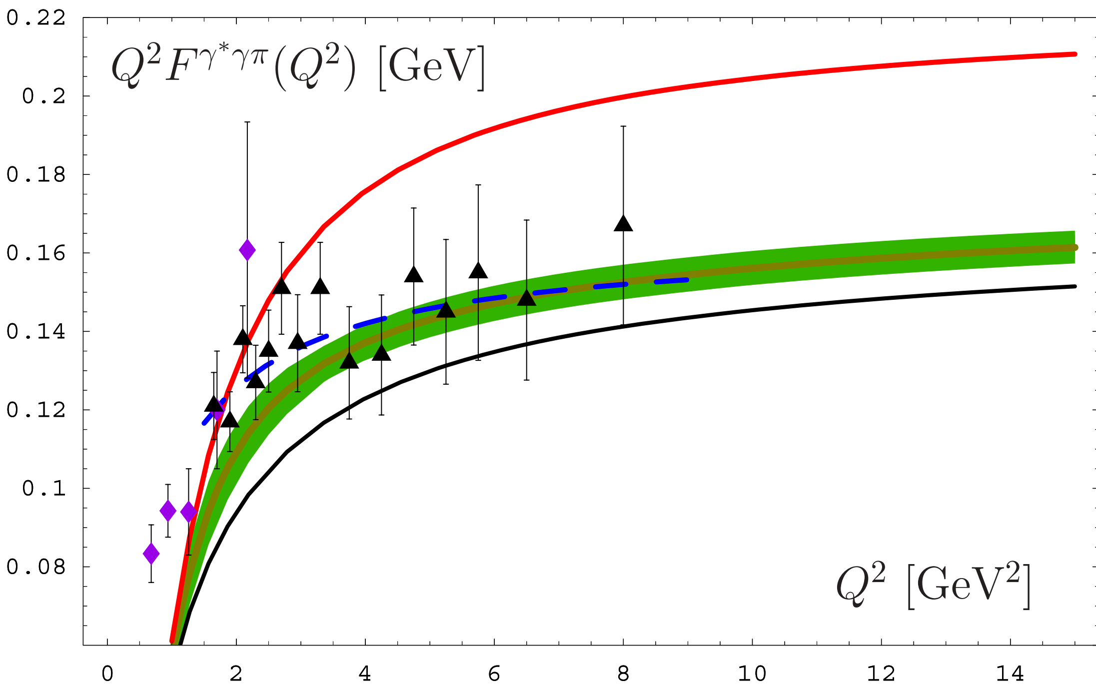
S. Mikhailov&N. Stefanis [NPB, 821 (2009) 291]



BMS bunch describes rather well all data for $Q^2 \gtrsim 1.5$ GeV^2.

$NNLO_{\beta_0}$ LCSR vs. NLO LCSR

S. Mikhailov&N. Stefanis [NPB, 821 (2009) 291]



curve	DA
red line	CZ
blue dashed line	CLEO best fit
green band	BMS bunch
black line	Asymp.

BMS bunch describes rather well all data for $Q^2 \gtrsim 1.5$ **GeV**^{2}.

PRE-CONCLUSIONS

- **QCD SR** method with **NLC** for pion DA **gives us** admissible sets (**bunches**) of DAs for each λ_q value.

PRE-CONCLUSIONS

- **QCD SR** method with **NLC** for pion DA **gives us** admissible sets (**bunches**) of DAs for each $\boldsymbol{\lambda_q}$ value.
- NLO LCSR method **produces new constraints** on pion DA parameters ($\boldsymbol{a_2, a_4}$) in conjunction with CLEO data.

PRE-CONCLUSIONS

- **QCD SR** method with **NLC** for pion DA **gives us** admissible sets (**bunches**) of DAs for each λ_q value.
- NLO LCSR method **produces new constraints** on pion DA parameters (a_2, a_4) in conjunction with CLEO data.
- Comparing**NLC SRs** with new CLEO **constraints** **allows to fix** the value of QCD vacuum nonlocality $\lambda_q^2 = 0.4$ **GeV^2**.

PRE-CONCLUSIONS

- **QCD SR** method with **NLC** for pion DA **gives us** admissible sets (**bunches**) of DAs for each λ_q value.
- NLO LCSR method **produces new constraints** on pion DA parameters (a_2, a_4) in conjunction with CLEO data.
- Comparing**NLC SRs** with new CLEO **constraints** **allows to fix** the value of QCD vacuum nonlocality $\lambda_q^2 = 0.4$ **GeV^2**.
- **This bunch** of pion DAs **agrees well** with **JLab F(pi)** data on pion EM form factor and with recent **lattice** data.

