Glueballs, gluon condensate, and pure glue QCD below T_c

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Brief introduction

Thermal QCD: basic facts

- $T = 0$
  - « Usual QCD »
  - Confinement

- $T > T_c$
  - QGP
  - Deconfinement

This work

- $T = T_c$
  - Phase transition
  - Weakly first-order

J. C. Collins and M. J. Perry, PRL **34**, 1353 (1975)
E. V. Shuryak, Phys. Rep. **61**, 71 (1980)

Quark-gluon plasma

- QGP: Dominant effort
  - Strongly coupled phase
    - Color interactions still strong
    - Experimental evidences at RHIC

      Nucl. Phys. A **757**, 1 (2005)
    - Surviving bound states?
  - Perturbative QCD, lattice QCD, phenomenological models, etc.

  - Stefan-Boltzmann limit

T_c

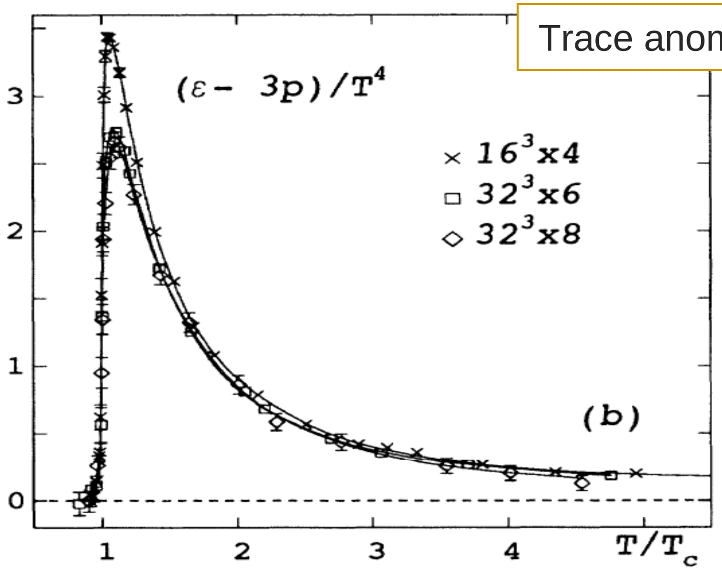
$T \sim (2-3)T_c$

$T \gg T_c$

Equation of state

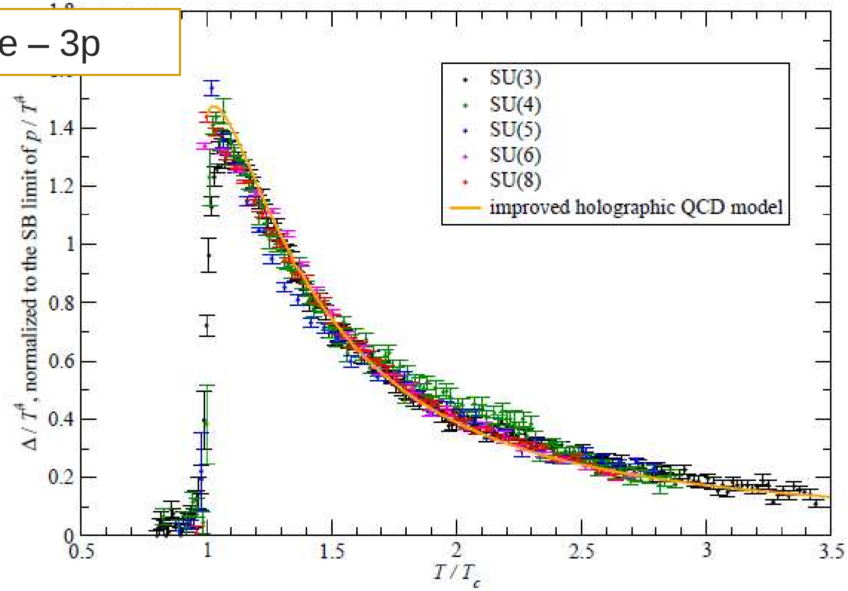
- First approach: pure glue QCD
  - Results from the lattice

G. Boyd *et al*., PRL **75**, 4169 (1995)



Trace anomaly ~ e – 3p

M. Panero, PRL **103**, 232001 (2009)



  - Much more available (with quarks, chemical potential, etc.)

