Analytic approach to the study of the electric-magnetic asymmetry of the dimension two condensate

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Overview

1. The dimension two condensate and its asymmetry
   - The dimension two condensate
   - The asymmetry in the dimension two condensate
   - The LCO formalism
2. The LCO formalism extended to $A_\mu A_\nu$
   - Which operator to take
   - The $A_\mu A_\nu$ operator
   - Towards the asymmetry
   - The effective potential
3. Finite temperature
   - Low-T behavior
   - Numerical analysis
   - High-T behavior
4. Conclusions

Overview

- work done in collaboration with D. Dudal, J.A. Gracey, N. Vandersickel and H. Verschelde
- Phys.Rev.D**80** (2009) 065017, and work in preparation
- motivated by work by M.N. Chernodub and E.–M. Ilgenfritz (Phys.Rev.D**78** (2008) 034036)

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Theoretical considerations

- $\langle A_\mu^2 \rangle$ is not gauge invariant, can it be relevant?
- existence of $1/q^2$ power corrections advertized by **Zakharov, Narison, et al.** from QCD phenomenology/(ultraviolet) renormalon analysis/topological considerations
- approaches based on AdS/QCD usually also predict $1/q^2$ corrections in e.g. (gauge invariant!) glueball correlators (**Andreev, Forkel, Gherghetta, Zakharov, et al.**)

Making $\langle A_\mu^2 \rangle$ gauge invariant

Consider the minimum of $\langle A_\mu^2 \rangle$ on the gauge orbit:

$$A^2_{\text{min}} = \min_{u \in SU(N)} \mathcal{V}^{-1} \int d^4x (A_\mu^u)^2$$

- is gauge invariant by construction
- however: very non-local operator!

Making A^2_{min} local

- iterative search for $u \in \text{SU(N)}$ such that A^2 gets minimized

$$A^2_{\text{min}} = \int d^4x \left[A^a_\mu \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A^a_\nu - g f^{abc} \left(\frac{\partial_\nu}{\partial^2} \partial A^a \right) \left(\frac{1}{\partial^2} \partial A^b \right) A^c_\nu \right] + \mathcal{O}(A^4)$$

- if we choose the Landau gauge $\partial_\mu A_\mu = 0$, then:

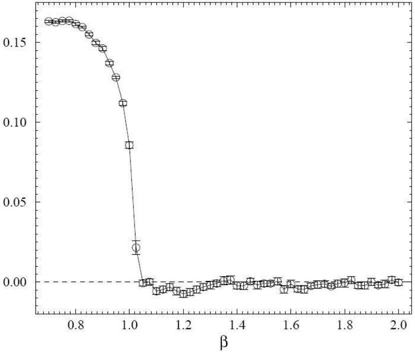
$$A^2_{\text{min}} = \mathcal{V}^{-1} \int d^4x A^2_\mu$$

More practical motivations

$\langle A_\mu^2 \rangle$ is a good observer of topological excitations (e.g. monopoles, vortices...)

- assume a thin vortex, carrying magnetic flux $\propto \int \vec{B} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{x} \neq 0$
- A_μ cannot be zero everywhere $\Rightarrow A^2_{\text{min}} \neq 0$

Compact QED on the lattice



- compact QED shows confinement for $e^2 > e_c^2 \sim 1$ (monopole condensation, mass gap, see literature)
- related to $\langle A_\mu^2 \rangle$ (cfr. **Gubarev and Zakharov**)

In QCD

- **Gubarev and Zakharov** provide 2 component picture for $\langle A_\mu^2 \rangle$
- soft (infrared) part, $\langle A_\mu^2 \rangle_{\text{IR}}$ can enter OPE for gauge variant things (like propagators, see lattice work by **Boucaud et al.**)
- hard (ultraviolet) part $\langle A_\mu^2 \rangle_{\text{UV}}$ can enter physical correlators (modeled in phenomenology with **gluon masses**), see work by **Ruiz-Arriola, Megias et al., Zakharov et al.**
- $\Rightarrow$ example of nonperturbative UV effects in gauge theory.
- effective gluon masses are nothing new, see work by **Cornwall, Parisi & Petronzio, Field, Bernard. . .**
- of course, no perturbative unitarity, but gluons are not the physical degrees of freedom after all