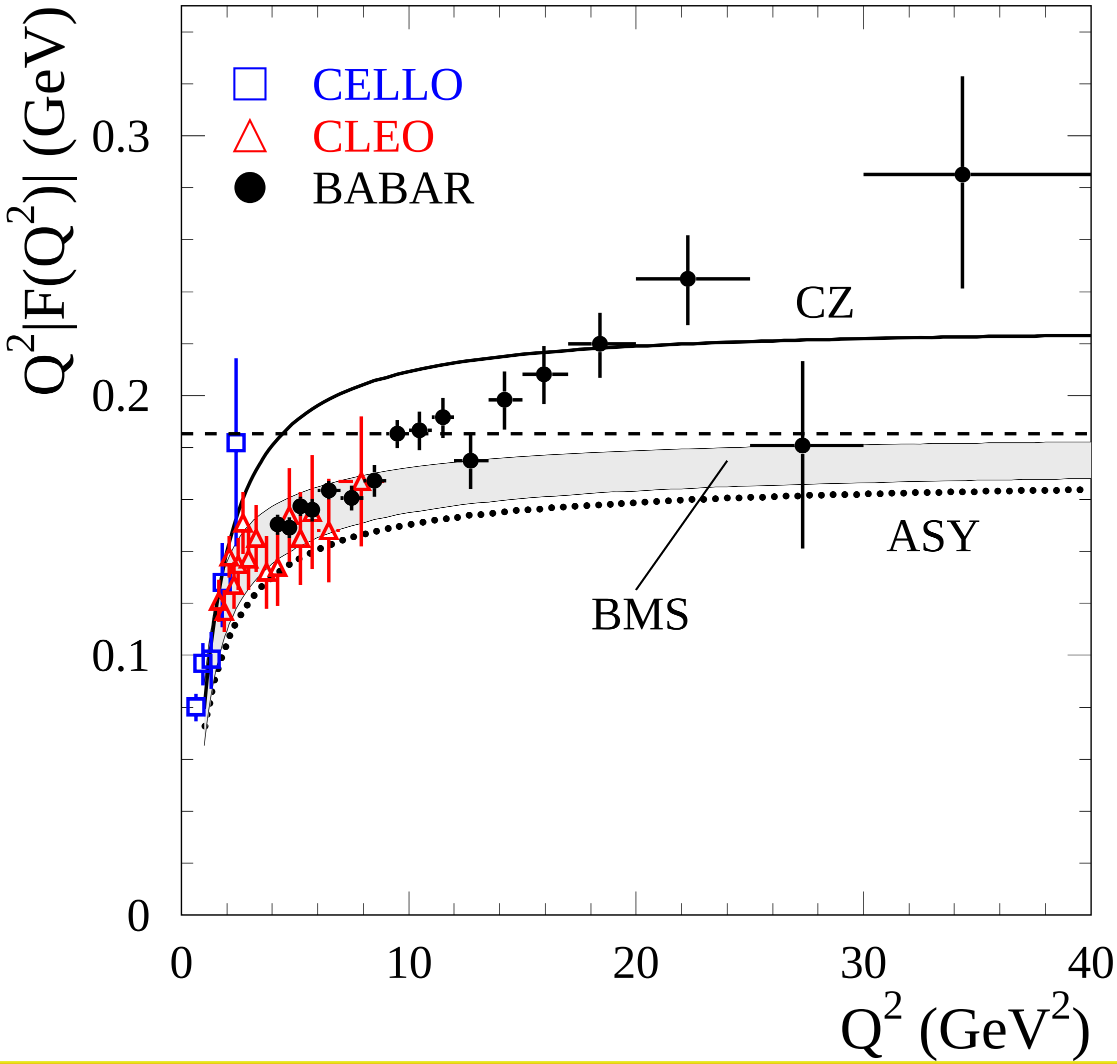
BaBar data on $F_{\gamma\gamma^*\pi}(Q^2)$

$$\Longrightarrow$$

Challenge for QCD?

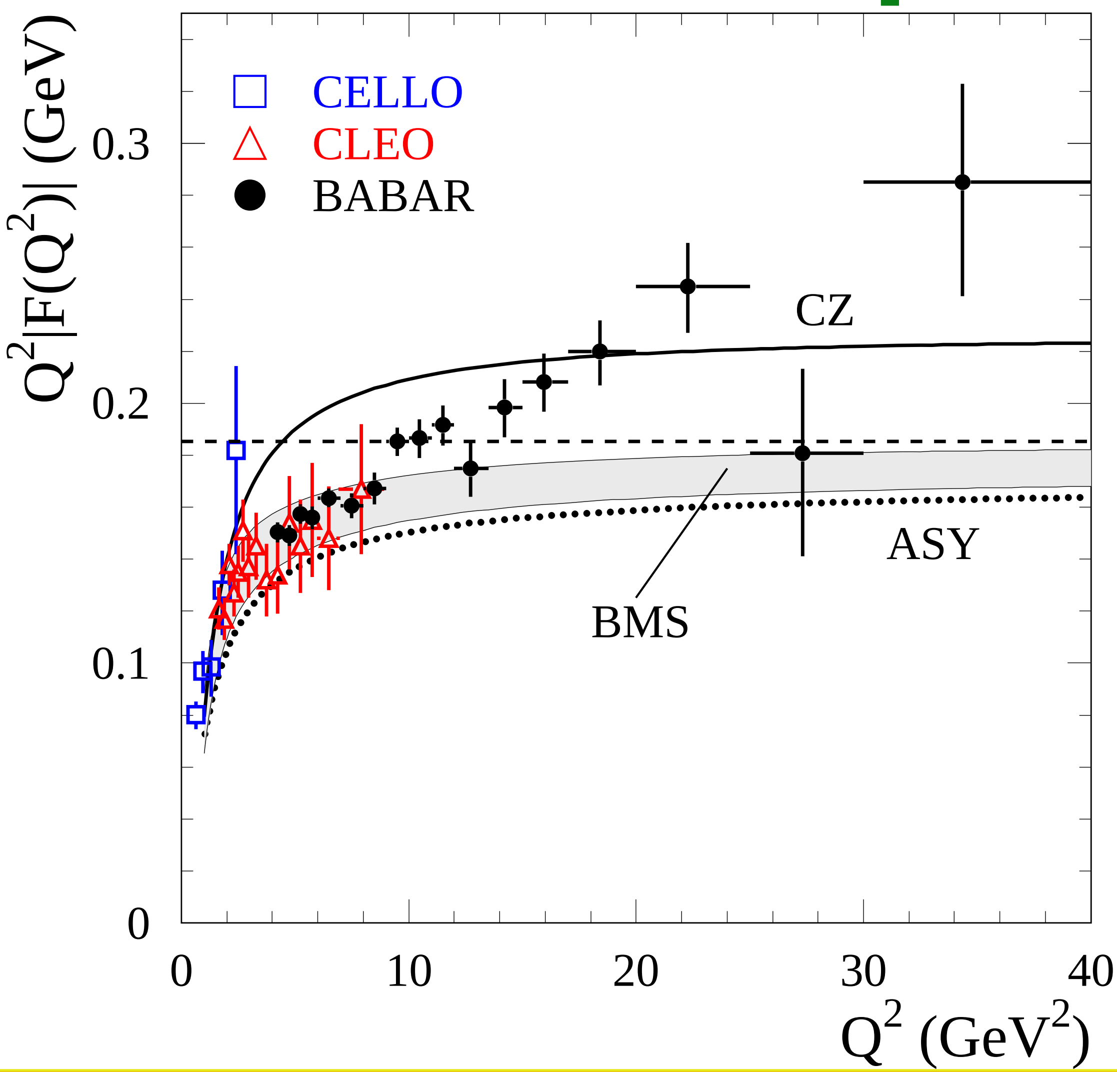
BaBar data on $F_{\gamma^*\gamma\to\pi}(Q^2)$

