Holographic study of magnetically induced QCD effects:

split between deconfinement and chiral transition, and evidence for rho meson condensation.

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Excited QCD 2011, Les Houches

Overview

1. Introduction
2. Holographic set-up
   - The Sakai-Sugimoto model
   - Introducing the magnetic field
3. The rho meson mass
   - First approximation: Landau levels
   - Also taking into account the chiral magnetic catalysis effect
4. The chiral symmetry restoration temperature
   - The Sakai-Sugimoto model at finite T
   - The Sakai-Sugimoto model at T and B

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Introduction

- Growing interest in magnetic-field induced QCD effects, mainly due to possible creation of strong magnetic fields in heavy ion collisions in LHC.
- Prediction of M. Chernodub [PRD **82** (2010) 085011, 1008.1055[hep-ph]; 1101.0117[hep-ph]]: the QCD vacuum is unstable towards condensation of charged rho mesons in the presence of a very strong magnetic field $eB \sim m_\rho^2$, turning into a superconducting vacuum.
- Goal of our work: study the QCD vacuum in strong magnetic field, in search of a possible rho meson condensation, using a holographic approach.
- Paper to appear, main results in proceeding 1102.3103[hep-ph]

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Holographic QCD

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QCD $\stackrel{dual}{=}$ supergravitation in a higher-dimensional background: four-dimensional QCD 'lives' on the boundary of a five-dimensional space where the supergravitation theory is defined

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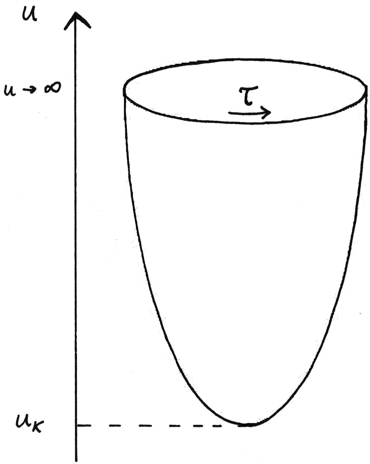
- **Origin of the QCD/gravitation duality idea?**

Anti de Sitter / Conformal Field Theory (AdS/CFT)-correspondence (Maldacena 1997):
conformal $\mathcal{N}=4$ SYM theory $\overset{dual}{=}$ supergravitation in AdS_5 space

The D4-brane background

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right),$$

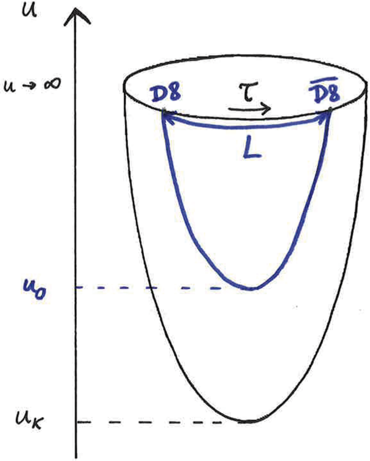
with $R \sim N_c^{1/3}$ and $f(u) = 1 - \frac{u_K^3}{u^3}$



The Sakai-Sugimoto model

To add flavour degrees of freedom to the theory, N_f pairs of D8-$\overline{\text{D8}}$ flavour branes are added to the D4-brane background.

SAKAI and SUGIMOTO, Prog. Theor. Phys. **113** (2005) 843, hep-th/0412141



D-branes

• Dp-brane $=$ $(p+1)$-dimensional hypersurface in spacetime in which an endpoint of a string (with Dirichlet boundary conditions) is restricted to move.

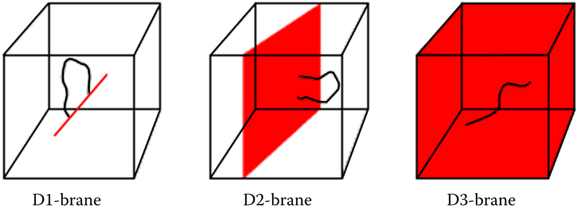


Figure from Peeters and Zamaklar, hep-ph/0708.1502

• The spectrum of vibrational modes of an open string with endpoints on the Dp-brane contains p massless photon states: "On a D-brane lives a Maxwell field."

The flavour D8-branes

- "On a D-brane lives a Maxwell field."
- "On a stack of N coinciding D-branes lives a $U(N)$ YM theorie."

