The QCD Equation of State — From Nuclear Physics to Perturbative QCD

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Equilibrium thermodynamics

Conceptually simple goal: Evaluate grand potential of QCD

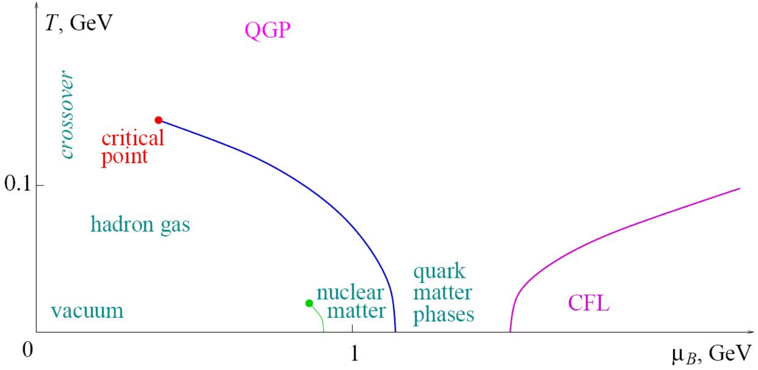
$$\Omega(T,\{\mu_f\},\{m_f\}) = -T\log\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int_0^\beta d\tau\int d^3x\mathcal{L}_{\rm QCD}}$$
$$\mathcal{L}_{\rm QCD} = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\bar{\psi}_f(\gamma_\mu D_\mu+m_f-\mu_f\gamma_0)\psi_f$$

Equilibrium thermodynamics from derivatives of Ω:

$$pV = -\Omega$$
$$sV = -\partial_T\Omega$$
$$n_fV = -\partial_{\mu_f}\Omega$$
$$\varepsilon = -p+Ts+\mu_f n_f$$

Equilibrium thermodynamics

In practice, no single method covers entire phase diagram ⇒ Need combination of (and interpolation between) several



Corners of the phase diagram

Parametrize the phase diagram by radial and angular variables:

$\rho \equiv \sqrt{T^2 + \frac{\mu_B^2}{6\pi^2}}$, $\theta \equiv \arctan \frac{\mu_B}{T}$

- ρ measures, how strongly coupled the system is
  - $\rho \lesssim 100$ MeV: Confinement; nuclear physics methods
  - 100 MeV $\lesssim \rho \lesssim 1$ GeV: Phase transition region, non-perturbative; lattice, effective theories,...
  - $\rho \gtrsim 1$ GeV: Weakly interacting, deconfined quasiparticles; weak coupling methods
- As function of θ, two separate, physically interesting regimes
  - $\theta \lesssim 1$: Quark-gluon plasma, heavy ion collisions, early universe
  - $\theta \approx \pi/2$: Cold nuclear matter, neutron stars

Challenge: Determine EoS throughout the phase diagram

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Hadron resonance gas models

For low densities and $T \ll T_c$, a good approximation is to treat the system as a (nearly ideal) gas of hadrons $\Rightarrow$ Hadron Resonance Gas Models (HRG)

Typical ingredients:

- Non-interacting, pointlike hadrons — finite volumes and interactions taken into account at higher energies
- Baryonic and mesonic states (from PDG) up to $M \sim$ 2-3 GeV; altogether typically $\mathcal{O}(100)$ stable hadrons and resonances included
- Overall strangeness neutrality

Hadron resonance gas models

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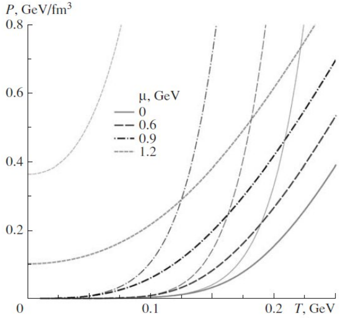
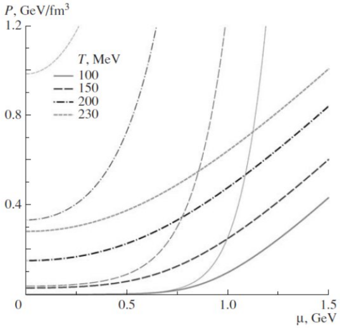
Typical result (Satarov *et al.*, 0901.1430):

