Themodynamics at strong coupling from Holographic QCD

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APC, U. Paris VII

Excited QCD
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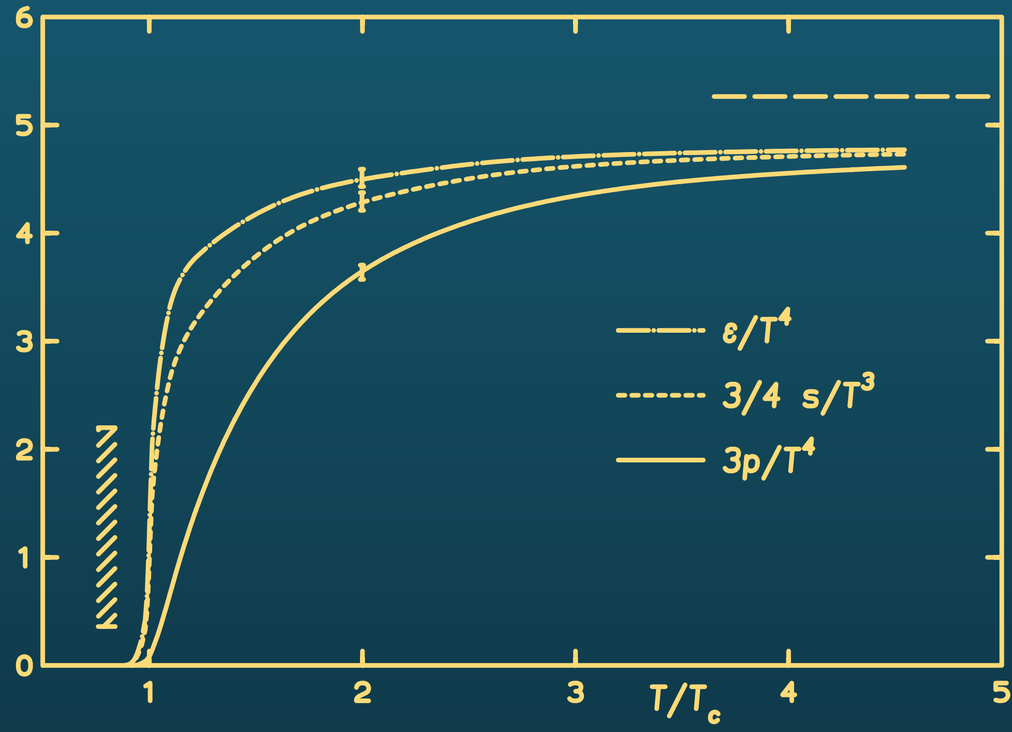
Work with E. Kiritsis, U. Gursoy, L. Mazzanti, G. Michalogiorgakis, '07-'10

Motivation: Thermodynamics of 4D YM

4D Pure YM theory exhibits a first order deconfining phase transition around $T_c \sim 260 MeV$.