$\Longrightarrow$ "On the stack of N_f coinciding pairs of D8-$\overline{\text{D8}}$ flavour branes lives a $U(N_f)_L \times U(N_f)_R$ theory, to be interpreted as the chiral symmetry in QCD."

The U-shaped embedding of the flavour branes models spontaneous chiral symmetry breaking
$U(N_f)_L \times U(N_f)_R \to U(N_f)$.

The flavour gauge field

The $U(N_f)$ **gauge field** $A_\mu(x^\mu, u)$ that lives on the flavour branes describes **a tower of vector mesons** $V_\mu, n(x^\mu)$ in the dual QCD-like theory:

$U(N_f)$ gauge field

$$A_\mu(x^\mu, u) = \sum_{n\geq 1} V_{\mu,n}(x^\mu)\psi_n(u)$$

with $V_{\mu,n}(x^\mu)$ a tower of vector mesons with masses m_n, and $\{\psi_n(u)\}_{n\geq 1}$ a complete set of functions of u, satisfying the **eigenvalue equation**

$$u^{1/2}\gamma_B^{-1/2}(u)\partial_u \left[u^{5/2}\gamma_B^{-1/2}(u)\partial_u\psi_n(u)\right] = -R^3 m_n^2\psi_n(u),$$

Approximations and choices of parameters

Approximations:

- quenched approximation
- chiral limit ($m_\pi = 0$)

We choose

- $N_c = 3$
- $N_f = 2$ to model charged mesons
- $u_0 > u_K$ to model non-zero constituent quark mass
  AHARONY *et.al.*, Annals Phys. **322** (2007) 1420, hep-th/0604161

Numerical fixing of holographic parameters

There are three unknown free parameters (u_K, u_0 and $\kappa(\sim \lambda N_c)$). In order to get results in physical units, we fix the free parameters by matching to

- the constituent quark mass $m_q = 0.310$ GeV,
- the pion decay constant $f_\pi = 0.093$ GeV and
- the rho meson mass in absence of magnetic field $m_\rho = 0.776$ GeV.

Results:

$$u_K = 1.39 \text{ GeV}^{-1}, \quad u_0 = 1.92 \text{ GeV}^{-1} \text{ and } \quad \kappa = 0.00678$$

How to turn on the magnetic field

Dual QCD theory:

Under a global chiral symmetry transformation $(g_L, g_R) \in U(N_f)_L \times U(N_f)_R$ a quark ψ transforms as $\psi = \psi_L + \psi_R \to g_L\psi_L + g_R\psi_R$.

Duality

$$(g_L, g_R) \leftrightarrow (g_+, g_-) = (lim_{z\to+\infty} g, lim_{z\to-\infty} g)$$

with $g \in U(N_f)$ a residual gauge symmetry transformation for the gauge field on the D8-branes, in the gauge $A_\mu(x^\mu, \pm\infty) = 0$.

Sakai-Sugimoto bulk theory:

Under a gauge symmetry transformation $g \in U(N_f)$ the flavour gauge field $A_\mu(x^\mu, z)$ in the bulk transforms as $A_\mu(x^\mu, z) \to gA_\mu(x^\mu, z)g^{-1} + g\partial_\mu g^{-1}$.

How to turn on the magnetic field

Lightly gauging the global symmetry $U(N_f)_L \times U(N_f)_R$, the Dirac action for the quarks transforms to $(g_L = g_R = g(z = \pm\infty, x^\mu) = g(x^\mu))$:

$$\overline{\psi} i \gamma_\mu \partial_\mu \psi \to \overline{\psi} i \gamma_\mu (\partial_\mu + \underbrace{g^{-1}\partial_\mu g}_{-A_\mu(z\to\pm\infty)})\psi = \overline{\psi} i \gamma_\mu \mathcal{D}_\mu \psi,$$

so corresponds to applying an external gauge field in the field theory that couples to the quarks via a covariant derivative

$$\mathcal{D}_\mu = \partial_\mu - A_\mu(z \to \pm\infty).$$

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so corresponds to applying an external gauge field in the field theory that couples to the quarks via a covariant derivative

$$\mathcal{D}_\mu = \partial_\mu - A_\mu(z \rightarrow \pm\infty).$$

To apply an external electromagnetic field A_μ^{em}, put

$$A_\mu(z \rightarrow +\infty) = A_\mu(z \rightarrow -\infty) = eQ_{em}A_\mu^{em} = \overline{A}_\mu$$