Effective degrees of freedom

- Aim of phenomenological models
  - Description of the QCD thermodynamics using effective degrees of freedom

- $T < T_c$
  - Hadrons

- $T > T_c$
  - Quarks, antiquarks
  - Gluons

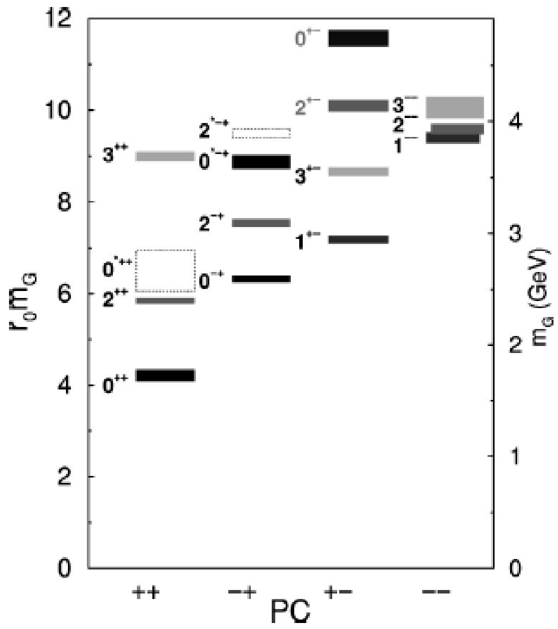
- First approach: pure glue QCD
  - Equation of state below T_c ~ Glueball gas

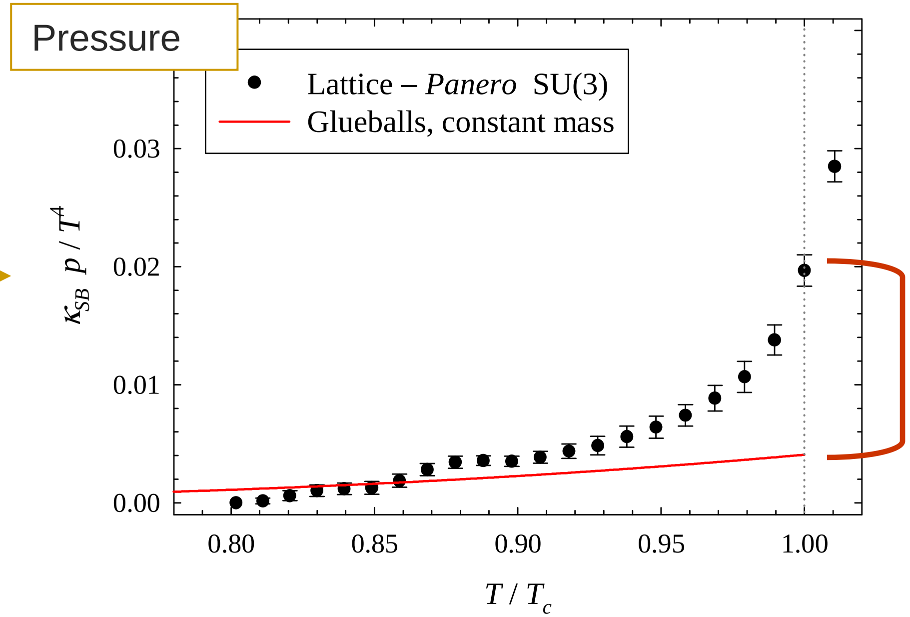
Pure glue QCD below T_c

Simple glueball gas (I)

- Basic model: Ideal Bose gas
  - Interactions between glueballs in $1/N_c^2$
  - Input: lattice spectrum, $T_c \sim 300$ MeV

C. J. Morningstar and M. J. Peardon, PRD **60**, 034509 (1999)





Simple glueball gas (II)

- Pressure severely underestimated
  - Glueball of mass m_g , spin J_g :

$$p_g = -\frac{(2J_g+1)T}{2\pi^2}\int_0^\infty dk\, k^2 \ln\left(1 - \mathrm{e}^{-\sqrt{k^2+m_g^2}/T}\right)$$

Suppression in $\mathrm{e}^{-m_g/T}$

  - How to avoid it?

