Finite temperature lattice QCD with GPUs

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work done in collaboration with:

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Feynman Path Integral

$$Z = \int DA_\mu D\psi D\overline{\psi}\, e^{-S_E}$$

$$S = \int dx^4 \left(\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \overline{\psi} \Big(i\gamma^\mu D_\mu - m \Big) \psi \right)$$

Operator Mean Value

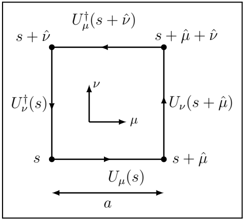
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DA_\mu D\psi D\overline{\psi} \mathcal{O}\left(\psi, \overline{\psi}, A_\mu\right)\, e^{-S_E}$$

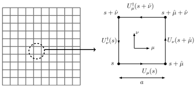
- We use the quenched approximation (neglect the fermionic part):
  - Lattice Gluon Action:

$$S_G = \beta \sum_{s \in \text{lattice}} \sum_{\mu<\nu} P_{\mu\nu}(s)$$

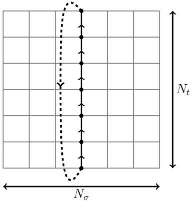
  - where the plaquette, $P_{\mu\nu}(s)$, is given by

$$P_{\mu\nu}(s) = 1 - \frac{1}{N} \mathrm{Re}\ \mathrm{Tr} \left[U_\mu(s) U_\nu(s+\hat{\mu}) U^\dagger_\mu(s+\hat{\nu}) U^\dagger_\nu(s) \right]$$

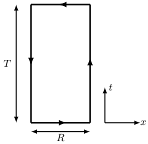




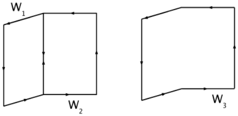
- Plaquette $\rightarrow E_i^2$ and B_i^2



- Polyakov Loop $\rightarrow$
  $\langle L(x) \rangle = \prod_{t=0}^{N_\tau - 1} U_{\mu=0}(x,t) \propto \exp\left(-\frac{F_q}{T}\right)$
  $\langle L \rangle = 0 \rightarrow$ **confinement**
  $\langle L \rangle \neq 0 \rightarrow$ **deconfinement**
- Two Polyakov Loops $\rightarrow$ color averaged free energy:
  $e^{-F_{\mathbf{avg}}(r,T)/T + C} = \frac{1}{N^2} \left\langle \mathrm{Tr}\, L(y) \mathrm{Tr}\, L^\dagger(x) \right\rangle$



- Wilson Loop $\rightarrow$ quark-antiquark potential



- Wilson Loop of the Static gluon-quark-antiquark system $\rightarrow W_{gq\bar{q}} = W_1 W_2 - \frac{1}{3} W_3$

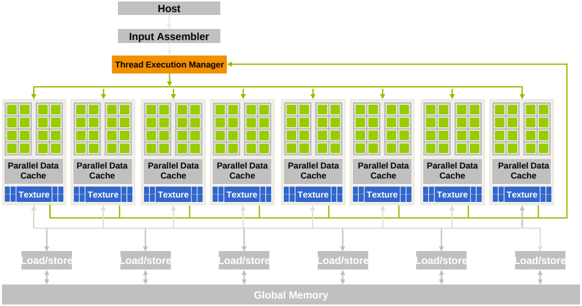
- Wilson loop:

$$
\begin{aligned}
W_3 &= \mathrm{Tr}\, U \\
W_8 &= \left(|W_3|^2 - 1\right) \\
W_6 &= \frac{1}{2}\left[(\mathrm{Tr}\, U)^2 + \mathrm{Tr}\, U^2\right] \\
W_{15a} &= \mathrm{Tr}\, U\, W_6 - \mathrm{Tr}\, U \\
W_{10} &= \frac{1}{6}\left[(\mathrm{Tr}\, U)^3 + 3\mathrm{Tr}\, U \mathrm{Tr}\, U^2 + 2\mathrm{Tr}\, U^3\right] \\
W_{24} &= \mathrm{Tr}\, U\, W_{10} - W_6 \\
W_{27} &= |W_6|^2 - |W_3|^2 \\
W_{15s} &= \frac{1}{24}\left[(\mathrm{Tr}\, U)^4 + 6\,(\mathrm{Tr}\, U)^2\, \mathrm{Tr}\, U^2 + 3\left(\mathrm{Tr}\, U^2\right)^2 + \right. \\
&\quad \left. 8\mathrm{Tr}\, U \mathrm{Tr}\, U^3 + 6\mathrm{Tr}\, U^4\right]
\end{aligned}
$$