Sakai and Sugimoto, Prog. Theor. Phys. **114** (2005) 1083, hep-th/0507073

How to turn on the magnetic field

To apply a magnetic field along the x_3-axis,

$$A_2^{em} = x_1 eB,$$

in the $N_f = 2$ case,

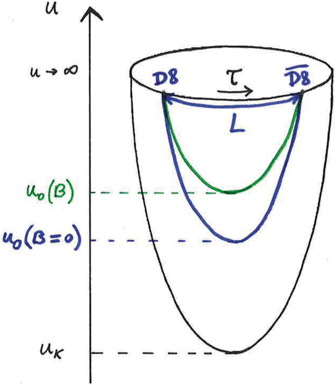
$$Q_{em} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1}{6}\mathbf{1}_2 + \frac{1}{2}\sigma_3,$$

we set

$$\overline{A}_2^3 = x_1 eB$$

Effect of B on the embedding

$L = 1.57 \text{ GeV}^{-1}$ (corresponding to $u_0(B=0) = 1.92 \text{ GeV}^{-1}$) kept fixed: u_0 rises with B.



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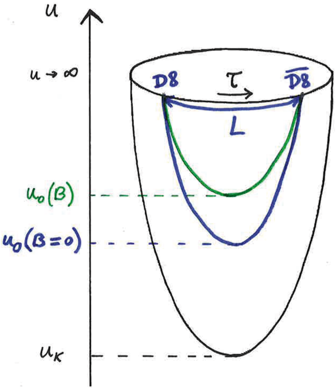

This models **magnetic catalysis of chiral symmetry breaking**.

Johnson and Kundu, JHEP **0812** (2008) 053, 0803.0038[hep-th]

Effect of B on the embedding of the flavour branes: chiral magnetic catalysis

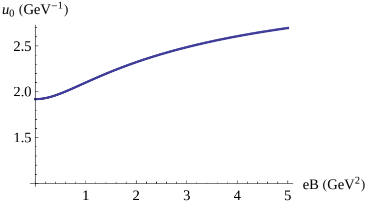

Figure: u_0 as a function of the magnetic field.

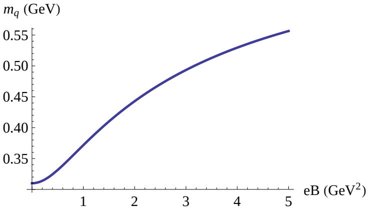

Figure: The constituent quark mass as a function of the magnetic field.

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Landau levels

Our gauge field ansatz is

$$A_\mu(x^\mu, u) = \overline{A}_\mu + \sum_{n\geq 1} V_{\mu,n}(x^\mu)\psi_n(u).$$

We plug it it into the non-abelian DBI-action for the flavour gauge field

$$S_{DBI} = -T_8 \int d^4x d\tau\, \epsilon_4 e^{-\phi} \mathrm{STr} \sqrt{-\det\left[g_{mn}^{D8} + (2\pi\alpha')F_{mn}\right]}$$

(with T_8 the D8-brane tension, STr the symmetrized trace, g_{mn}^{D8} the induced metric on the D8-branes, α' the string tension, and F_{mn} the field strength),

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expand the action to order $(V_{\mu,n})^2$,
use the normalization condition for the $\psi_n(u)$'s to integrate out the u-dependence,

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(with T_8 the D8-brane tension, STr the symmetrized trace, g_{mn}^{D8} the induced metric on the D8-branes, α' the string tension, and F_{mn} the field strength), expand the action to order $(V_{\mu,n})^2$,
use the normalization condition for the $\psi_n(u)$'s to integrate out the u-dependence, and derive the mass equation for the rho meson $\rho_\mu = V_{\mu,1}$.

Landau levels

We find

$$m_\rho^2 \rho_\mu^- - D_\nu^2 \rho_\mu^- + 2i(\partial_\mu A_\nu^{em} - \partial_\nu A_\mu^{em})\rho^{\nu -} = 0, \quad \text{with } D_\mu = \partial_\mu - ieA_\mu^{em}$$

for the negatively charged rho meson $\rho_\mu^- = \rho_\mu^1 - i\rho_\mu^2$.