Add higher-lying states whose degeneracy grows like $\mathrm{e}^{+m_g/T}$

Hagedorn spectrum

Keep the low-lying states but with m_g decreasing near T_c

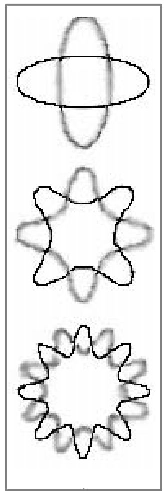
N. Ishii and H. Suganuma, EPJA **17**, 77 (2003)

Hagedorn spectrum

- Number of states growing like e^{+m/T_H}
  - Result of string theory
  - Might be suggested by experimental data (mesons and baryons)

    W. Broniowski, W. Florkowski and L. Y. Glozman, PRD **70**, 117503 (2004)

- Glueballs: Closed string picture
  - Fitted Hagedorn spectrum leads to an agreement with lattice data



H. B. Meyer, PRD **80**, 051502(R) (2009)

Alternative mechanism

- Unexplored: decreasing glueball mass near T_c
  - Found in dual Ginzburg-Landau theory

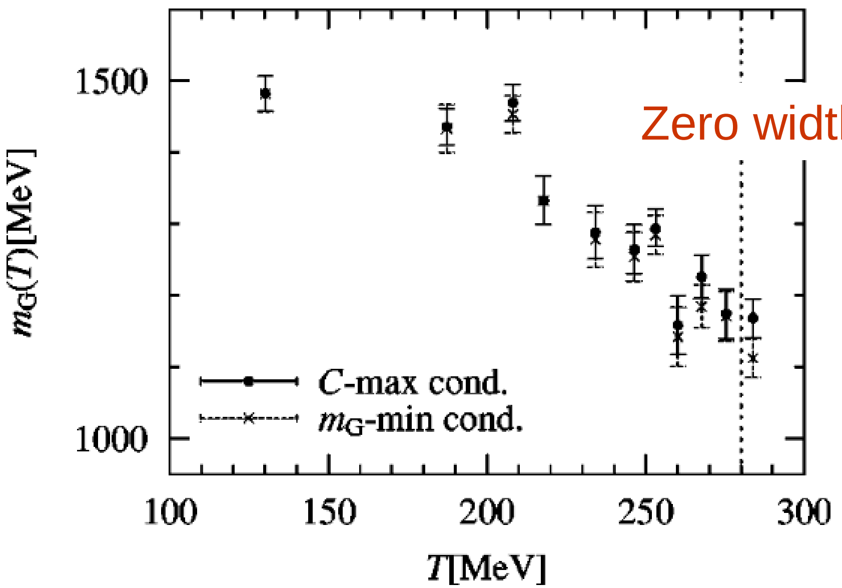
    H. Ichie, H. Suganuma and H. Toki, PRD **52**, 2944 (1995)
  - Found in lattice QCD for the 0^{++} and 2^{++} states

    N. Ishii, H. Suganuma and H. Matsufuru, PRD **66**, 094506 (2002)
- Can the equation of state be described by using such a starting point?
  - Schematic view: Bose-gas formalism