New data appeared at the end of May 2009 due to BaBar Collaboration **[PRD 80 (2009) 052002]**



BaBar data on $F_{\gamma^*\gamma\to\pi}(Q^2)$

New data appeared at the end of May 2009
due to BaBar Collaboration **[PRD 80 (2009) 052002]**

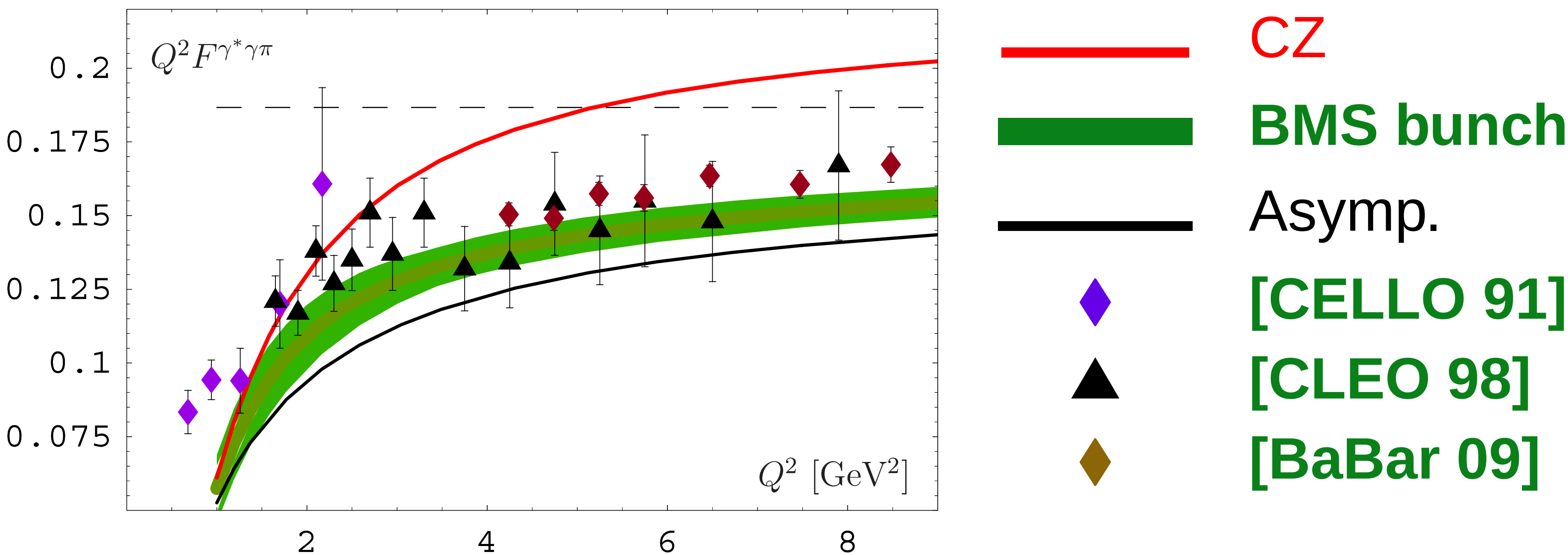


BaBar reservedly claimed:

"BaBar data contradicts most models for the pion DA..."

$\gamma^*\gamma \longrightarrow \pi$-*transition FF in CLEO region*

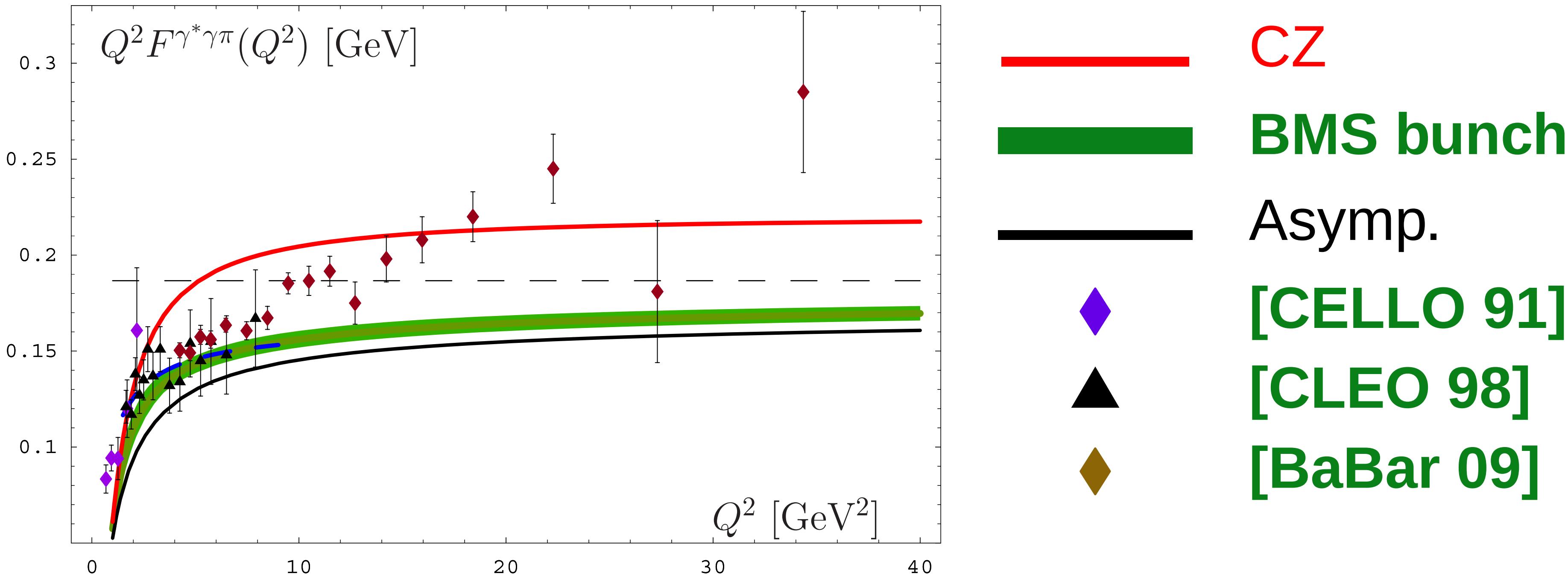
NNLO predictions vs. all data below 9 GeV^2 [M&S, 2009]



- **BMS "bunch"** provides best agreement with *all* data within QCD paradigm.