- Casimir scaling, $d_D = \mathcal{H}_D / \mathcal{H}_3$:

$$
\begin{aligned}
d_8 &= 2.25 \\
d_6 &= 2.5 \\
d_{15a} &= 4 \\
d_{10} &= 4.5 \\
d_{27} &= 6 \\
d_{24} &= 6.25 \\
d_{15s} &= 7
\end{aligned}
$$

- Using GPUs to generate pure gauge lattice configurations by heat bath method
- All the computation is done in the GPUs
- The CPU only controls de GPUs and saves the results to files.
- This was done with CUDA and OPENMP (allowing to use several GPUs on the same PC.

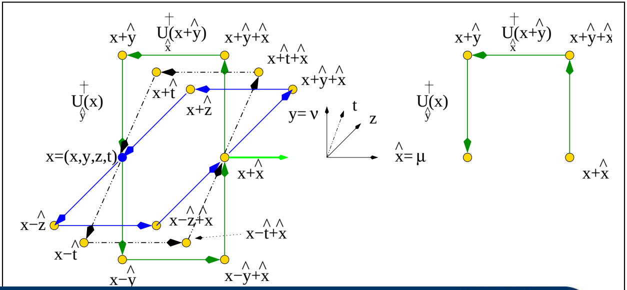


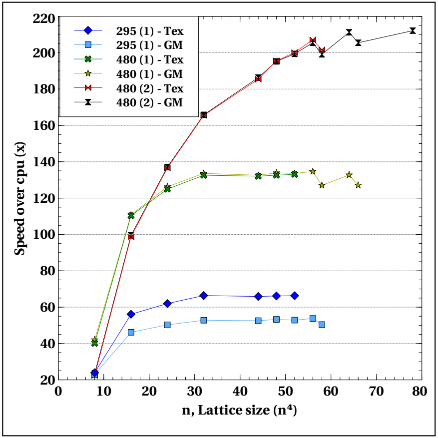
GPUs Tested

NVIDIA Geforce GTX	295 (GT200)	480 (Fermi)
Number of GPUs	2	1
CUDA Capability	1.3	2.0
Number of cores	2×240	480
Global memory	1792 MB GDDR3 (896MB per GPU)	1536 MB GDDR5
Number of threads per block	512	1024
Registers per block	16384	32768
Shared memory (per SM)	16KB	48KB or 16KB
L1 cache (per SM)	None	16KB or 48KB
L2 cache (per SM)	None	768KB
Clock rate	1.37 GHz	1.40 GHz

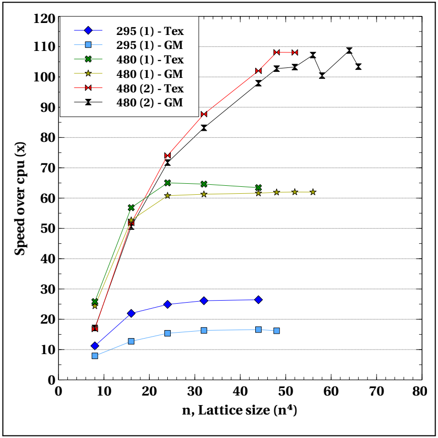
- **CPU:**
  - Intel(R) Core(TM) i7 CPU 920, 2.67GHz
  - L2 Cache 8 MB
  - RAM - 12GB

- Method for each link on the SU(2) lattice:
  1. Calculate the normalized staple, $\overline{U} = V/k$, where $k = \sqrt{\det V}$;
  2. Evaluate $a_0 = 1 + \frac{ln(x)}{\beta k}$ with $x \in \left[e^{-2\beta k}, 1\right]$;
  3. Accept a_0 with probability $\sqrt{1 - a_0^2}$;
  4. End or go to step (2) if a_0 is not accepted;
  5. Choose **a** randomly in a sphere of radius $\sqrt{1 - a_0^2}$;
  6. $U' = a_0\mathbf{1} + i\vec{a} \cdot \vec{\sigma}$;
  7. $U \rightarrow U'\overline{U}$.
- SU(3): Similar to SU(2) heat bath, make the heat bath in 3 steps, using 3 blocks with 2x2 from the SU(3) matrix.