Glueball masses below T_c

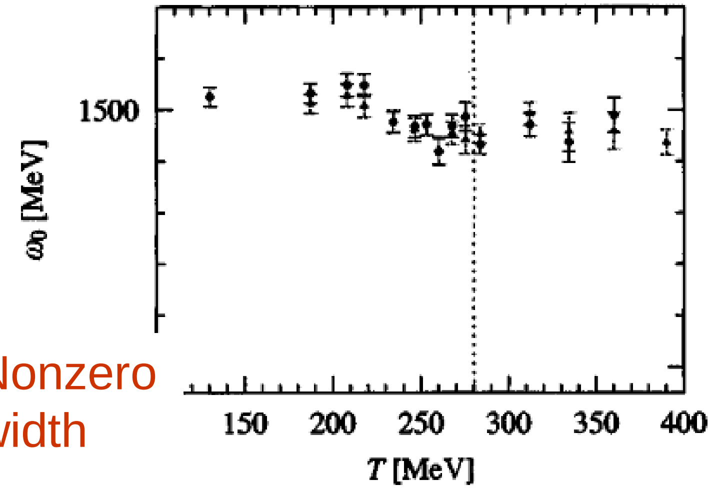
Results from the lattice (I)

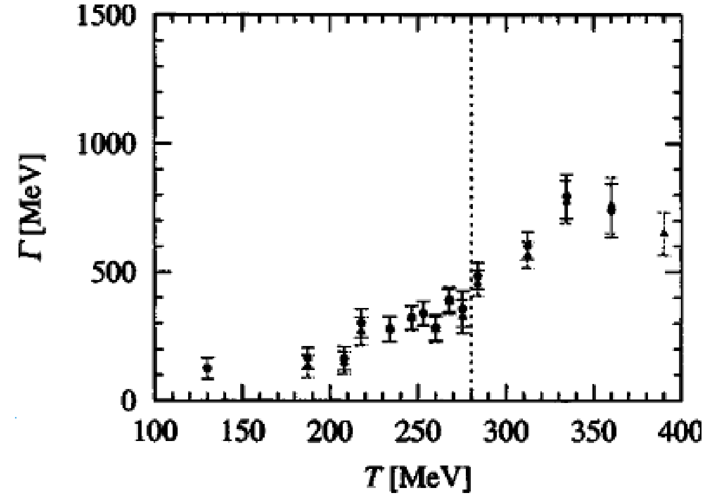
- Two fits for the glueball correlator



Zero width

Nonzero width





  - Same conclusions for the 0^{++} and 2^{++} states

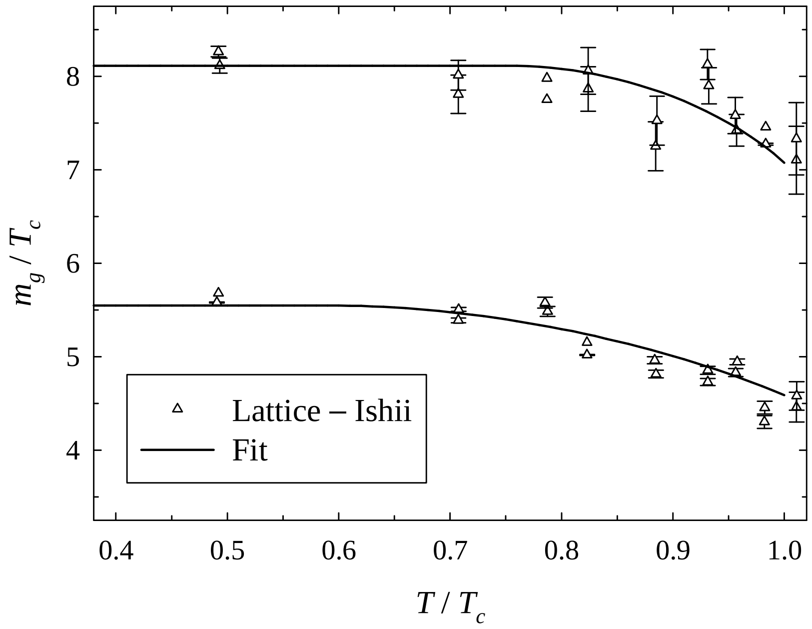
N. Ishii, H. Suganuma and H. Matsufuru, PRD **66**, 094506 (2002)

Results from the lattice (II)