The asymmetry in the dimension two condensate

- Yang–Mills phase transition is at finite T
- let's consider A_4^2 and A_i^2 separately
- the sum A_μ^2 has been investigated before
- **Chernodub and Ilgenfritz** considered the difference

The asymmetry in the dimension two condensate

- **Chernodub and Ilgenfritz** considered the difference
- at $T = 0$, the asymmetry vanishes
- only consider temperature contributions (which are finite)
- define:

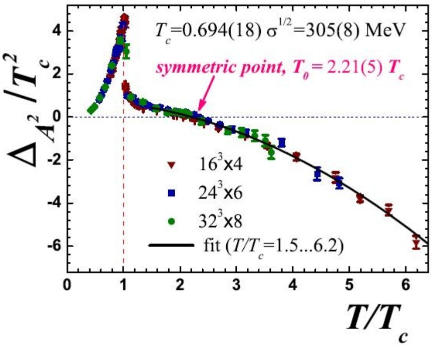
$$\begin{aligned} \Delta_{A^2}(T) &= \langle g^2 A_E^2 \rangle - \frac{1}{3} \langle g^2 A_M^2 \rangle \\ &= \langle g^2 A_d^2 \rangle_T - \frac{1}{d-1} \sum_{i=1}^{d-1} \langle g^2 A_i^2 \rangle_T \end{aligned}$$

Naive expectations

We would naively expect:

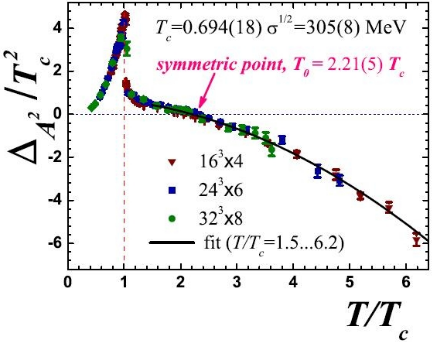
- at high T: $\Delta_{A^2}(T) \propto T^2$
- at low $T = \Delta_{A^2}(T) \propto e^{-m/T}$ (m lowest mass in spectrum, in SU(2) Yang–Mills: $m_{\mathrm{gl}} \sim 1700\,\mathrm{MeV}$)
- at low T, Abelian Higgs model gives $\Delta_{A^2}(T) \propto T^4/m^2$ (due to massless pole of longitudinal dof's)

Lattice results



- $\Delta_{A^2} = 0$ at $T = 0$
- jump at deconfinement phase transition
- at $T = 2.2T_c$ the asymmetry flips sign, at higher T a monopole gas is expected (cfr. **Chernodub & Zakharov, Shuryak & Liao**)

Lattice results



- at high T: $\Delta_{A^2}(T) \propto T^2$ (no surprise)
- at low T: $\Delta_{A^2}(T) \propto e^{-m/T}$ with $m \sim 200\,\mathrm{MeV}$
- why such a low mass?

Conclusion

Doing a continuum investigation?