Is the new BaBar data a "pion puzzle"?

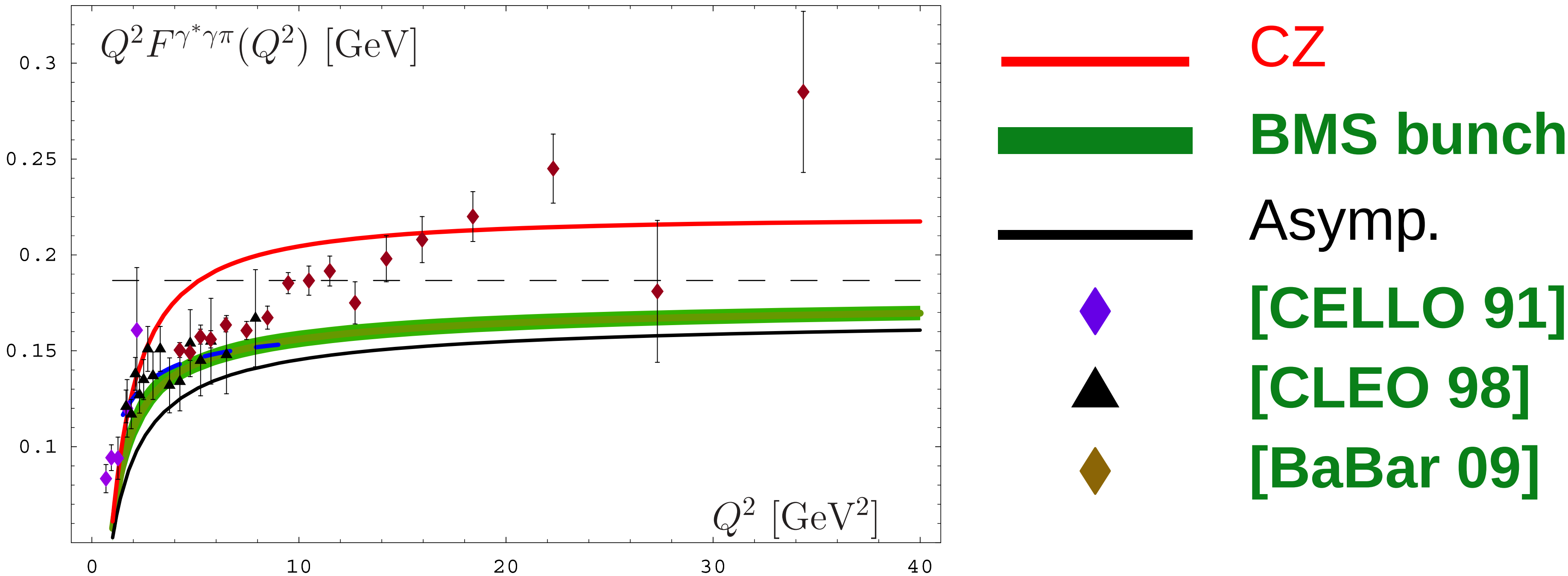
Analysis by Mikhailov&Stefanis **[NPB 09]**:



Pion DAs	CLEO and BaBar $\overline{\chi}^2$	BaBar $\overline{\chi}^2$	BaBar $\overline{\chi}^2$ ($Q^2 > 10$ **GeV**^{2})
Asy	$\mathbf{11.5}$	$\mathbf{19.2}$	$\mathbf{19.8}$
BMS	$\mathbf{4.4}$	$\mathbf{7.8}$	$\mathbf{11.9}$
CZ	$\mathbf{20.9}$	$\mathbf{36.0}$	$\mathbf{6.0}$

Is the new BaBar data a “pion puzzle”?

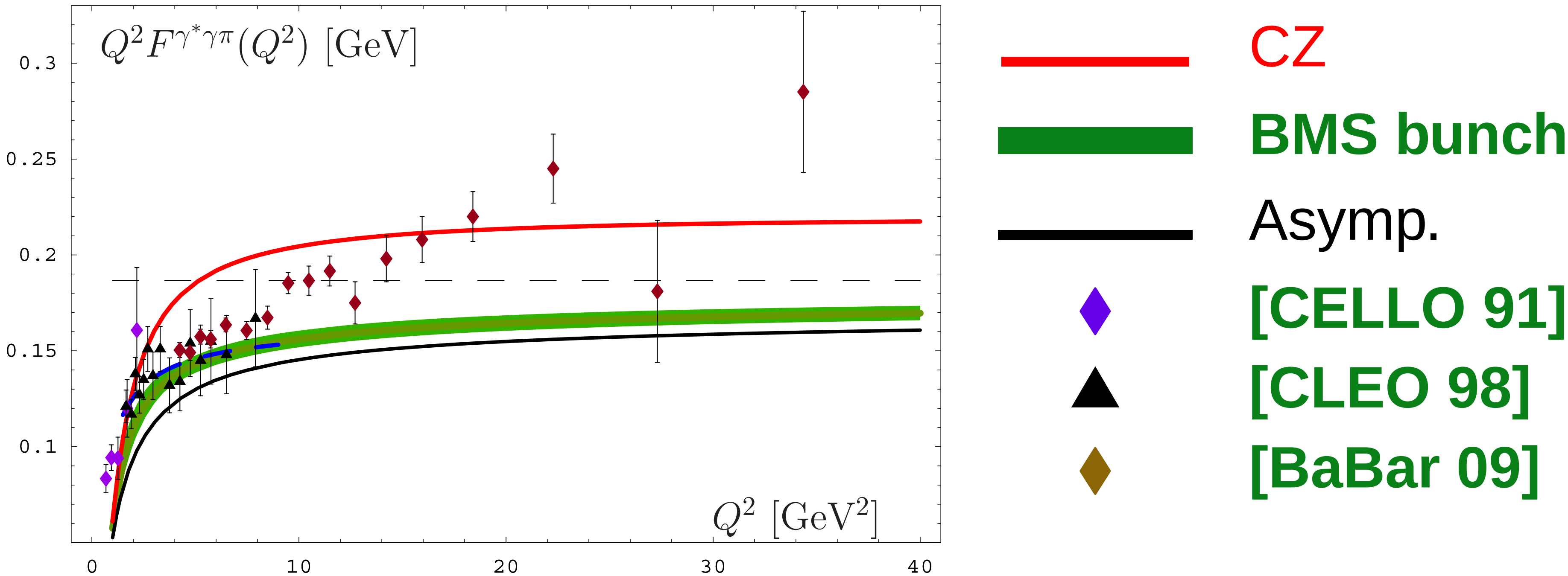
Analysis by Mikhailov&Stefanis **[NPB 09]**:



The growing with Q^2 BaBar data points ($Q^2 \gtrsim 15$ **GeV^2**)
contradict all pion DA models
that vanish at the endpoint $x = 0$.

Is the new BaBar data a “pion puzzle”?

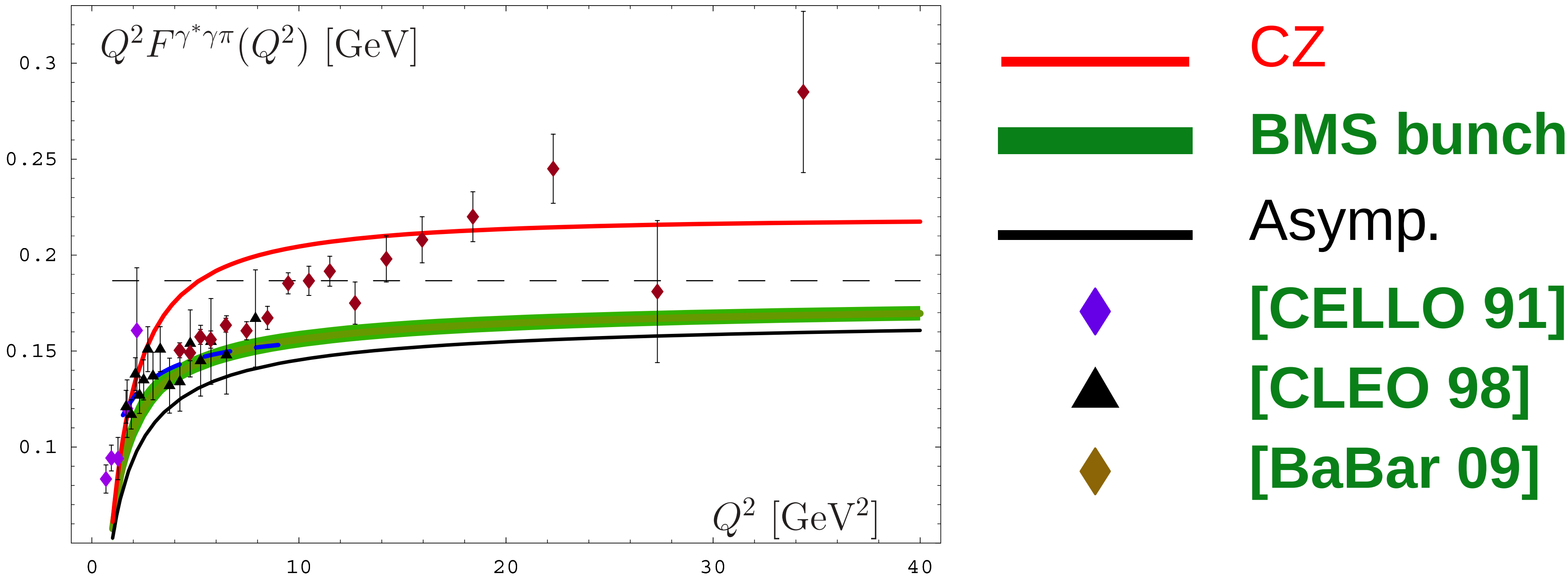
Analysis by Mikhailov&Stefanis **[NPB 09]**:



- They contradict the **Collinear Factorization** per se.

Is the new BaBar data a “pion puzzle”?

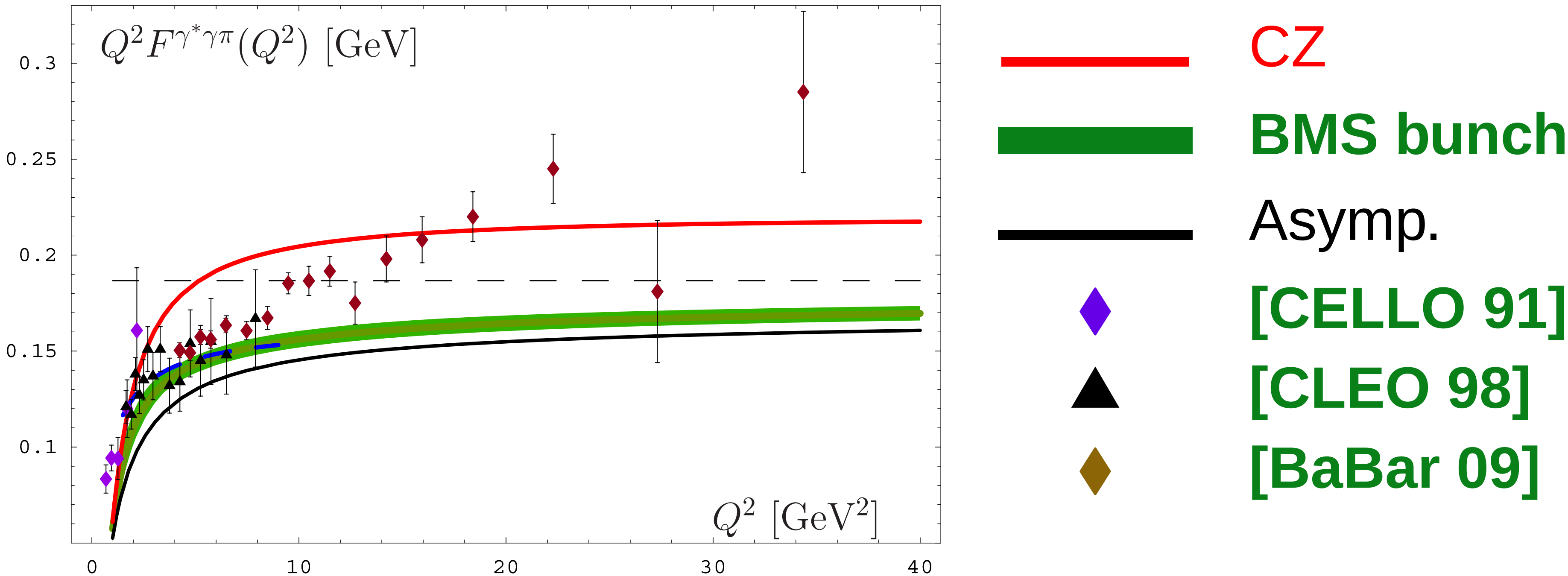
Analysis by Mikhailov&Stefanis **[NPB 09]**:



- They contradict the **Collinear Factorization** per se.
- Even more, they contradict the **“counting rules”** — the most reliable method up to now.

Is the new BaBar data a “pion puzzle”?

Analysis by Mikhailov&Stefanis **[NPB 09]**:



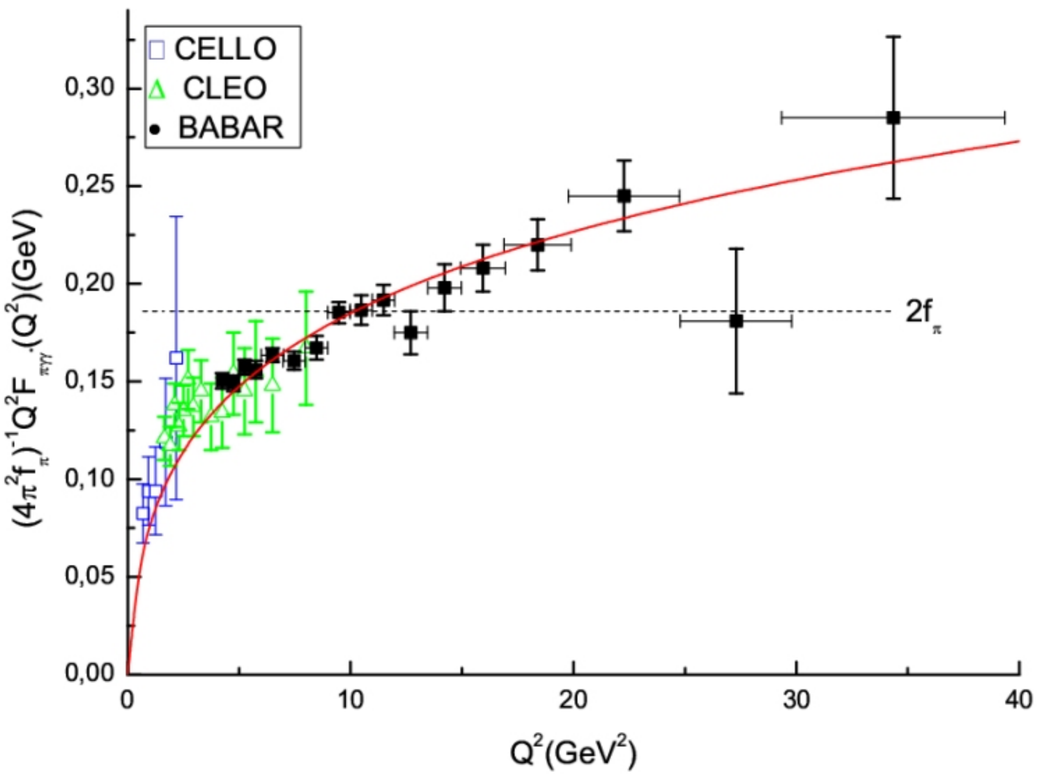
- They contradict the **Collinear Factorization** per se.
- Even more, they contradict the **“counting rules”** — the most reliable method up to now.
- If the BaBar data are correct, they constitute **a challenge** for QCD.

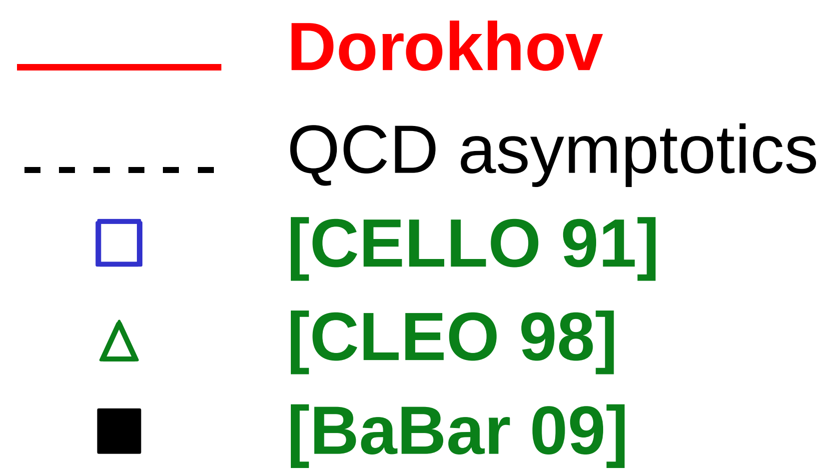
First attempts to overcome the “pion puzzle”

A possible scenarios to explain the BaBar data

- A. Dorokhov **[0905.3577]** with constituent quark model

$Q^2\, F_{\gamma^*\gamma\to\pi}(Q^2) \sim \ln^2(Q^2/M_q^2)$ with $M_q \simeq 135$ **MeV**.





Note $M_q \simeq 135$ **MeV** < 300 **MeV**. No trace of QCD...