- Breit-Wigner fit
  - Constant mass
  - Linearly increasing width
- Zero-width fit
  - Decreasing mass
  - Linked to increasing width

$$m_g(T) \approx \bar{m}_0 - 2T + \sqrt{4T^2 - \Gamma_g}$$

  - Used in an ideal-gas framework



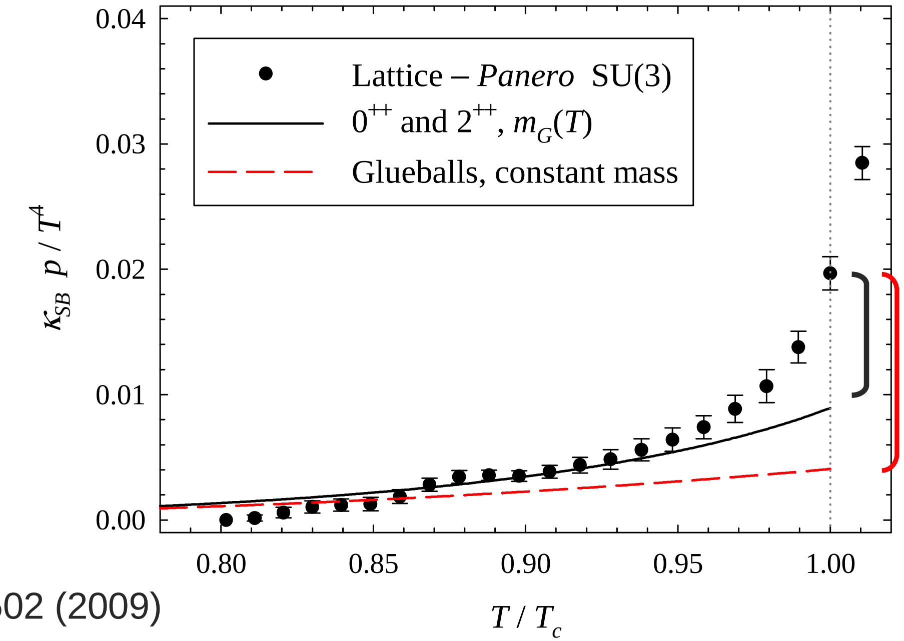
The pressure

- Glueball gas model $p \approx p_{0^{++}} + p_{2^{++}}$

$$p_g = -\frac{(2J_g+1)T}{2\pi^2}\int_0^\infty dk\, k^2 \ln\left(1 - e^{-\sqrt{k^2+m_g(T)^2}/T}\right)$$

  - Agreement improved

- Higher-lying states
  - Not dominant if they stay higher than the 2^{++}
    - Ok for the 0^{-+}



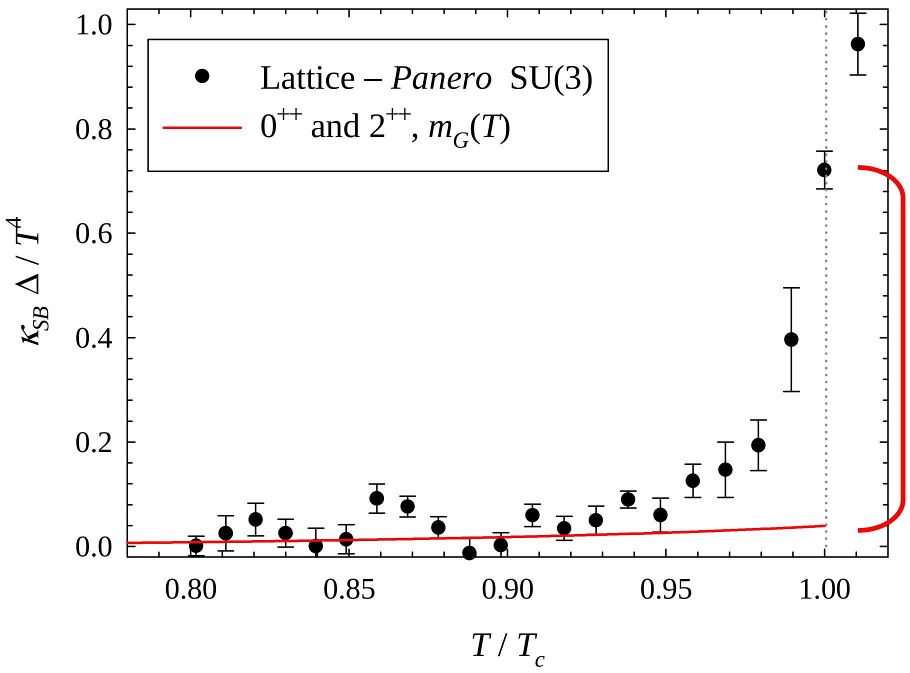
X. F. Meng *et al.*, PRD **80**, 114502 (2009)