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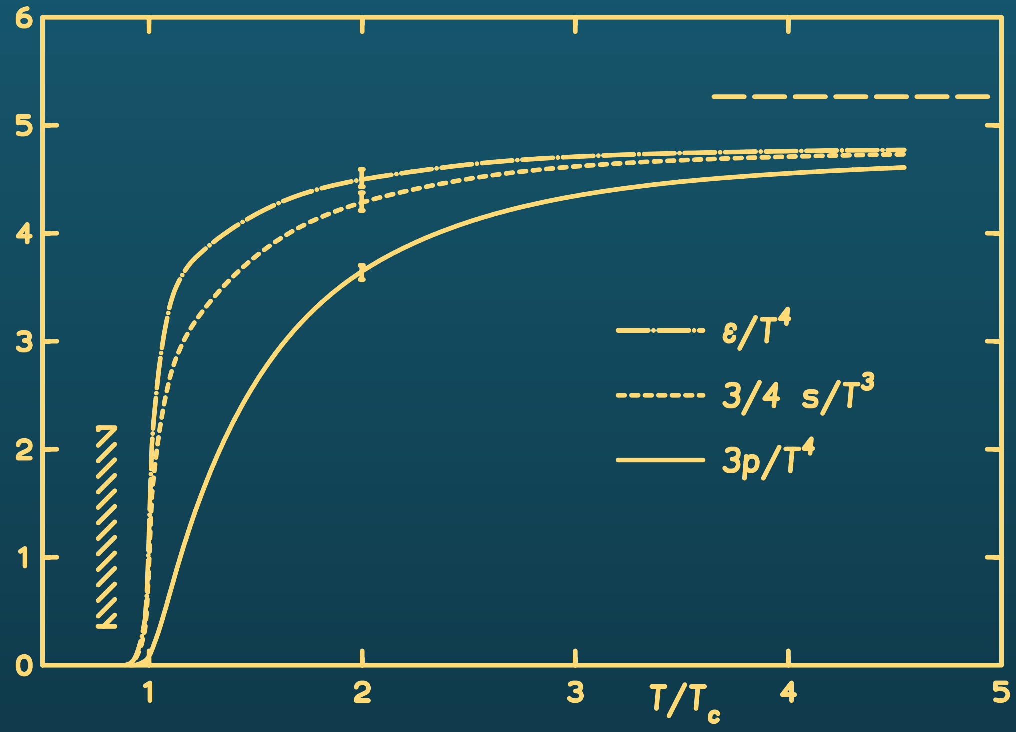

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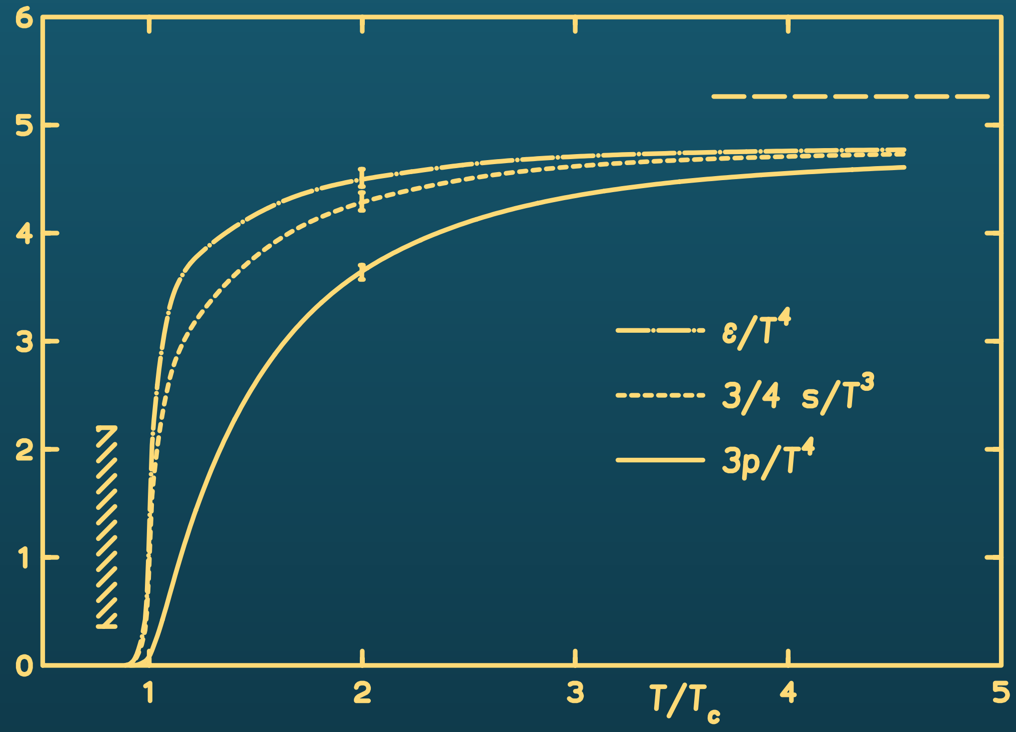

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- Can AdS/CFT techniques reproduce the phase transition and the YM equation of state found on the lattice?
- If yes, can it compute hydrodynamic and transport coefficient of the QCD plasma?