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for the negatively charged rho meson $\rho_\mu^- = \rho_\mu^1 - i\rho_\mu^2$.

Inserting the background gauge field ansatz $A_2^{em\,3} = x_1 eB$, and Fourier transforming ρ_μ^-, we obtain the Landau energy levels

$$E^2(\rho_1^- \mp i\rho_2^-) = \left(eB(2N+1) + p_3^2 + m_\rho^2 \mp 2eB \right)(\rho_1^- \mp i\rho_2^-)$$

with p_3 the momentum of the meson in the direction of the magnetic field.

Landau levels

Landau levels

$$E^2\rho = \Big(eB(2N+1) + p_3^2 + m_\rho^2 - 2eB\Big)\,\rho$$

The combinations $\rho = (\rho_1^- - i\rho_2^-)$ and $\rho^\dagger = (\rho_1^+ + i\rho_2^+)$ have spin $s_3 = 1$ parallel to $\vec{B} = B\vec{e}_3$.

In the lowest energy state ($N = 0$, $p_3 = 0$) their effective mass,

$$m_{eff}^2 = m_\rho^2 - eB,$$

can become zero if the magnetic field is strong enough.

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can become zero if the magnetic field is strong enough.
$\Longrightarrow$ The fields ρ and $\rho^\dagger$ condense at the critical magnetic field

$$eB_c = m_\rho^2.$$

Taking into account chiral magnetic catalysis

In the above we have neglected any dependence on the magnetic field of the mass eigenvalue m_ρ itself, but actually

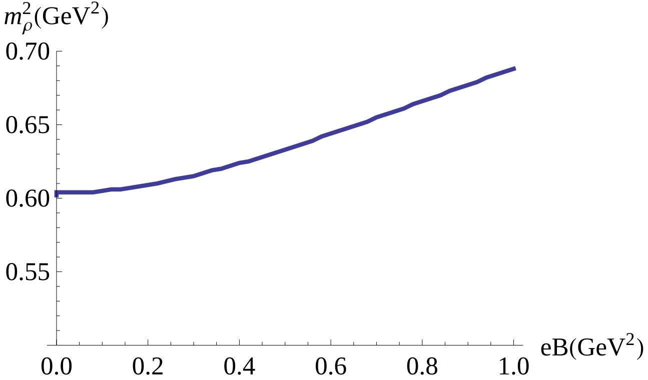
$$m_{eff}^2 = m_\rho^2(B) - eB.$$

$m_\rho^2(B)$ is the lowest eigenvalue of the eigenvalue equation

$$u^{1/2}\gamma_B^{-1/2}(u)\partial_u\left[u^{5/2}\gamma_B^{-1/2}(u)\partial_u\psi_n(u)\right] = -R^3 m_n^2(B)\psi_n(u),$$

which depends on B both explicitly and implicitly through the changed embedding of the probe branes, represented by the value of $u_0(B)$.

Taking into account chiral magnetic catalysis



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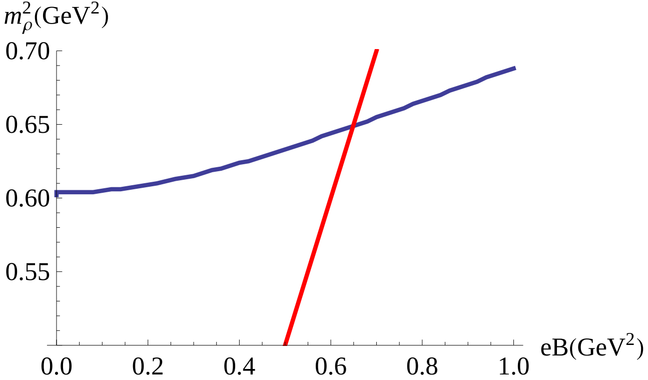
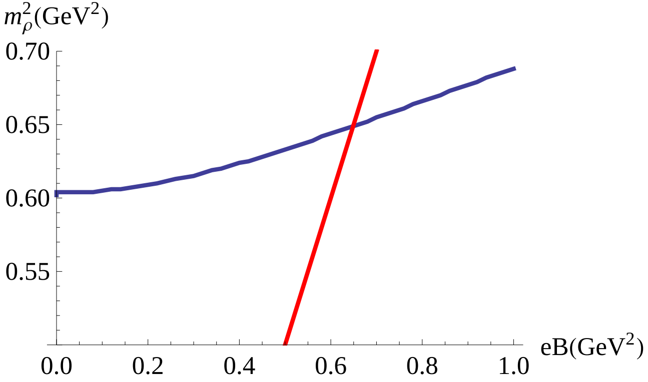


