Solving the sign problem in a QCD related model

Y. Delgado[1], H. G. Evertz[2], C. Gattringer[1]

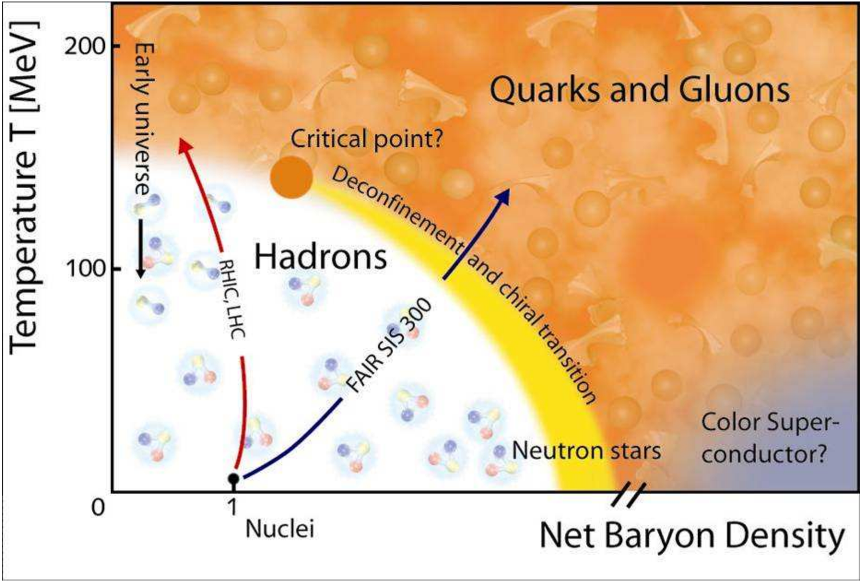
[1]Karl-Franzens University Graz
[2]Graz University of Technology

Les Houches, February 2011

Der Wissenschaftsfonds.

Y. Delgado, H. G. Evertz, C. Gattringer [arXiv:1102.3096]

Something we would like to understand



Sign problem of lattice QCD at finite density

- Lattice QCD has problems with finite chemical potential μ.

$$Z = \int D[q,\overline{q},A] e^{-S_G[A]-S_F[q,\overline{q},A;\mu]}$$
$$= \int D[A] e^{-S_G[A]} (detD[A;\mu])^{N_f}$$

- $detD[A;\mu]$ is complex for $\mu > 0$.

- Cannot be used as a probability in a Monte Carlo simulation.

Perspectives

- Only very little progress for LQCD with chemical potential due to the sign problem (complex phase problem).
- Some progress for QCD related effective theories.
- New concepts are developed, e.g. worm algorithms.

An effective theory for QCD thermodynamics

$$S_{eff} = -\sum_n \left(\tau \sum_{\nu=1}^{3} \left[P(n)P(n+\hat{\nu})^* + c.c. \right] \; + \; \kappa \left[e^{\mu} P(n) \; + \; e^{-\mu} P(n)^* \right] \right)$$

- The degrees of freedom are:

$$P(n) \in \mathbb{Z}(3) = \{1, e^{+i2\pi/3}, e^{-i2\pi/3}\}$$

- P is related to the Polyakov loop, which is a static source quark.

  $T < T_c$: $\langle P \rangle = 0 \; \rightarrow$ quarks confined

  $T > T_c$: $\langle P \rangle \neq 0 \; \rightarrow$ quarks deconfined

- τ increases with the temperature, and κ decreases with the quark mass.

Remarks

- The deconfined transition of pure gluodynamics can be understood through the spontaneous breaking of center symmetry.

$$\sum_{n} \tau \sum_{\nu=1}^{3} \left[P(n) P(n+\hat{\nu})^{*} + c.c. \right]$$

- Description by an effective 3d center symmetric spin model. Yaffe and Svetitsky (1981).

- Our theory also contains center symmetry breaking terms and chemical potential. These terms come from the fermion determinant.

$$\sum_{n} \kappa \left[e^{\mu} P(n) \; + \; e^{-\mu} P(n)^{*} \right]$$

- The structure of the new terms can be obtained from hopping expansion.