Outline

- AdS/CFT 101
- Holographic description of deconfinement transition
  - AdS examples
  - 5D Phenomenological models

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I will restrict the discussion to the 5D (x^μ, r) directions, and consider general non-conformal backgrounds.

Field/Operator correspondence

- Gravity dual described by an action

$$S_{grav} = M_p^2 \int d^5x \sqrt{g} R - (\partial\Phi)^2 + \ldots$$

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Gravity description valid at large N, large $g^2 N$

Finite Temperature

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The partition function is given by the gravity action evaluated at its extremum (solution of the of the gravity-side field equation)

$$\mathcal{Z}(\beta) = e^{-S_{grav}[g_0, \Phi_0]}$$

where g_0 and Φ_0 are periodic with period β.

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- Phase transition happens at T_c where $\mathcal{F}_1(T_c) = \mathcal{F}_2(T_c)$
- From $\mathcal{F}_i(T)$ we can compute other equilibrium quantities (entropy, pressure, speed of sound) using standard thermodynamic formulae.

Plasma/Black Hole correspondence

Holographic description of a deconfined plasma in thermal equilibrium at temperature $T \equiv 1/\beta$:

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- Due to AdS boundary conditions, BH is stable.
- Polyakov loop gets a vev, signaling deconfinement.
- Spectrum of quasinormal modes.

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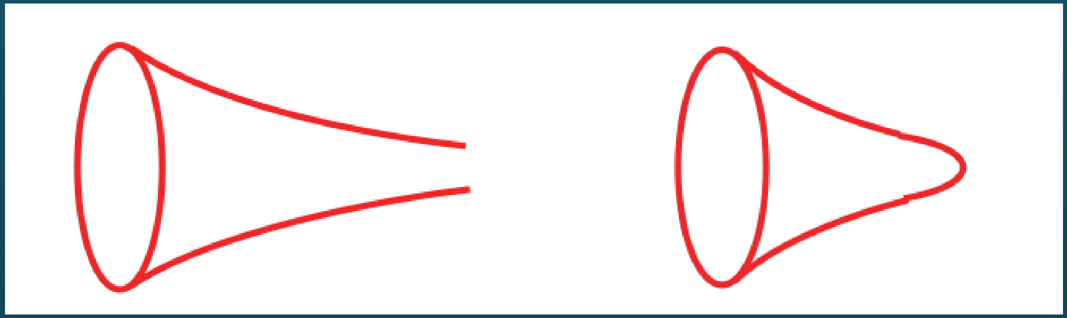
Thermal gas in AdS AdS Black Hole



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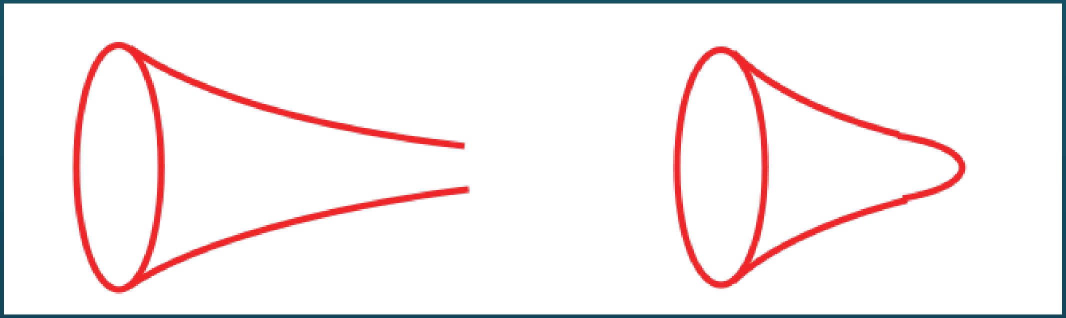

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$\mathcal{F}_{AdS} > \mathcal{F}_{BH}$ for all $T > 0$: BH always dominates;
Thermodynamics of a conformal gas, $\mathcal{F} = c\,T^4$

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To get a phase transition at nonzero T we need to introduce a scale,, e.g. put $\mathcal{N} = 4$ on an S^3.