The trace anomaly

- The trace anomaly, $\bar{\Delta} = T^5 \partial_T \left(\frac{p}{T^4} \right)$, is greatly underestimated
  - Something is missing
  - Trace anomaly

QCD

Gluon condensate



The gluon condensate

Known results

- The trace anomaly at finite T involves the gluon condensate $\langle G^2 \rangle_T = -\left\langle \frac{\beta}{g} G^a_{\mu\nu} G^{\mu\nu}_a (T) \right\rangle$

$$\Delta_{G^2} = \langle G^2 \rangle_0 - \langle G^2 \rangle_T$$

H. Leutwyler, Lecture given at the Workshop on Effective Field Theories, Dobogoko, Hungary, 22-26 August 1991

- Lattice results

M. D'Elia, A. Di Giacomo and E. Meggiolaro, PRD **67**, 114504 (2003)

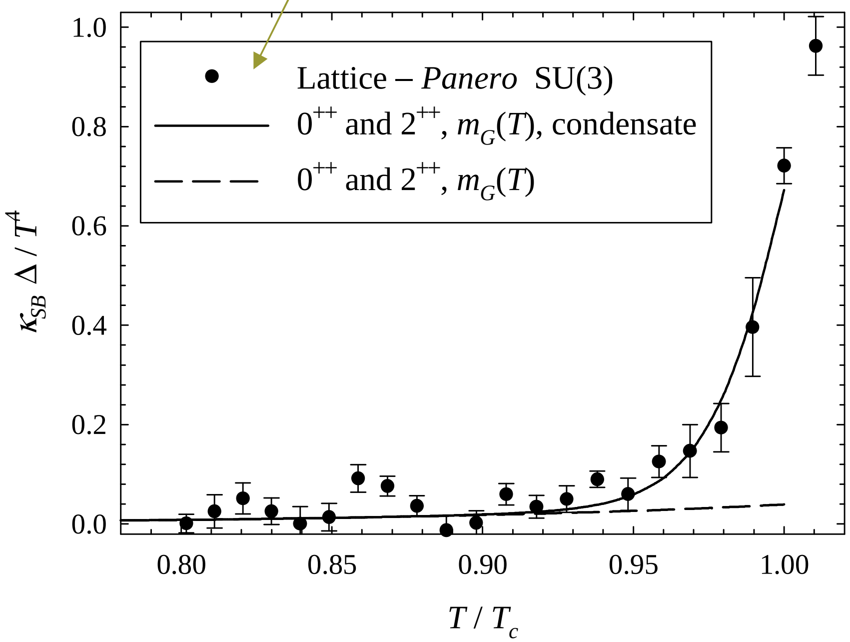
$$\langle G^2 \rangle_T = \langle G_e^2 \rangle_T + \langle G_m^2 \rangle_T$$