Flux representation - 1

- Effective center model still has complex action $\Rightarrow$ new variables!
- For the neighbor interaction and magnetic term, we use the Ansatz:

$$e^{\tau[P(n)P(n+\hat{\nu})^* + c.c.]} = C \sum_{b_{n,\nu}=-1}^{+1} B^{|b_{n,\nu}|} \left(P(n)P(n+\hat{\nu})^* \right)^{b_{n,\nu}}$$

$$e^{\kappa e^{\mu} P(n) + \kappa e^{-\mu} P(n)^*} = \sum_{s_n=-1}^{+1} M_{s_n} P(n)^{s_n}$$

- New variables:
  - **dimers** : $b_{n,\nu} \in \{-1, 0, +1\}$ on the link (n, ν).
  - **monomers**: $s_n \in \{-1, 0, +1\}$ on the site n.

Flux representation - 2

- The partition function in the flux representation:

$$Z = \sum_{\{b,s\}} \left(\prod_{n,\nu} B^{|b_{n,\nu}|} \right) \left(\prod_{n} M_{s_n} \right) \prod_{n} T \left(\sum_{\nu} [b_{n,\nu} - b_{n-\hat{\nu},\nu}] + s_n \right)$$

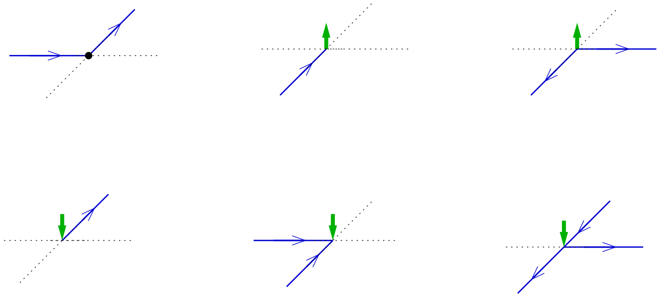
- Only real and positive contributions.

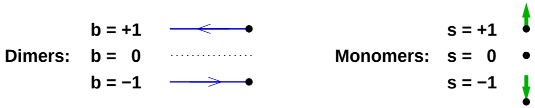
- Constraint $T(n)$: flux conservation modulo 3 at every site.

F. Karsch et al.(1984)

A. Patel,T. DeGrand,C. DeTar(1983)

Graphical representation of admissible terms





Goal of the numerical analysis

- Study the $\tau - \mu$ phase diagram.
- Identify the phase boundaries between confinement and deconfinement.
- Analyze the nature of the transitions.
- Location of the transition lines defined by the maxima of the Polyakov loop susceptibility χ_P and heat capacity C.

Worm Algorithm

- **Definition:**

  The worm algorithm is a MC simulation of the closed-path configurations of monomers and dimers from the flux representation.

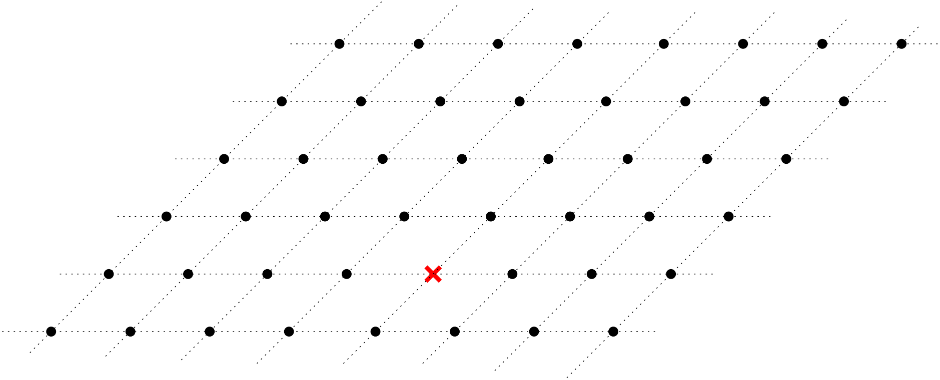
- **Advantages:**
  - Most suitable algorithm
  - Small autocorrelation time in critical regions (outperforms the cluster algorithm)