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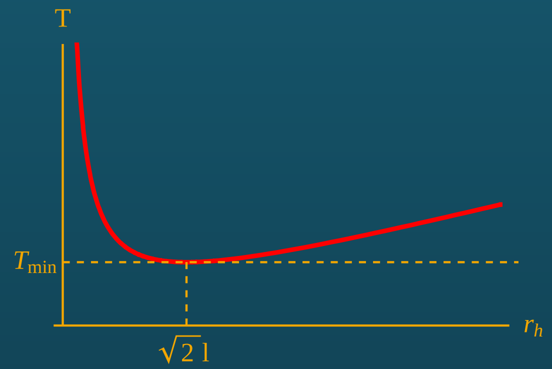

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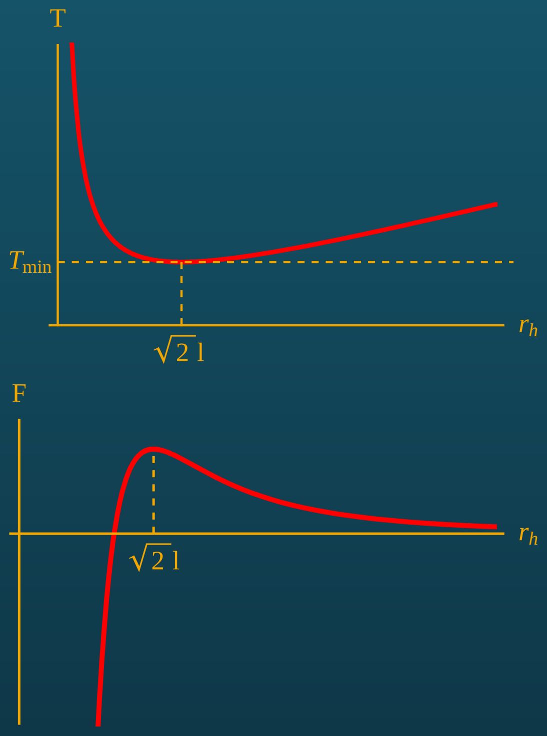
- "Large" BH dominant for $T_c > T_{min}$



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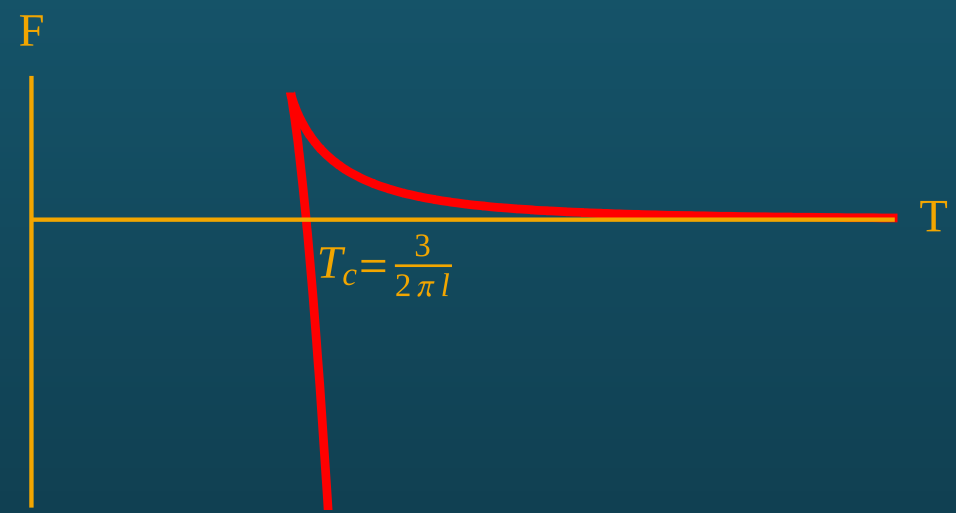

Hawking-Page transition.