Hoping to get more insight in

- the three-phase diagram
- the low-T behavior

Need a continuum formalism to study the A_μ^2 asymmetry.
Use the Local Composite Operator (LCO) formalism developed by **Verschelde et al.**

The LCO formalism

Developed by H. Verschelde *et al.* in Phys.Lett.B**516** (2001) 307

- add a term $\frac{1}{2}JA_\mu^2$
- non-renormalizable $\longrightarrow$ add a term $-\frac{\zeta}{2}J^2$
- new parameter ζ: must be determined in some way
- choose ζ to be a unique meromorphic function of g^2
- the J^2 term: Hubbard–Stratanovich transformation
- result found for $SU(N)$ Yang–Mills: $\frac{1}{2}\langle g^2 A_\mu^2\rangle \sim -(500\,\mathrm{MeV})^2$ (both at one- and two-loop order)

Overview

Which operator to take

- separate access to A_0^2 and A_i^2 possible?
- maintain a clear view on renormalization (group)?
- at least maintain Lorentz invariance when $T = 0$. A replacement like $\frac{1}{2}JA_\mu^2 \to \frac{1}{2}J_1A_0^2 + \frac{1}{2}J_2A_i^2$ does not look as a very good choice.
- favourably, recover the $T = 0$ results?

Which operator to take

- good choice might be

$$\frac{1}{2} J A_\mu^2 \to \frac{1}{2} J_{\mu\nu} A_\mu A_\nu$$

- gives rise to renormalization troubles, also $J_{\mu\mu} A_\nu^2$ comes to life, difficult to control
- the right thing:

$$\frac{1}{2} J A_\mu^2 \to \frac{1}{2} J A_\mu^2 + \frac{1}{2} \rho_{\mu\nu} \left(A_\mu A_\nu - \frac{\delta_{\mu\nu}}{d} A_\sigma^2 \right)$$

- we can consider A_μ^2 ($=$ trace of $A_\mu A_\nu$) and the traceless part of $A_\mu A_\nu$, no mixing between the trace and "off-trace" operator

The $A_\mu A_\nu$ operator

- couple a source $K_{\mu\nu}$ to traceless part of $A_\mu A_\nu$
- add a source $\eta_{\mu\nu}$ in a BRST doublet with $K_{\mu\nu}$:

$$
\begin{aligned}
\mathcal{L}_K &= s\left(\frac{1}{2}\eta_{\mu\nu}A^a_\mu A^a_\nu - \frac{1}{2d}\eta_{\mu\mu}A^2 - \frac{\omega}{2}\eta_{\mu\nu}K_{\mu\nu} + \frac{\omega}{2d}\eta_{\mu\mu}K_{\nu\nu}\right) \\
&= \frac{1}{2}K_{\mu\nu}A^a_\mu A^a_\nu + \eta_{\mu\nu}A^a_\mu \partial_\nu c^a - \frac{1}{2d}K_{\mu\mu}A^2 \\
&\quad -\frac{1}{d}\eta_{\mu\mu}A^a_\nu \partial_\nu c^a - \frac{\omega}{2}K_{\mu\nu}K_{\mu\nu} + \frac{\omega}{2d}K^2_{\mu\mu}
\end{aligned}
$$

- ω is new parameter

Algebraic renormalization

- introduce two further external sources K_μ^a and L^a:

$$\mathcal{L}_{\text{ext}} = -K_\mu^a D_\mu^{ab} c^b + \frac{1}{2} g f^{abc} L^a c^b c^c$$

- we have the BRST nilpotent operator acting as

$$\begin{aligned} sA_\mu^a &= -D_\mu^{ab} c^b & sc^a &= \frac{1}{2} g f^{abc} c^b c^c \\ s\overline{c}^a &= b^a & sb^a &= 0 \\ s\eta_{\mu\nu} &= K_{\mu\nu} & sK_{\mu\nu} &= 0 \\ sK_\mu^a &= 0 & sL_\mu &= 0 \end{aligned}$$

Ward identities

The complete action obeys:

- Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = \int d^d x \left(\frac{\delta \Sigma}{\delta A_\mu^a} \frac{\delta \Sigma}{\delta K_\mu^a} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta \Sigma}{\delta L^a} + b^a \frac{\delta \Sigma}{\delta \overline{c}^a} + K_{\mu\nu} \frac{\delta \Sigma}{\delta \eta_{\mu\nu}} \right) = 0$$