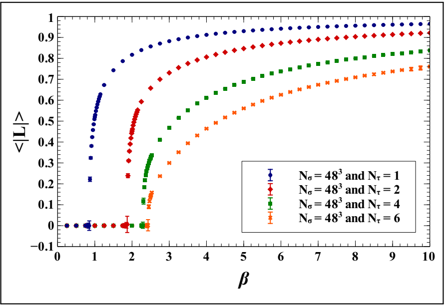


- Single precision

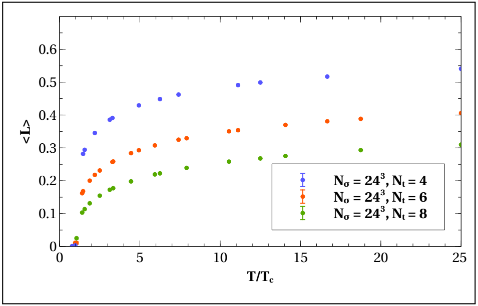


- Double precision

- Polyakov Loop versus $\beta(g^2)$

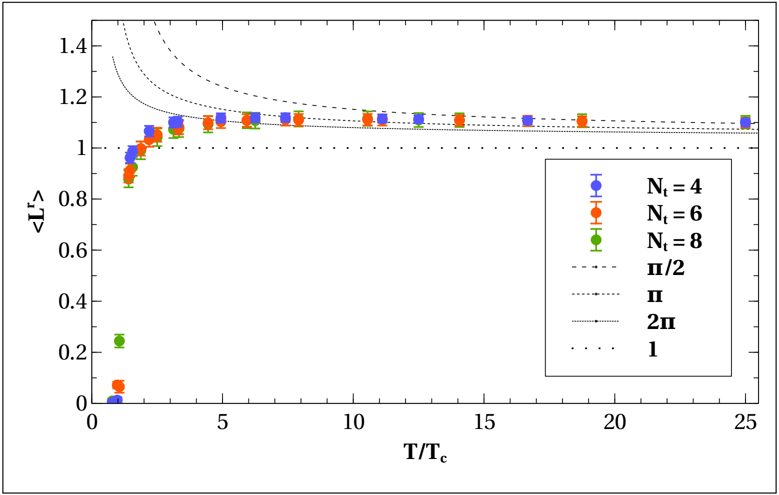


- Polyakov Loop versus T/T_c



There is a dependence on the extension of the lattice in time direction. This is due to the self-energy contribution of the static quark source used as order parameter. Elimination of this self energy term is necessary to obtain an order parameter which is a function of the temperature alone.

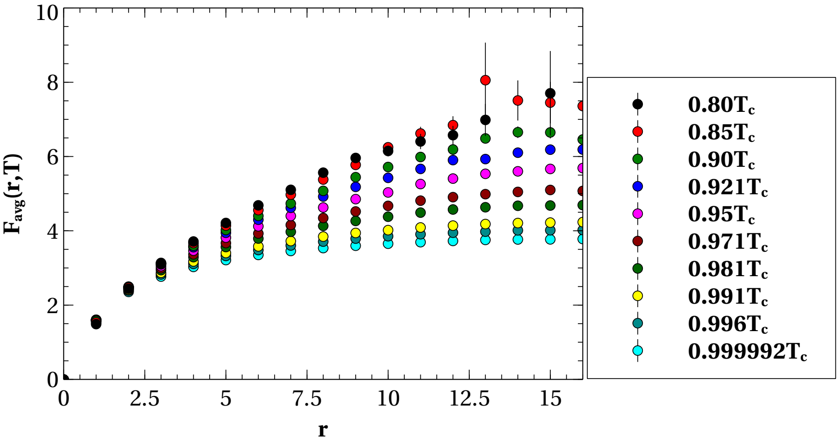
$$\langle L^r \rangle = Z^{N_\tau} \langle L \rangle$$



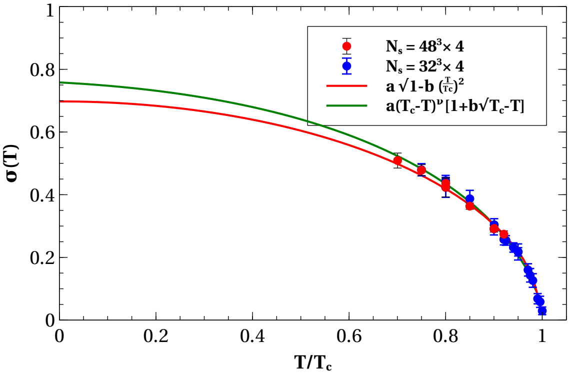
- $\pi/2$, π, 2π: Polyakov loop in HTL perturbation theory