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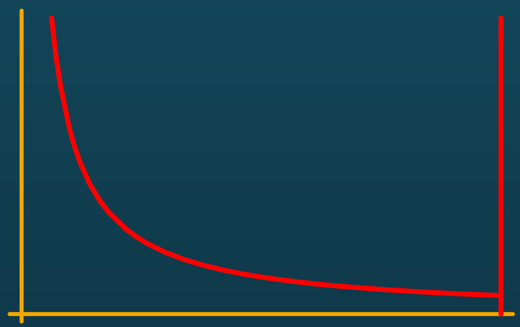

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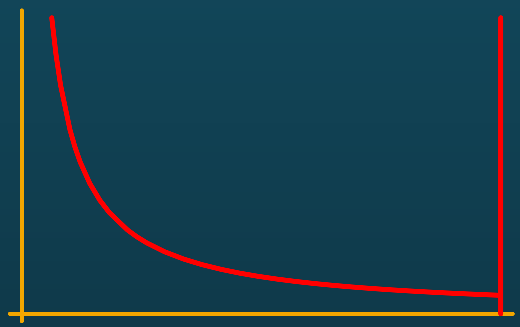

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  - still "too" conformal, except for the transition;
  - issue of boundary conditions at the IR cutoff

5D Einstein-Dilaton Theories

A simple class of models that displays realistic thermodynamics

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- Black holes (Deconfined phase - Gluon Plasma):

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$$b(r) \underset{r\to 0}{\longrightarrow} b_0(r) \sim \ell/r, \quad f(r) \underset{r\to 0}{\longrightarrow} 1, \quad f(r_h) = 0,$$

Dilatonic Black holes

Gürsoy, Kiritsis, Mazzanti, F.N. '08

Consider a potential that in the IR (large Φ) asymptotes:

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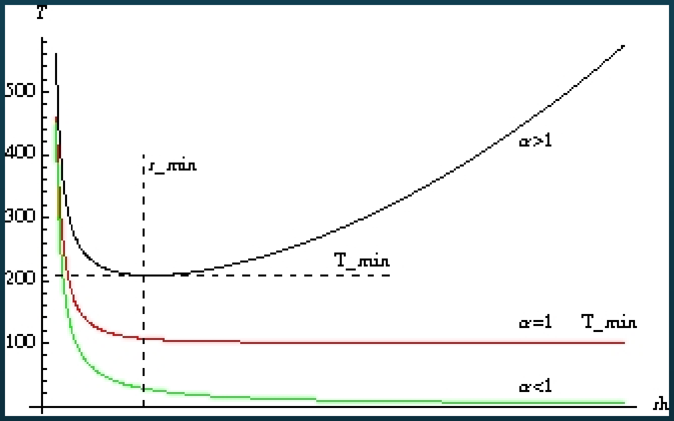

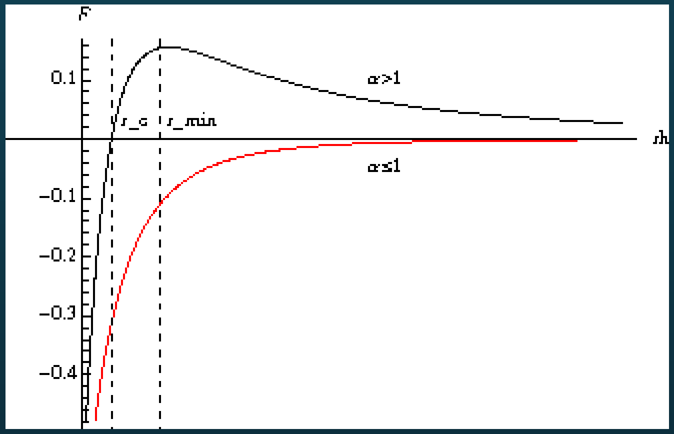

Situation as in the global AdS case ($\alpha > 1$) or Poincaré AdS ($\alpha < 1$)