N. Prokof'ev and B. Svistunov (2001)

Y. Deng, M. Garoni and A. Sokal (2007)

Step 1

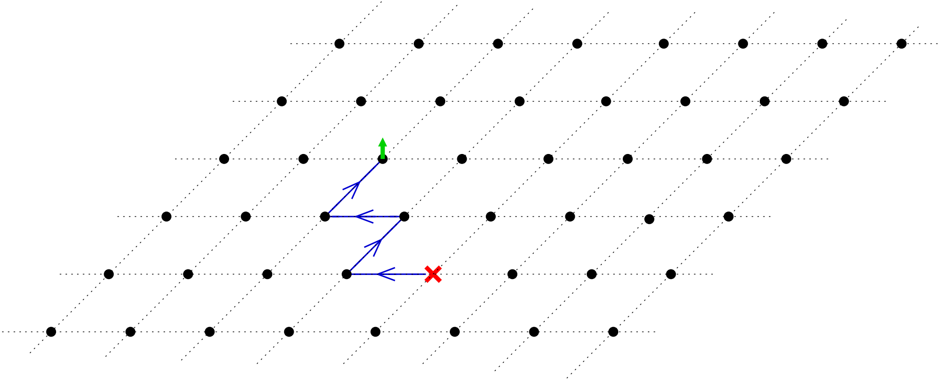
The worm starts at a random position of the lattice.



Step 2

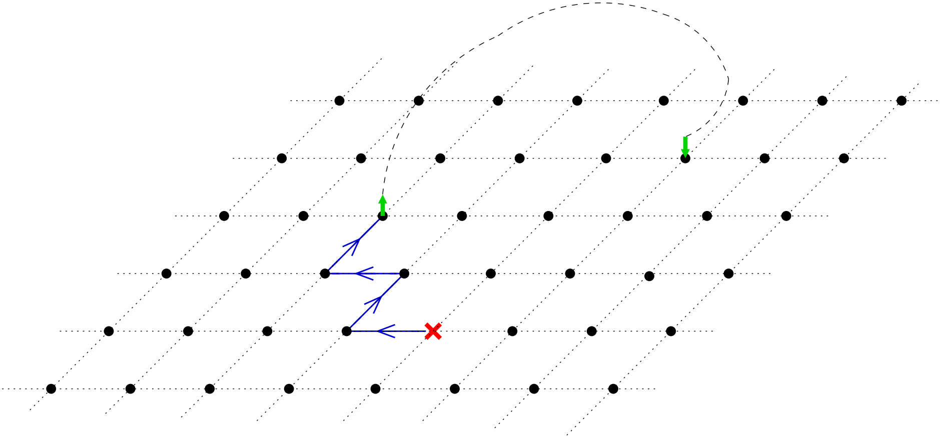
It may decide to insert dimers or monomers. Each update is accepted or rejected using the Metropolis criterion:

$$rand() \leq min(1, \frac{Weight_{new\ configuration}}{Weight_{old\ configuration}})$$



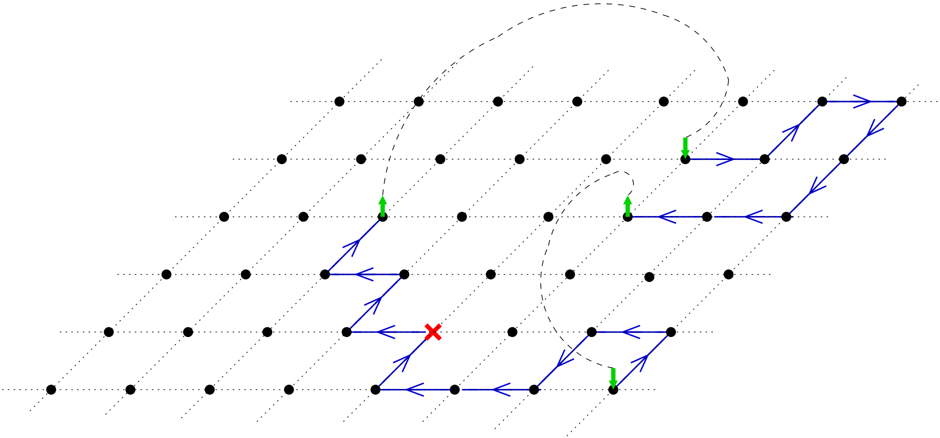
Step 3

The insertion of a monomer is followed by a random hop to another position, where again a monomer is inserted.