$$\begin{aligned} P &= \sum_i \frac{g_i}{6\pi^2 T} \int_{m_i}^{\infty} \mathrm{d}E \frac{(E^2 - m_i^2)^{3/2}}{\exp[(E - \tilde{\mu}_i)/T] \pm 1}, \\ g_i &= \text{degeneracy factor for species } i, \\ \tilde{\mu}_i &\equiv \mu_i - v_i P \end{aligned}$$

Hadron resonance gas models

For low densities and $T \ll T_c$, a good approximation is to treat the system as a (nearly ideal) gas of hadrons $\Rightarrow$ Hadron Resonance Gas Models (HRG)





Satarov *et al.*, 0901.1430

Perturbative physics at high T

At $T \gg T_c$, with $g \ll 1$, degrees of freedom of QCD quarks and gluons.

Clear hierarchy between three length scales $\left[n_b(E) \equiv 1/(\mathrm{e}^{E/T} - 1)\right]$:

- $\lambda \sim 1/(\pi T)$: Wavelength of thermal fluctuations, inverse effective mass of non-static field modes ($p_0 \neq 0$)
  - $n_b(E)g^2(T) \sim g^2(T) \Rightarrow$ Contributes perturbatively at high T
- $\lambda \sim 1/(gT)$: Screening length of static color electric fluctuations, inverse thermal mass of A_0
  - $n_b(E)g^2(T) \sim g(T) \Rightarrow$ Physics somewhat perturbative at high T
- $\lambda \sim 1/(g^2 T)$: Screening length of static color magnetic fluctuations, inverse "magnetic mass"
  - $n_b(E)g^2(T) \sim g^0(T) \Rightarrow$ Physics non-perturbative at high T

No more longer length scales due to confinement

Perturbative physics at high T

Scale hierarchy $\Rightarrow$ Efficient description of long-distance physics through dimensionally reduced 3d effective theory (EQCD)

- Natural separation of contributions from hard, soft and ultrasoft scales to the EoS
- Not expanding effective theory parameters in powers of g_4 leads to improved convergence very similarly to HTL
- Current status $\mathcal{O}(g^6 \ln g)$, Kajantie *e*t al., hep-ph/0211321

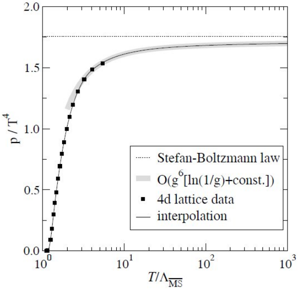
Generalization to finite μ straightforward; AV, hep-ph/0305183

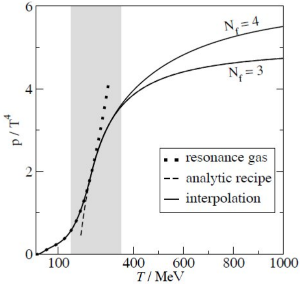
- Only minor technical complication
- Dimensional reduction approach valid as long as $g\mu \lesssim T$; Ipp *et al.*, hep-ph/0604060

Perturbative physics at high T

$$
\begin{aligned}
p_{\text{QCD}} \;=\;& T^4 \Big\{ p_0(\mu/T) + g^2\, p_2(\mu/T) + g^3\, p_3(\mu/T) + g^4 \ln g\, p_4'(\mu/T) \\
&+ g^4\, p_4(\mu/T) + g^5\, p_5(\mu/T) + g^6 \ln g\, p_6'(\mu/T) + g^6\, p_6(\mu/T) + \cdots \Big\}
\end{aligned}
$$

Contribution of scale πT to $\mathcal{O}(g^6)$ term still partially unknown; fitting coefficient to lattice results gives an almost perfect match down to $2T_c$ (Laine, Schröder, hep-ph/0603048)





Lattice QCD: Recent developments

Phase transition region both technically and conceptually most difficult to access: For quantitative results the only reliable method is lattice field theory

