

Glueball Spectroscopy

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V. M., N. Kochelev, V. Vento,
“The Physics of Glueballs”,
INT. J. MOD. PHYS. E18 (2009) 1

A. C. Aguilar, D. Ibáñez, V. M. and J. Papavassiliou,
“The Schwinger Mechanism in QCD,”
in preparation

C. Degrande, J.-M. Gérard and V. M.,
in preparation,

PHENOMENOLOGY - STRONG COUPLING CONSTANT

Deur, arXiv:0901.2190

Saturation of $\alpha_s(Q^2)$ at large distances

Gribov, Eur. Phys. J., C10,71 (1999)

Dokshitzer and Webber,

Phys. Lett., B352, 451 ('95)

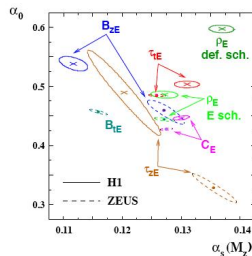
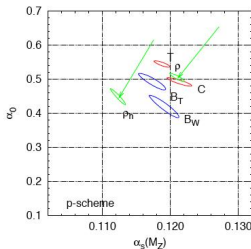
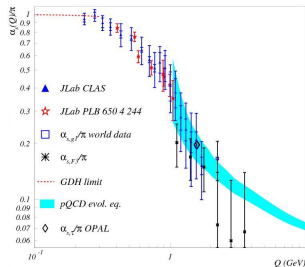
Dasgupta and Salam,

J. Phys. G 30, R143 (2004)

Power correction in event shapes

($\mu_I = 2 \text{ GeV}$)

$$\alpha_0 = \mu_I^{-1} \int_0^{\mu_I} \alpha(Q^2) dQ^2 \sim 0.5$$



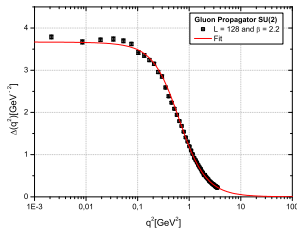
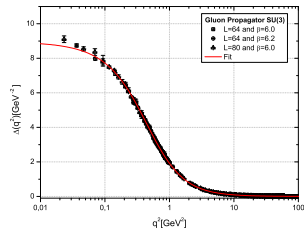
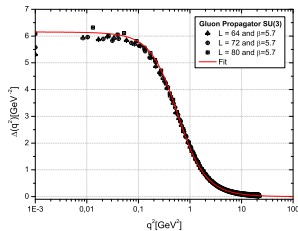
Gluon propagator from
lattice QCD in Landau
gauge

Cucchieri and Mendes,
PoS (Lattice 2007) 297

Bogolubsky *et al.*,
PoS (Lattice 2007) 290

Oliveira and Silva,
PoS (QCD-TNT 2009) 033

Predicted by Cornwall Phys. Rev. D**26**, 1453 (1982)
by introducing a **dynamically generated gluon mass**



DYNAMICAL MASS GENERATION

Not a violation of **gauge invariance**

Schwinger Mechanism like in
(1 + 1)-QED

Phys. Rev. **128** (1962) 2425

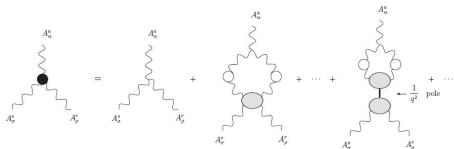
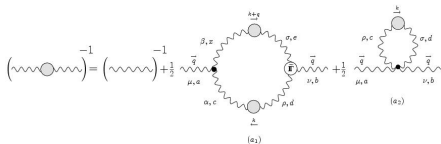
Self-energy gains a **dynamical pole**

$$\Delta(q^2)_{\mu\nu} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{-i}{q^2 - q^2 \Pi(q^2)}$$

Mass is the residue

$$\Pi(q^2)|_{\text{pole}} = \frac{m^2(q^2)}{q^2}$$

Composite bound state not present in
physical processes



R. Jackiw and K. Johnson, *Phys. Rev. D* **8**, 2386 (1973)

D. Binosi and J. Papavassiliou, *Phys. Rept.* **479**, 1 (2009)

A. C. Aguilar, D. Ibáñez, V. M. and J. Papavassiliou, “The Schwinger Mechanism in QCD”,
in preparation

GLUEBALL ?