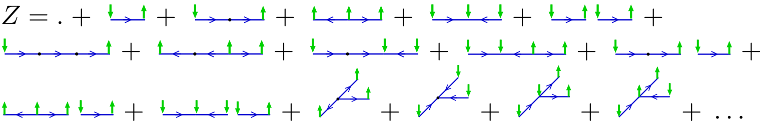
Step 4

These steps are continued until the worm closes.



Limiting case for testing the algorithm

- For small τ we determined the power series perturbatively for the partition sum, taking into account the terms up to τ^3:



Limiting case for testing the algorithm

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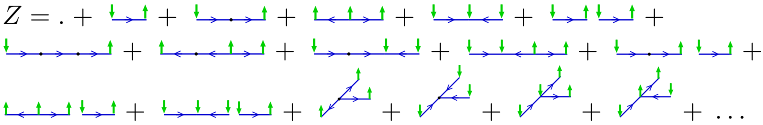
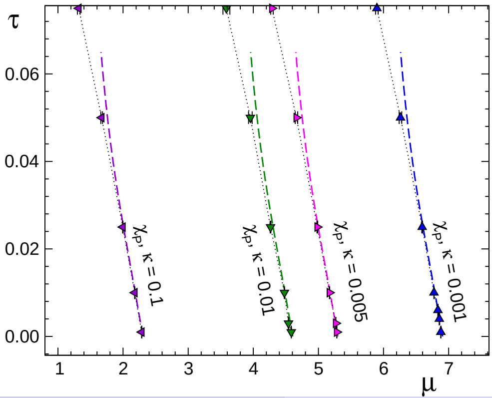
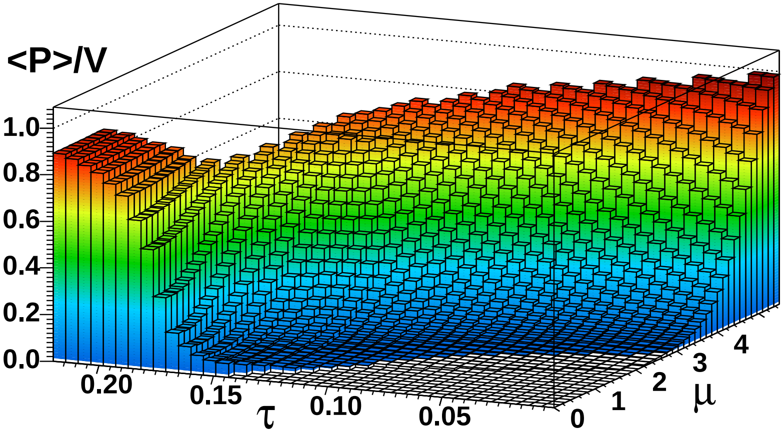


- Excellent agreement between the MC and the power series was found:



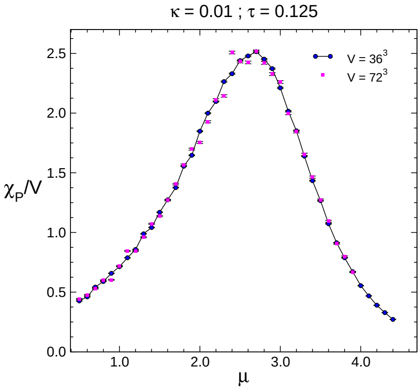
Order parameter $\langle P \rangle / V$

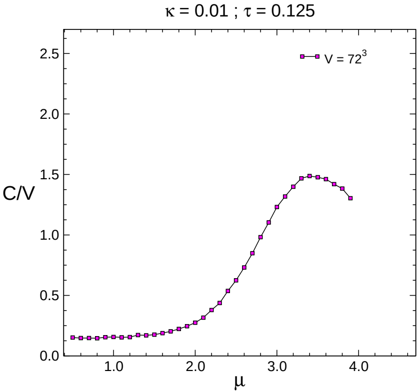


- $\langle P \rangle / V \rightarrow 0$, center symmetry is broken very mildly $\Rightarrow$ confined phase
- $\langle P \rangle / V \rightarrow 1$, center symmetry broken $\Rightarrow$ deconfined phase