First attempts to overcome the "pion puzzle"

A possible scenarios to explain the BaBar data

- A. Radyushkin **[0906.0323]** with "flat" DA $\varphi_\pi(x) \approx 1$ and using Light-Front Gaussian model:

$$Q^2 F^{\text{LFG}}_{\gamma^*\gamma\to\pi}(Q^2) \sim \int_0^1 \frac{\varphi_\pi(x)}{x}\left[1 - \mathbf{exp}\left(-\frac{x\,Q^2}{2\,\bar{x}\,\sigma}\right)\right] dx$$

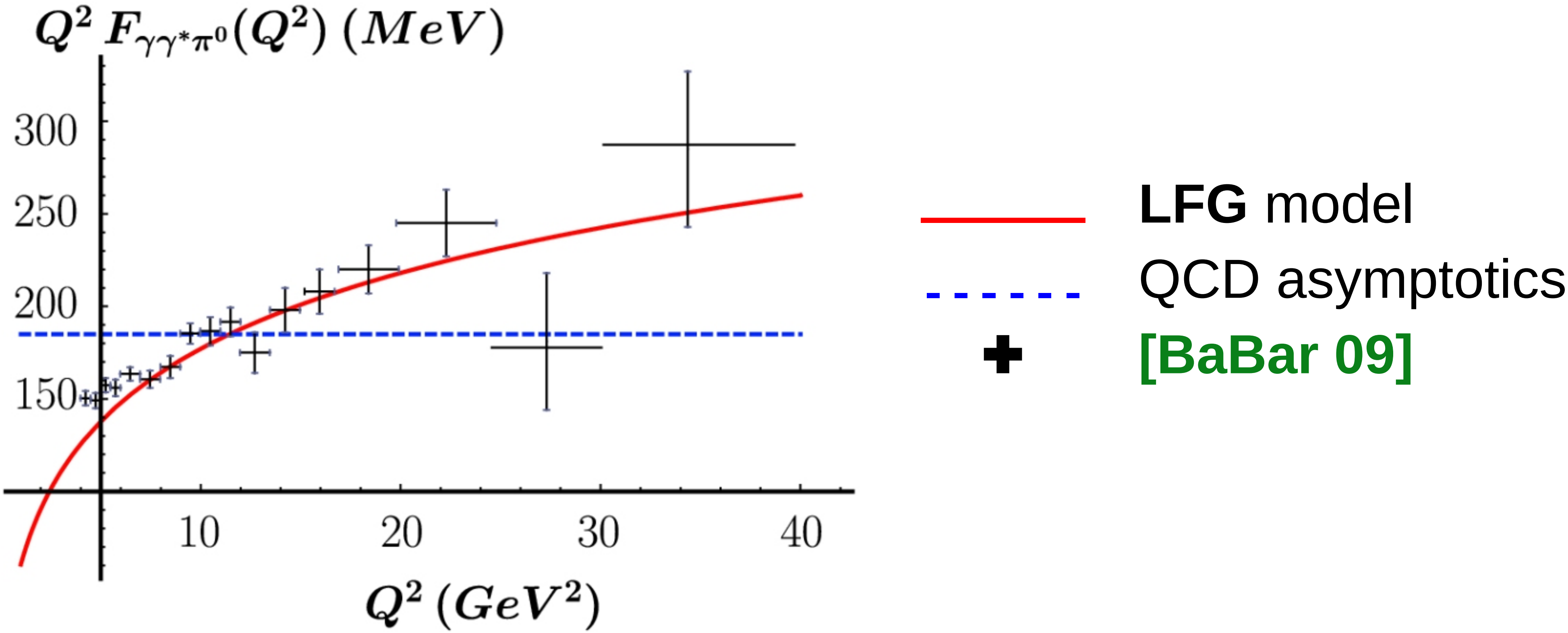
Here $\sigma \simeq 0.53$ **GeV^2**.

First attempts to overcome the "pion puzzle"

A possible scenarios to explain the BaBar data

- A. Radyushkin **[0906.0323]** with "flat" DA $\varphi_\pi(x) \approx 1$:

$$Q^2\, F^{\textsf{LFG}}_{\gamma^*\gamma\to\pi}(Q^2,0) \sim \ln\left[\frac{Q^2}{M^2}\right], \quad M^2 = 2\sigma e^{-\gamma_E} \simeq 0.6\ \textbf{GeV}^2.$$



No Factorization. Rad. Corrs. removed **by hand!**

First attempts to overcome the “pion puzzle”

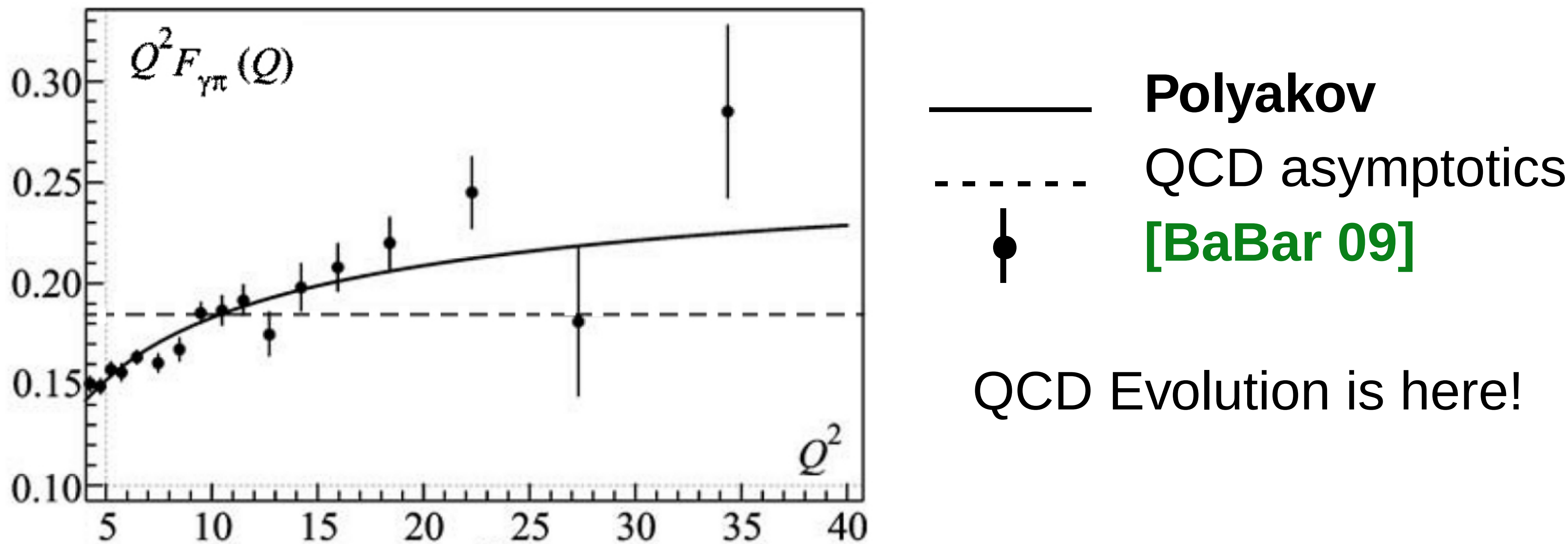
A possible scenarios to explain the BaBar data