- To eliminate the trivial temperature dependence due to the color trace normalization

$$F_{avg}(r,T) \to F_{avg}(r,T) - \frac{T}{T_c}\log N^2$$



- Fitting $F_{\text{avg}}(r, T)$ with $V(r, T) = a_0(T) - \frac{a_1(T)}{r} + \sigma(T) r$



- Fit with $\sigma(T) = a\sqrt{1 - b\,(T/T_c)^2}$
  - $a = 0.6976 \pm 0.0176$
  - $b = 0.9990 \pm 0.0059$
  - $\chi^2/dof = 0.732$

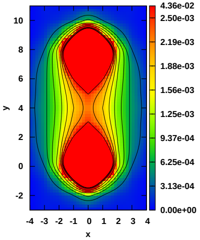
- Fit with $\sigma(T) = a\,(T_c - T)^{\nu}[1 + b\sqrt{T_c - T}]$
  - $\nu = 0.63$ fixed
  - $a = 1.5541 \pm 0.0435$
  - $b = -0.5122 \pm 0.0576$
  - $\chi^2/dof = 0.598$

- colour fields:
  - gluon-quark-antiquark system;
  - SU(3) representations and Casimir scaling.

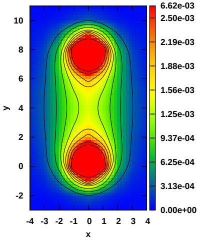
Flux tube: two gluon glueball and quark-antiquark

- results with APE smearing only and 287 configurations.
- **static gluon-gluon system**

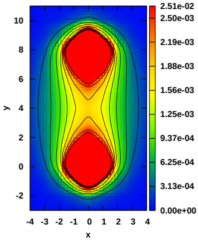
- $\langle E^2 \rangle$



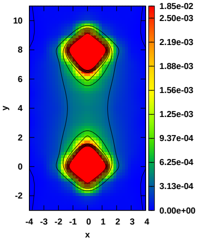
- $-\langle B^2 \rangle$



- $\mathcal{L}$

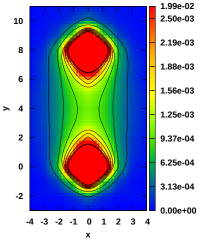


- $\mathcal{H}$

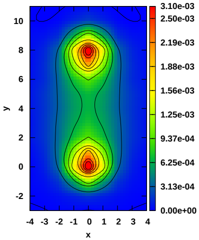


- **static quark-antiquark system**

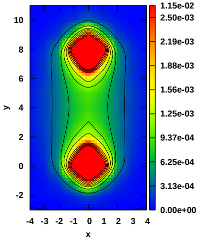
- $\langle E^2 \rangle$



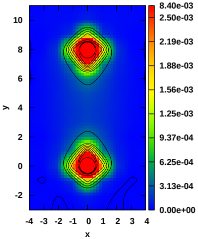
- $-\langle B^2 \rangle$



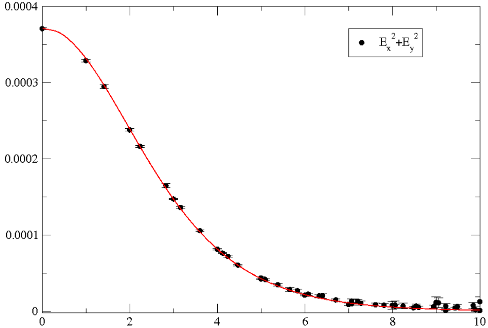
- $\mathcal{L}$



- $\mathcal{H}$



- Dual Superconductor Model
  - Quark/antiquark in a meson ↔ Magnetic monopoles in a superconductor
  - Chromoelectric field ↔ Magnetic Field
  - Confinement ↔ Meissner Effect
  - Dual gluon mass ↔ Effective Photon mass
  - Both give a constant force at large quark-antiquark (monopole-antimonopole) distances



ξ	λ	μ (MeV)	κ	χ^2/dof
2.38 ± 0.13	3.45 ± 0.14	789 ± 32	1.44 ± 0.14	1.10

(a) $\langle E^2 \rangle$

(b) $-\langle B^2 \rangle$

(c) $\mathcal{L}$

(d) $\mathcal{H}$

- results with APE smearing in space and HYP in time direction.