Taking into account chiral magnetic catalysis



$\Longrightarrow$ The fields ρ and $\rho^\dagger$ condense at the critical magnetic field

$$eB_c = m_\rho^2(B_c) = 1.08 m_\rho^2.$$

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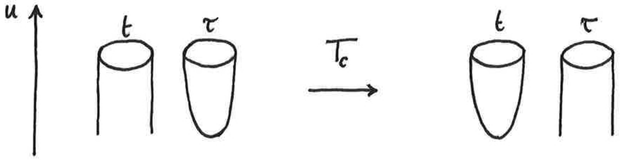
The Sakai-Sugimoto model at finite T

- At low temperature $T < T_c$, the D4-brane background of the Sakai-Sugimoto model is wick-rotated and the Euclidean time is periodic with period $\beta = 1/T$.
- At the deconfinement temperature $T_c = 115$ MeV (for our numerical values of the holographic parameters) this background changes to the high-T background ($T \geq T_c$), which only differs from the low-T one in that the time and τ dimensions are interchanged.



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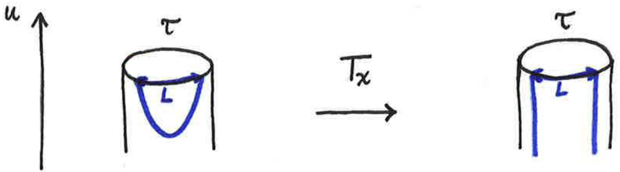


The chiral symmetry restoration temperature T_χ is determined as the value of the temperature for which the difference in actions

$$\Delta S = S[\text{U-shaped embedding}] - S[\text{straight embedding}]$$

becomes zero.

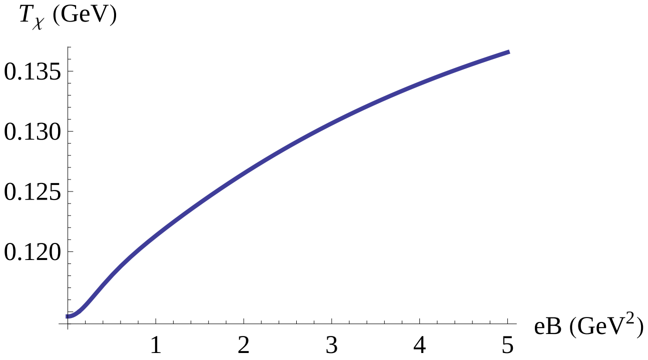
Aharony *et.al.*, Annals Phys. **322** (2007) 1420, hep-th/0604161

The Sakai-Sugimoto model at T and B

Keeping $L(u_0, T, B)$ fixed to the value for $B = 0$ ($\Rightarrow u_0(B, T)$), we determine T_χ from

$$\Delta S(B, u_0, T) = 0$$

and the assumption that $T_\chi(B = 0) = T_c$.



The Sakai-Sugimoto model at T and B

We find a split $T_\chi(B) - T_c$ of

$$1.2\% \text{ at } eB = 15 m_\pi^2 \approx 0.28 \text{ GeV}^2$$

and

$$3.2\% \text{ at } eB = 30 m_\pi^2 \approx 0.57 \text{ GeV}^2.$$

The percent estimation of the magnitude of the split is

- in quantitative agreement with Gatto and Ruggieri, 1012.1291[hep-ph];D'Elia *et.al.*,PRD **82** (2010) 051501, 1005.5365[hep-lat]
- in qualitative agreement with Mizher,Chernodub and Fraga,PRD **82** (2010) 105016, 1004.2712[hep-ph]; Gatto and Ruggieri, Phys. Rev. **D82** (2010) 054027, 1007.0790[hep-ph].

The Sakai-Sugimoto model at T and B

Results in the chiral limit can be extracted from the work of Fukushima *et.al.* wherein T_c seems to depend only marginally on B, at least up to $eB = 20m_\pi^2$, the maximum value considered there.

FUKUSHIMA *et.al.*, Phys. Rev. **D81** (2010) 114031, 1003.0047[hep-ph]

Thank you for your attention!

Questions?