- Landau gauge fixing condition $\dfrac{\delta \Sigma}{\delta b^a} = \partial_\mu A_\mu^a$
- antighost equation $\dfrac{\delta \Sigma}{\delta \overline{c}^a} + \partial_\mu \dfrac{\delta \Sigma}{\delta K_\mu^a} = 0$

Ward identities

The complete action obeys:

- The ghost Ward identity $\mathcal{G}^a\Sigma = \Delta^a_{\mathrm{cl}}$ with

$$\mathcal{G}^a = \int d^dx \left(\frac{\delta}{\delta c^a} + gf^{abc}\left(\overline{c}^b \frac{\delta}{\delta b^c}\right)\right)$$

  and the classical breaking

$$\Delta^a_{\mathrm{cl}} = g\int d^dx f^{abc}\left(K^b_\mu A^c_\mu - L^b c^c\right)$$

- additional identities $\delta_{\mu\nu}\dfrac{\delta}{\delta K_{\mu\nu}}\Sigma = 0$ and $\delta_{\mu\nu}\dfrac{\delta}{\delta \eta_{\mu\nu}}\Sigma = 0$

Ward identities

The counterterms obey similar relations:

- linearized Slavnov-Taylor identity $\mathcal{B}_\Sigma \Sigma^{\text{count}} = 0$ where

$$\mathcal{B}_\Sigma = \int d^d x \left(\frac{\delta \Sigma}{\delta A^a_\mu} \frac{\delta}{\delta K^a_\mu} + \frac{\delta \Sigma}{\delta K^a_\mu} \frac{\delta}{\delta A^a_\mu} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta}{\delta L^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + b^a \frac{\delta}{\delta \overline{c}^a} + K_{\mu\nu} \frac{\delta}{\delta \eta_{\mu\nu}} \right)$$

  obeys $\mathcal{B}_\Sigma \mathcal{B}_\Sigma = 0$
- Landau gauge fixing condition $\dfrac{\delta \Sigma^{\text{count}}}{\delta b^a} = 0$

Ward identities

The counterterms obey similar relations:

- antighost equation $\frac{\delta \Sigma^{\text{count}}}{\delta \overline{c}^a} + \partial_\mu \frac{\delta \Sigma^{\text{count}}}{\delta K_\mu^a} = 0$
- ghost Ward identity $\mathcal{G}^a \Sigma^{\text{count}} = 0$
- additional identities
  $\delta_{\mu\nu} \frac{\delta}{\delta K_{\mu\nu}} \Sigma^{\text{count}} = 0$ and $\delta_{\mu\nu} \frac{\delta}{\delta \eta_{\mu\nu}} \Sigma^{\text{count}} = 0$

Most general counterterm

- quantum numbers:

	A_μ	c	$\bar{c}$	b	$\eta_{\mu\nu}$	$K_{\mu\nu}$	K_μ	L
dimension	1	0	2	2	2	2	3	4
ghost number	0	1	-1	0	-1	0	-1	-2

- most general counterterm is

$$\Sigma^{\text{count}} = \frac{a_0}{4}\int d^d x F^a_{\mu\nu} F^a_{\mu\nu} + \mathcal{B}_\Sigma \Delta^{-1}$$

with Δ^{-1} the most general local polynomial with dimension 4 and ghost number -1

Most general counterterm

Writing down the most general local polynomial with dimension 4 and ghost number -1 and demanding it obey all necessary identities gives:

$$\Delta^{-1} = \int d^d x \Bigg(a_1 \left(K_\mu^a A^{a\mu} + \partial_\mu \overline{c}^a A^{a\mu} \right) + a_2 \left(\frac{\eta_{\mu\nu}}{2} A_\mu^a A_\nu^a - \frac{1}{d} \eta_{\mu\mu} A_\nu^2 \right) + a_3 \left(\frac{\omega}{2} \eta_{\mu\nu} K_{\mu\nu} - \frac{1}{d} \frac{\omega}{2} \eta_{\mu\mu} K_{\nu\nu} \right) \Bigg)$$

Most general counterterm

- Computing the most general counterterm from Δ^{-1}, can we reabsorb this?

$$\Sigma(g,\omega,\phi,\Phi) + \vartheta\Sigma^{\text{count}} = \Sigma(g_0,\omega_0,\phi_0,\Phi_0) + O(\vartheta^2)$$

- Yes, we can!