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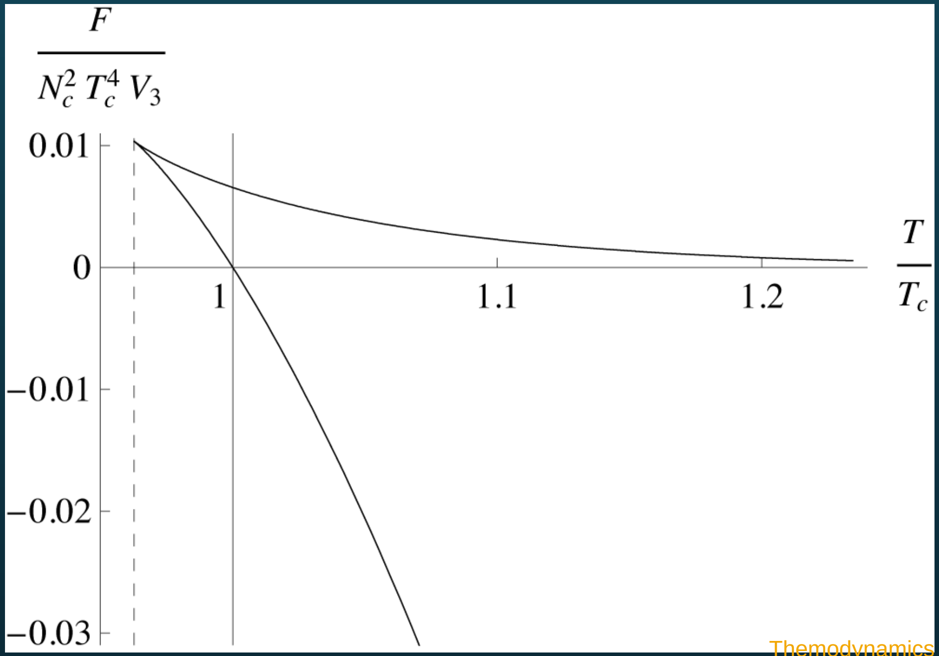
- confining case: Hawking-Page transition



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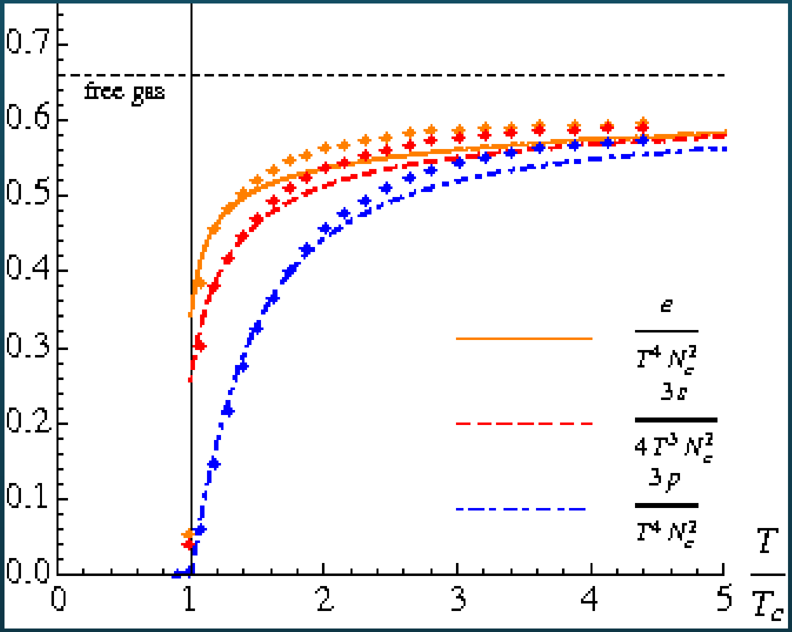
Simple parametrization for $V(\phi) \Rightarrow$ good numerical agreement with lattice Yang-Mills thermodynamics