(e) $\langle E^2 \rangle$

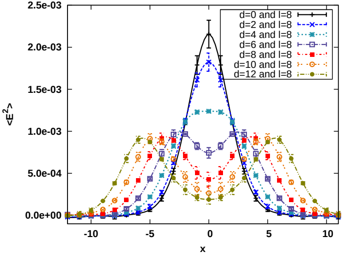
(f) $-\langle B^2 \rangle$

(g) $\mathcal{L}$

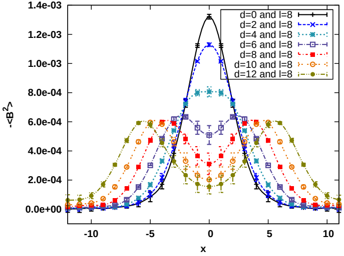
(h) $\mathcal{H}$

Results for the U geometry at $y = 4$ and $z = 0$.

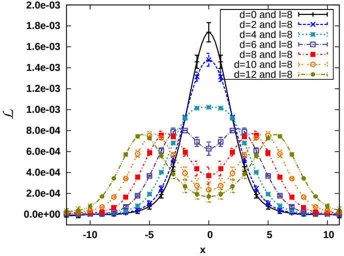
- $\langle E^2 \rangle$



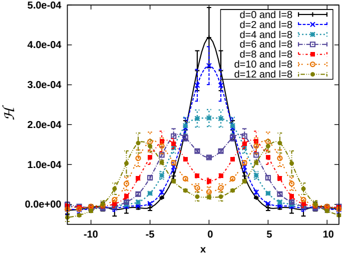
- $-\langle B^2 \rangle$



- $\mathcal{L}$

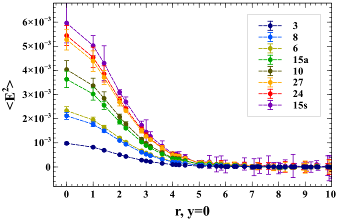


- $\mathcal{H}$

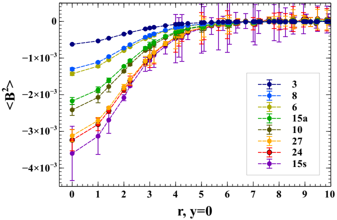


287 configurations with APE (space) and HYP (time) smearing.
Sources in the xy plane. Sources at (-4,0,0) and (4,0,0)
Results for the xz plane between the sources, in the middle of the flux tube.

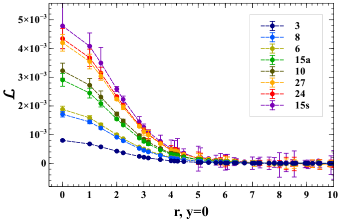
- $\langle E^2 \rangle$



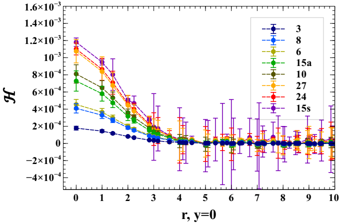
- $\langle B^2 \rangle$



- $\mathcal{L}$

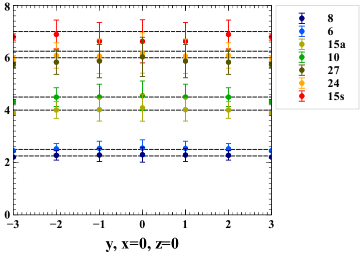


- $\mathcal{H}$

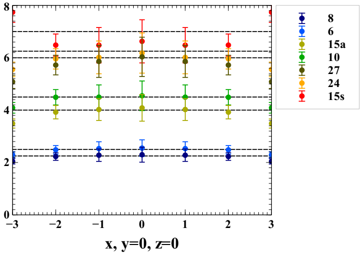


Sources in the xy plane. Sources at (-4,0,0) and (4,0,0)

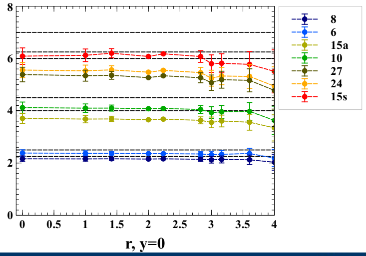
- xy plane at x=0



- xy plane at y=0



- xz plane at y=0



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