Conclusion

The Yang–Mills action with inclusion of the traceless part of $A_\mu A_\nu$ can be multiplicatively renormalized.

Trace and traceless part

To add the A_μ^2 operator itself is completely analogous. It is possible to prove the renormalizability of both the traceless part of $A_\mu A_\nu$ and the trace A_μ^2 together.

Conclusion

The Yang–Mills action with inclusion of the traceless part of $A_\mu A_\nu$ and of A_μ^2 can be multiplicatively renormalized.

Hubbard–Stratanovich transformation

In order to get rid of the terms quadratic in the sources, introduce the identities:

- $1 = \int [d\sigma] e^{-\frac{1}{2\zeta}\int d^dx \left(\frac{\sigma}{g}+\frac{1}{2}A_\mu^2-\zeta J\right)^2}$
- $1 = \int [d\varphi_{\mu\nu}] e^{-\frac{1}{2\omega}\int d^dx \left(\frac{1}{g}\varphi_{\mu\nu}+\frac{1}{2}A_\mu A_\nu-\omega(K_{\mu\nu}-\frac{\delta_{\mu\nu}}{d}K_{\mu\mu})\right)^2}$
- two new fields σ and (traceless) $\varphi_{\mu\nu}$ with $\langle\sigma\rangle = -\frac{g}{2}\langle A_\mu^2\rangle$ and $\langle\varphi_{\mu\nu}\rangle = -\frac{g}{2}\left\langle A_\mu A_\nu - \frac{\delta_{\mu\nu}}{d}A^2\right\rangle$

The action

Our action is:

$$S_{\mathrm{YM}} + S_{\mathrm{gf}} + \int d^d x \left[\frac{1}{2\zeta}\frac{\sigma^2}{g^2} + \frac{1}{2\zeta g}\sigma A_\mu^2 + \frac{1}{8\zeta}(A_\mu^2)^2 + \frac{1}{2\omega}\frac{\varphi_{\mu\nu}^2}{g^2} + \frac{1}{2\omega g}\varphi_{\mu\nu} A_\mu A_\nu + \frac{1}{8\omega}(A_\mu^a A_\nu^a)^2 \right]$$

Determination of ζ and ω

ζ was determined by Verschelde *et al.*, ω is analogous:

- write down renormalization group equation for J and $K_{\mu\nu}$
- consider ζ and ω function of g^2
  (for example for ζ: $\beta(g^2)\dfrac{\partial}{\partial g^2}\zeta(g^2) =$
  $2\gamma_J(g^2)\zeta(g^2) + \epsilon\delta\zeta - \beta(g^2)\dfrac{\partial}{\partial g^2}(\delta\zeta) + 2\gamma_J(g^2)\delta\zeta)$
- solve with a Laurent series

Determination of ζ and ω

In practice:

- compute anomalous dimensions and renormalization factors up to two-loop order (for one-loop computation of effective action)
- plenty of diagrams (best done computerized)
- solving the renormalization group equation
- $\zeta = \frac{N^2-1}{16\pi^2}\left[\frac{9}{13}\frac{16\pi^2}{g^2N}+\frac{161}{52}\right]$
- $\omega = \frac{N^2-1}{16\pi^2}\left[\frac{1}{4}\frac{16\pi^2}{g^2N}+\frac{73}{1044}\right]$