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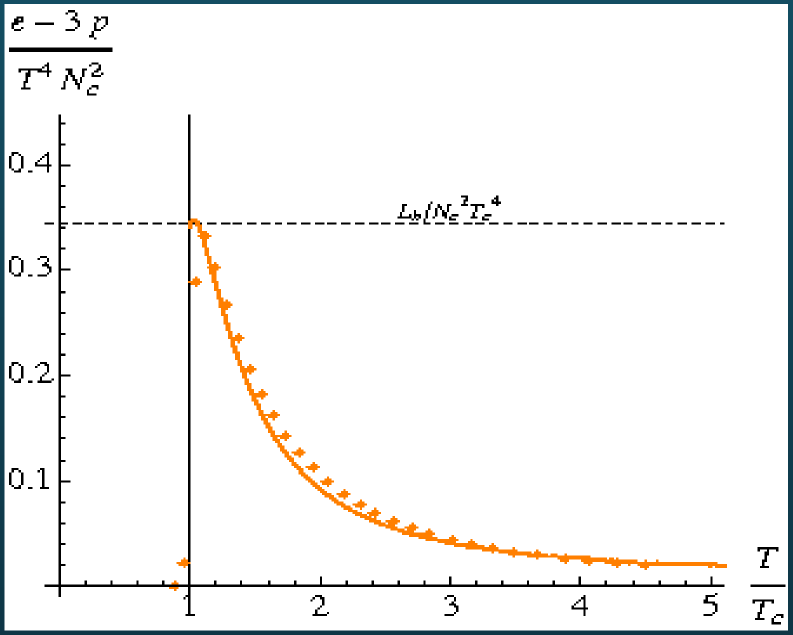

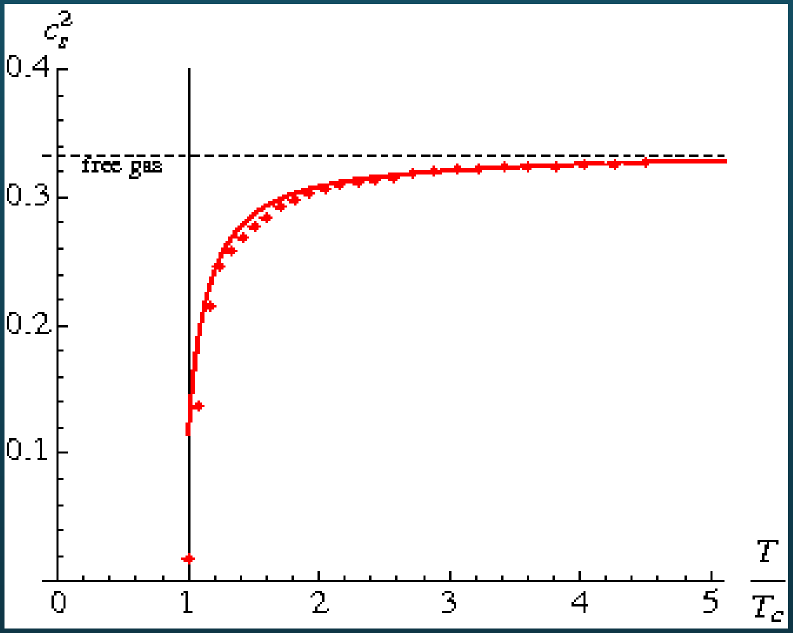

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- The potential $V(\Phi)$ is supposed to be fixed phenomenologically as a way of parametrizing some of the unknown features of the full holographic dual.

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- Natural to extend the description beyond equilibrium (e.g. diffusion)