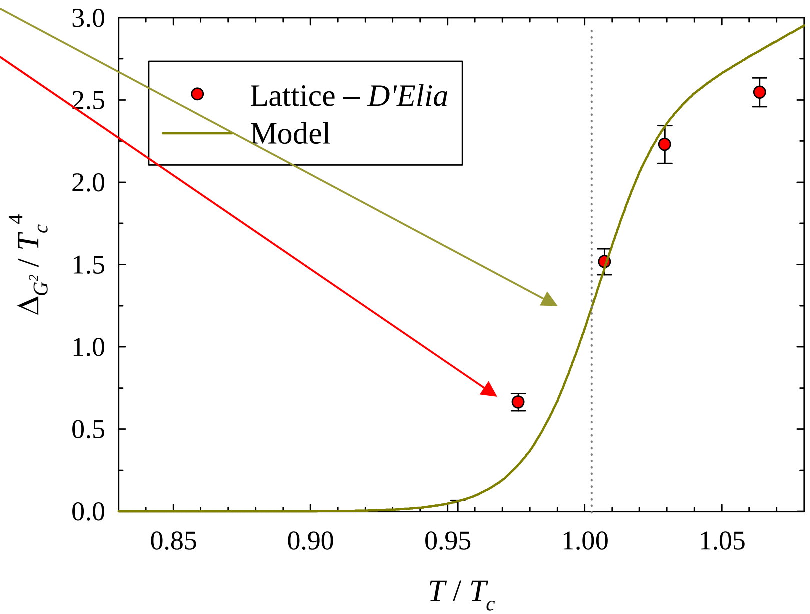
$$\langle G_m^2 \rangle_T \approx \langle G_m^2 \rangle_0$$

$\langle G_e^2 \rangle_0 \approx \langle G^2 \rangle_0 /2$ and abruptly vanishes near T_c

Return to the trace anomaly

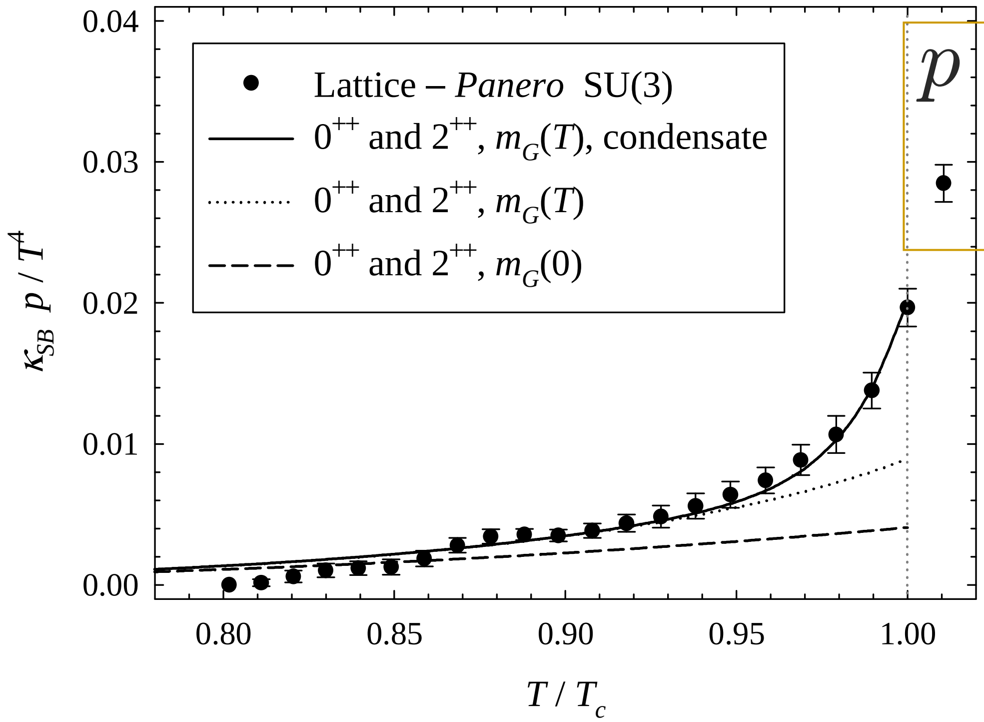
- The trace anomaly should be $\Delta = \bar{\Delta} + \Delta_{G^2}$
  - The term Δ_{G^2} needed to reproduce the equation of state fits the values obtained from lattice calculations