Conventions

- $m^2 = \frac{13}{9}\frac{N}{N^2-1} g\sigma = g\sigma'$
- $M_{\mu\nu} = 4\frac{N}{N^2-1} g\varphi_{\mu\nu} = g\varphi'_{\mu\nu}$
- take $M_{\mu\nu} = A \begin{pmatrix} 1 & & & \\ & -\frac{1}{d-1} & & \\ & & \ddots & \\ & & & -\frac{1}{d-1} \end{pmatrix}$
  (introduces an electric-magnetic asymmetry, preserves $3d$ rotational invariance)

The effective action

Up to one-loop order, we have

$$\Gamma^{(1)}(\sigma', \varphi'_{\mu\nu}) = \frac{1}{2 Z_\zeta Z_J^2 \zeta} \frac{\sigma^2}{g^2} + \frac{1}{2 Z_\omega Z_K^2 \omega} \frac{\varphi_{\mu\nu}^2}{g^2} + \frac{N^2-1}{2} \operatorname{tr} \ln \left(-\partial^2 \delta_{\mu\nu} + \left(1 - \frac{1}{\xi}\right) \partial_\mu \partial_\nu + \delta_{\mu\nu} m^2 + M_{\mu\nu} \right)$$

The tr log

The operator in the tr log can be easily diagonalized. In the limit $\xi \to 0$ it decomposes as

$$\operatorname{tr}\ln(-\partial^2) + (d-2)\operatorname{tr}\ln\left(-\partial^2 + m^2 - \frac{A}{d-1}\right) + \operatorname{tr}\ln\left(-\partial^2 + m^2 + A\left(1 - \frac{d}{d-1}\frac{\partial_0^2}{\partial^2}\right)\right)$$

The final result

Some "straightforward algebra" gives:

$$\Gamma^{(1)}(\sigma',\varphi'_{\mu\nu}) = \frac{N^2-1}{2(4\pi)^2}\Bigg\{\frac{9}{13}\frac{(4\pi)^2}{g^2N}m^4 + \frac{1}{3}\frac{(4\pi)^2}{g^2N}A^2 + \frac{1}{18}\ln\left(\frac{m^2-A/3}{\overline{\mu}^2}\right)\left[7A^2+27m^4\right] - \frac{155}{522}A^2 + \frac{11}{12}Am^2 - \frac{87}{26}m^4 + \frac{1}{4}\frac{m^6}{A} + \frac{1}{18}\left[5A^2+12Am^2+9m^4\right]\left[\ln\left(\frac{A}{A-3m^2}\right) + \ln\left(1+\sqrt{\frac{m^2+A}{m^2-\frac{A}{3}}}\right) - \ln\left(1-\sqrt{\frac{m^2+A}{m^2-\frac{A}{3}}}\right)\right] - \frac{(m^2-\frac{A}{3})}{12A}(6A^2+11Am^2+3m^4)\sqrt{\frac{m^2+A}{m^2-\frac{A}{3}}}\Bigg\}$$

The final result

Remarks:

- is real only if $m^2 \geq \frac{A}{3}$ and $m^2 \geq -A$
- in the limit $A \rightarrow 0$ it gives the result already found by Verschelde *et al.*
- its minimum is at $A = 0$ and with σ the value found by Verschelde *et al.*

Conclusion

We found the effective potential in the presence of A_μ^2 and of an electric-magnetic asymmetry Δ_{A^2}. It has its minimum where it should be.
(This is at $T = 0$.)