- Recent high-precision results for 2+1 quarks with physical masses: Convergence on T_c, EoS,...
- Extension of results to $\mu \lesssim T$ doable
- Low T matching to HRG results and high T matching to resummed perturbation theory very successful
  - Low T matching nearly perfect (after adjustment of hadron masses in some cases)
  - At high T, Endrodi *et al.*: Extension of lattice results to almost arbitrarily high T

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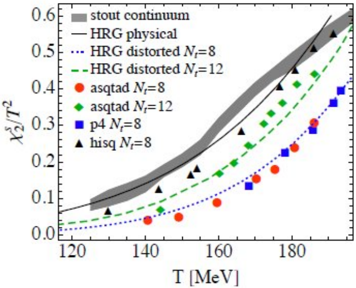

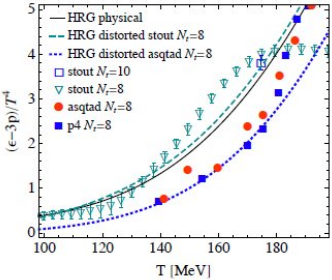

Borsanyi *et al.*, 1012.5215

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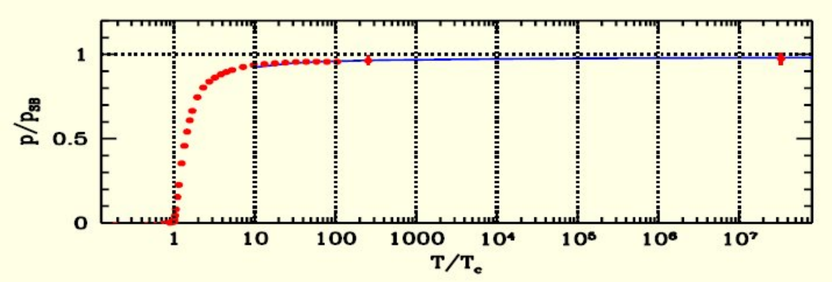

Endrodi *et al.*, 0710.4197

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Nuclear matter EoSs

Low energy nuclear physics not derivable from first principles, but experimentally under excellent control $\Rightarrow$ Several model EoSs, which agree well at low densities

Problems start with increasing density: Uncertainties from

- Composition of the matter: Hyperons, Kaon condensation,...
- Multi-nucleon interactions (typically neglected)
- Form of variational ansatz
- Details of hyperon interactions, kaon condensation potential, etc.

Nuclear matter EoSs

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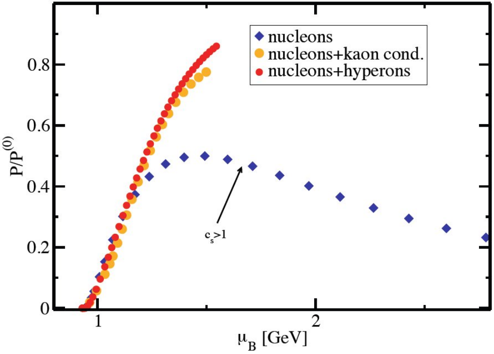


Quark matter EoS

At high densities, gauge coupling small $\Rightarrow$ Use weak coupling techniques to evaluate EoS

Problem: Color superconductivity non-perturbative in nature ($\Delta \sim e^{-\#/g}$), but formulating weak coupling calculations with anomalous propagators and vertices difficult

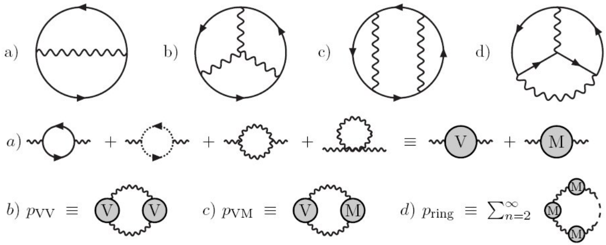
- At asymptopia, physical phase Color-Flavor-Locking (CFL)
- Physically, two competing effects: Pairing increases pressure, but deformation of Fermi surfaces decreases it

Leading order solution: Add condensation energy term to the pressure of unpaired quark matter

$$p = p_{\text{pert}} + \# \times \frac{\Delta^2 \mu_B^2}{3\pi^2}$$

Quark matter EoS

Complication in evaluation of p_{pert}: For practical applications, must keep strange quark mass non-zero — state of the art three loops (Kurkela, Romatschke, AV, 1012.1856)



Also need to enforce β-equilibrium and charge neutrality.