Return to the pressure

- Finally one adds the gluon condensate contribution to the pressure: $p_{G^2} = T^4 \int_0^T \frac{\Delta_{G^2}(x)}{x^5}\, dx$



$$p \approx p_{0^{++}} + p_{2^{++}} + p_{G^2}$$

Good agreement

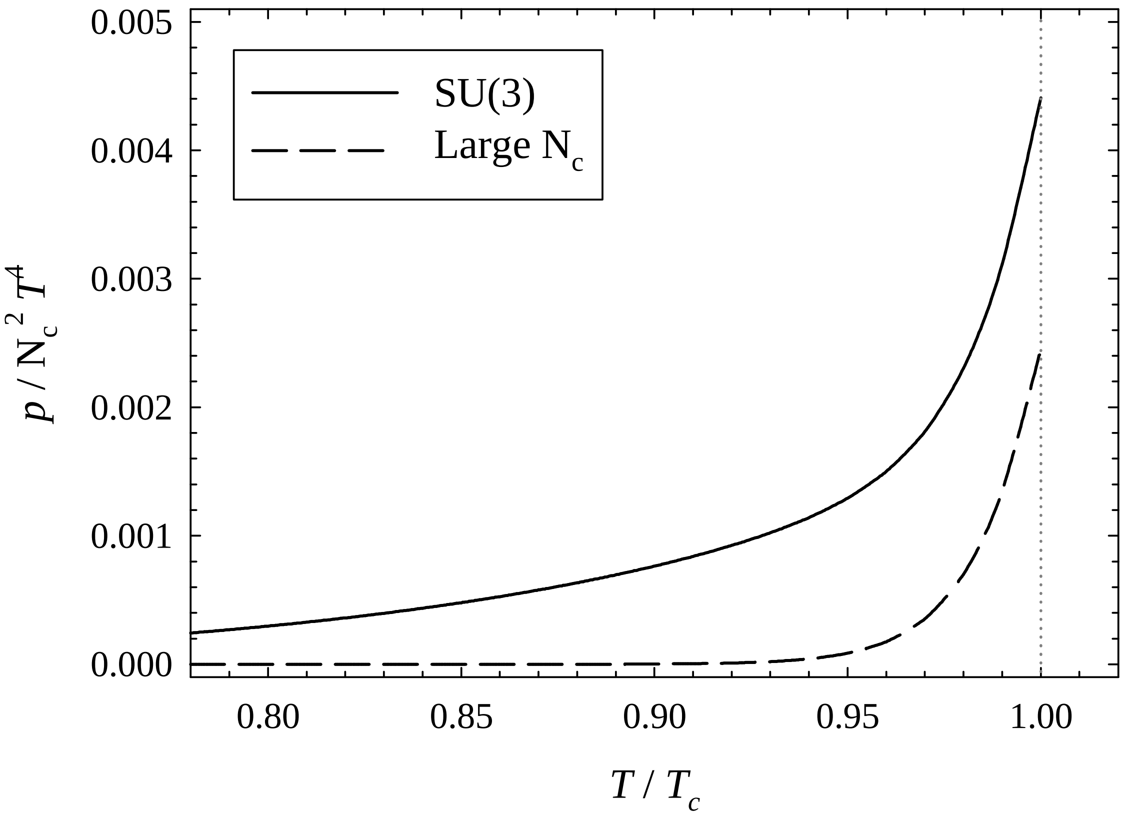
Large N_c limit

Scaling arguments

- Glueball masses are O(1)
  - Constant thermodynamical contribution from the glueballs
- The gluon condensate scales as N_c^2
  - Contribution in N_c^2
- Only the gluon condensate term survives in the equation of state normalised to N_c^2
  - Strong decrease of the pressure
  - Weak decrease of the trace anomaly

The pressure at large N_c

- Model prediction



Conclusions

Conclusion

- Description of the pure glue equation of state below T_c
  - Glueball gas + gluon condensate
  - Coherent mixing of several lattice results
    - Early stages of the phase transition
    - Glueball mass decrease
    - Gluon condensate behavior near T_c
- Large N_c limit, to be checked
  - Strong pressure suppression
  - Weak trace anomaly suppression