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Temperature corrections to the action

Finite-temperature corrections can be straightforwardly computed using standard techniques. The result is

$$(N^2-1)(d-2)T\int\frac{d^3k}{(2\pi)^3}\ln\left(1-\exp-\frac{\sqrt{\alpha}}{2T}\right)$$
$$+(N^2-1)T\int\frac{d^3k}{(2\pi)^3}\left(\ln\left(1-\exp-\frac{\sqrt{\frac{\alpha}{2}+\frac{\sqrt{\alpha^2-4\beta}}{2}}}{T}\right)\right.$$
$$\left.+\ln\left(1-\exp-\frac{\sqrt{\frac{\alpha}{2}-\frac{\sqrt{\alpha^2-4\beta}}{2}}}{T}\right)\right)$$

with $\alpha=m^2+2\vec{k}^2-A/3$ and $\beta=\vec{k}^2(\vec{k}^2+m^2+A)$.

Low-T expansion

For the lowest temperatures, we find:

$$\Delta_{A^2} = (N^2 - 1)\frac{g^2\pi^2}{30}\left(1 - \frac{85}{1044}\frac{g^2 N}{(4\pi)^2}\right)\frac{T^4}{m^2}$$

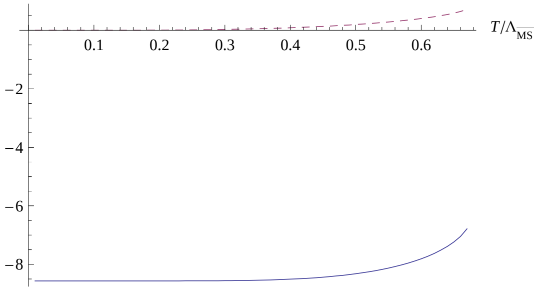
and no correction to $\langle A_\mu^2 \rangle$ up to this order.

Observation

We see a $\propto T^4$ behavior! This does not agree with the lattice results, but those are only for $T > 0.4T_c$.

Low T

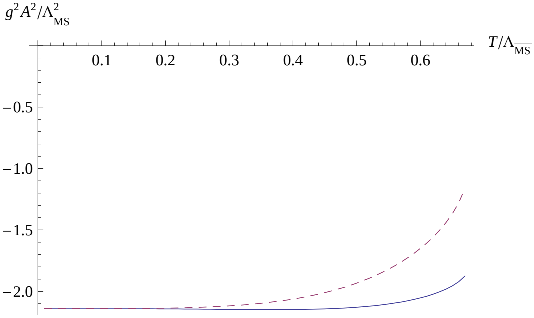
(We put $\bar{\mu}$ equal to $g\sigma'$ in the $T = 0$ minimum.)



(full line is $\langle A_\mu^2 \rangle$, dotted line is Δ_{A^2}, units $\Lambda_{\overline{\text{MS}}}$)

Low T

(We put $\bar{\mu}$ equal to $g\sigma'$ in the $T = 0$ minimum.)



(full line is magnetic part, dotted line is electric part, units $\Lambda_{\overline{\text{MS}}}$)

High T

- Once $T > 0.67\,\Lambda_{\overline{\mathsf{MS}}}$, no (real) solutions to the gap equation.
- There are solutions when ignoring the imaginary part of the effective potential.

High-T expansion

For the highest temperatures, we find

$$\langle A_\mu^2 \rangle = (N^2-1)\frac{T^2}{4} , \qquad \Delta_{A^2} = (N^2-1)\frac{T^2}{12} .$$

The effective gluon masses are negative and of order $g^2 T^2$ $\rightarrow$ need for resummation.

Resummation

- resum the cactus diagrams
- additional thermal mass
- This helps, but is not yet sufficient.

Conclusion

A full HTL resummation may be needed.

Overview

Conclusions

- It is possible to add the traceless part of $A_\mu A_\nu$ to the LCO formalism. The action is renormalizable.
- We computed the extra parameter ω.
- We computed the effective action in the presence of a nonzero $\langle A_\mu^2 \rangle$ and of an electric-magnetic asymmetry.
- At finite temperature, there is a non-vanishing asymmetry.
- At low temperature, the asymmetry behaves like $\propto T^4$ instead of exponentially.
- At high temperature, resummation is needed.
- For intermediary temperatures, a full treatment of confinement may be necessary.