Quark matter EoS

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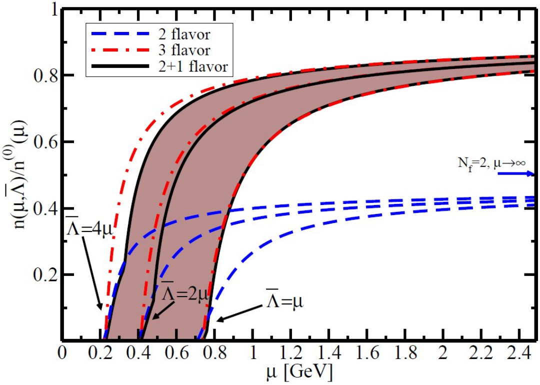


Interpolation to intermediate densities

For densities relevant for neutron star interiors, no quantitatively reliable method available

- Nuclear matter EoSs differ wildly
- Weak coupling expansions show poor convergence

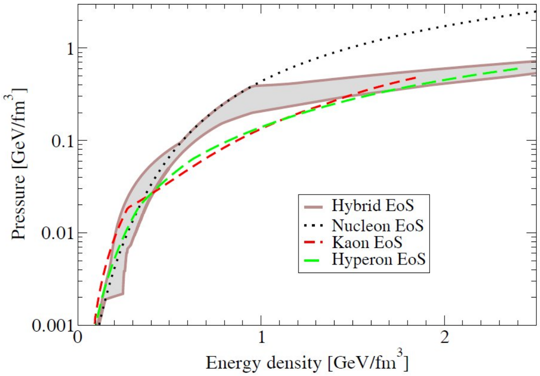
In particular, details of the phase transition remain unknown: μ_c, existence of mixed phase,...

Two possibilities:

1. Model calculations based on symmetries
    - Conceptually appealing but hard to estimate validity
2. Interpolation between trusted limits, requiring thermodynamically stable matching
    - Hope: Bulk thermo insensitive to details of phase structure

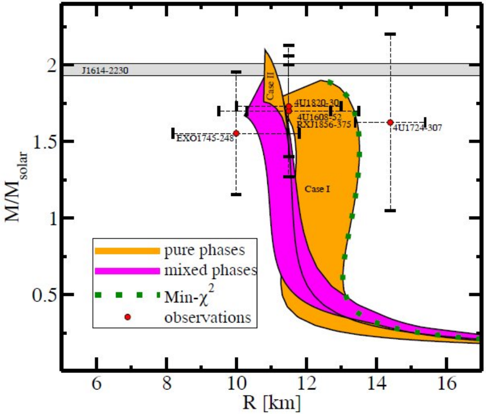
Interpolation to intermediate densities

Result of thermodynamic matching: EoS band for all densities;
Kurkela, Romatschke, AV, Wu (1006.4062)



Interpolation to intermediate densities

Ultimately, mass-radius measurements of neutron stars will determine the correct EoS; Kurkela, Romatschke, AV, Wu (1006.4062)



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Conclusions

$$\rho \equiv \sqrt{T^2 + \frac{\mu_B^2}{6\pi^2}}, \quad \theta \equiv \arctan\frac{\mu_B}{T} \ :$$

	$\rho \lesssim$ 100 MeV:	0.1 GeV $\lesssim \rho \lesssim$ 1 GeV	$\rho \gtrsim$ 1 GeV
$\theta \lesssim 1$	Gas of weakly int. hadrons; HRG	Non-perturbative; lattice QCD	Quasiparticles; perturb. theory
$\theta \approx \pi/2$	Nuclear matter; potential models	?????	CFL phase; weak coupling