- M. Polyakov **[0906.0538]** with “flat” DA

$\varphi_\pi(x, \mu_0 = 0.6\ \textbf{GeV}) = 1.3 - 0.3 \cdot 6\, x\, (1 - x)$ and using:

$$Q^2\, F^{\textsf{Pol}}_{\gamma^*\gamma\to\pi}(Q^2) \sim \int_0^1 \frac{\varphi_\pi(x, Q^2)}{x + m^2/Q^2}\, dx$$

Here $m \simeq 0.65\ \textbf{GeV}$ from BaBar data fit.



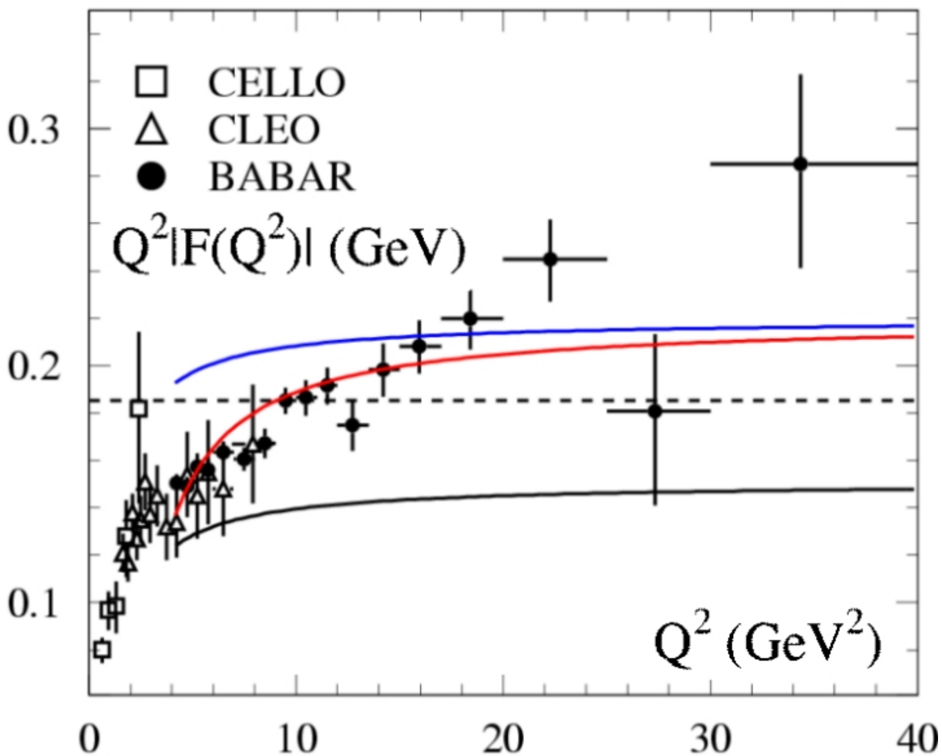
QCD Evolution is here!

Last attempt to overcome the "pion puzzle"

An "impossible" scenario to explain the BaBar data

- V. Chernyak **[0912.0623]** with standard pQCD $\oplus$ **CZ pion DA** $\oplus$ **"mild"** power corrs.:

$$Q^2\, F^{\text{Che}}_{\gamma^*\gamma\pi}(Q^2) = f_\pi \sqrt{2}\left[1.18 - \frac{\text{PC}_2}{Q^2} - \frac{\text{PC}_4}{Q^4}\right]$$



CZ pion DA

QCD asymptotics

CZ "mild" PC:

$\text{PC}_2 = 1.50\ \text{GeV}^2$

$\text{PC}_4 = 1.44\ \text{GeV}^4$

Preliminary conclusions about BaBar data

- There is a hint that the **“old” Cello$\oplus$CLEO data** and the **new BaBar data** cannot be fitted simultaneously — Mikhailov&Stefanis **[MPLA 24 (2009) 2858]**:

χ^2 **per d.f.**	Cello&CLEO	BaBar
Cello&CLEO	$\mathbf{1.22}$	$\mathbf{15.8}$
BaBar	$\mathbf{3.5}$	$\mathbf{1.8}$

Preliminary conclusions about BaBar data

- There is a hint that the **"old" Cello$\oplus$CLEO data** and the **new BaBar data** cannot be fitted simultaneously — Mikhailov&Stefanis **[MPLA 24 (2009) 2858]**:

χ^2 **per d.f.**	Cello&CLEO	BaBar
Cello&CLEO	$\mathbf{1.22}$	$\mathbf{15.8}$
BaBar	$\mathbf{3.5}$	$\mathbf{1.8}$

- From the theoretical side, it is not possible to describe these data with the inclusion of higher radiative corrections — as have been shown at the NNLO level by Mikhailov&Stefanis **[NPB 821 (2009) 291]**.

Preliminary conclusions about BaBar data

- There is a hint that the **“old” Cello$\oplus$CLEO data** and the **new BaBar data** cannot be fitted simultaneously — Mikhailov&Stefanis **[MPLA 24 (2009) 2858]**:

χ^2 **per d.f.**	Cello&CLEO	BaBar
Cello&CLEO	$\mathbf{1.22}$	$\mathbf{15.8}$
BaBar	$\mathbf{3.5}$	$\mathbf{1.8}$

- From the theoretical side, it is not possible to describe these data with the inclusion of higher radiative corrections — as have been shown at the NNLO level by Mikhailov&Stefanis **[NPB 821 (2009) 291]**.
- Let us wait for **Belle** verification of the **BaBar** data.