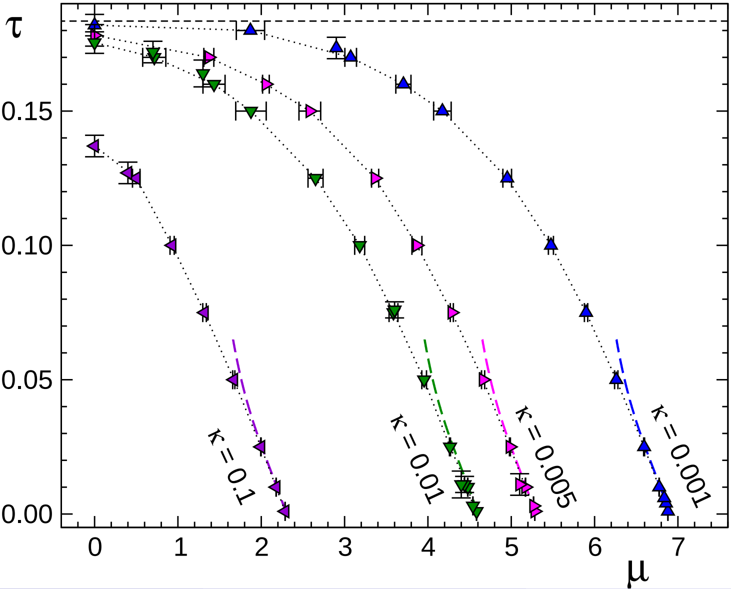
Determination of phase boundaries

- Location of phase boundaries determined by the maxima of χ_P and C.

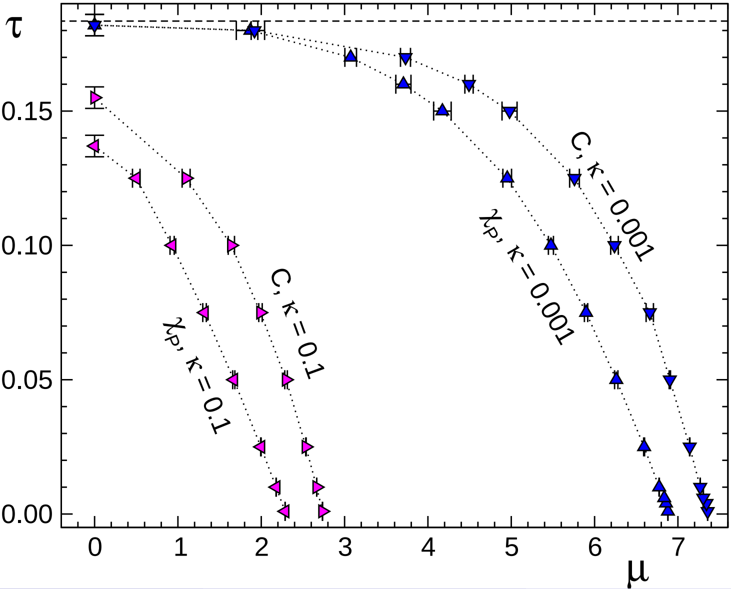




Phase diagram from χ_P



Comparison of phase boundaries from C and χ_P



Summary

- We have studied an effective center theory of QCD with finite quark density at non-zero temperature.
- In the flux representation the model is free of the complex phase problem and can be simulated with a generalized worm algorithm.
- The result is the phase diagram of QCD when only the center degrees of freedom are considered.
- Phase diagram qualitatively as expected for QCD.
- There are only crossover type transitions (unless $m_q >> 1$).
- We generated reference results at finite μ which can be used to test other approaches.
- **Future plans:**
  - Replace the spin variable $P(n)$ by more realistic degrees of freedom.
  - Implement the worm algorithm on GPUs.