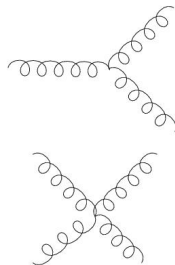
QCD = gauge theory with the **color group** $SU(3)$

$$\mathcal{L}_{QCD} = -\frac{1}{4}\text{Tr} G_{\mu\nu}G^{\nu\mu} + \sum \bar{q}(\gamma^\mu D_\mu - m)q$$
$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

Quark = fundamental representation **3**

Gluon = Adjoint representation **8**

Observable particles = color singlet **1**



Mesons: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$

Baryons: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$

Glueballs: $\mathbf{8} \otimes \mathbf{8} = (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}) \oplus (\mathbf{8} \oplus \mathbf{10} \oplus \bar{\mathbf{10}})$

$$\mathbf{8} \otimes \dots \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8} \oplus \dots$$

Three light quarks \rightarrow **nine** 0^\pm mesons : 3π ($I = 1$) $\oplus 4K$ ($I = 1/2$) $\oplus 2\eta$ ($I = 0$)

Glueball can mix with two isoscalars \rightarrow glue content in η 's wave function **and maybe** a third isoscalar

QUENCHED RESULTS

Investigation of the glueball spectrum (**pure gluonic operators**) on a lattice by Morningstar and Peardon, *Phys. Rev. D***60** (1999) 034509]

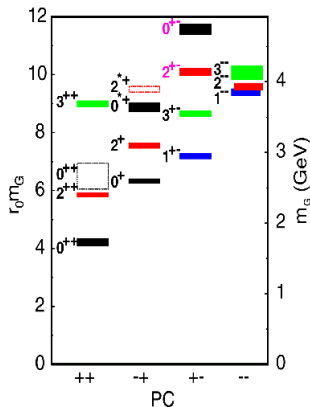
Identification of **15 glueballs** below 4 GeV

$$M(0^{++}) = 1.730 \pm 0.130 \text{ GeV}$$

$$M(0^{-+}) = 2.590 \pm 0.170 \text{ GeV}$$

$$M(2^{++}) = 2.400 \pm 0.145 \text{ GeV}$$

Quenched approximation (gluodynamics) \rightarrow mixing with quarks is neglected



UNQUENCHED RESULTS

Lattice studies with $n_f = 2$ exist. The lightest scalar would be sensitive to the inclusion of sea quarks but **no definitive conclusion**.

Theoretical status of glueballs :

V. M., N. Kochelev, V. Vento, "The Physics of Glueballs", *Int. J. Mod. Phys. E***18** (2009) 1

R. L. Jaffe and K. Johnson, *Phys. Lett. B* 60 (1976) 201

J. Kuti, *Nucl. Phys. Proc. Suppl.* 73 (1999) 72

Free Particles Confined in a Cavity

Gluonic Modes in a Cavity

$$\text{TE mode } J^P = 1^+ \quad x_{\text{TE}} = 2.74,$$

$$\text{TE mode } J^P = 2^- \quad x_{\text{TE}} = 3.96,$$

$$\text{TM mode } J^P = 1^- \quad x_{\text{TM}} = 4.49.$$

Mass Spectrum

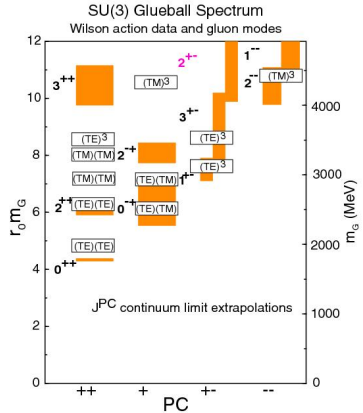
$$E = \frac{4\pi BR^3}{3} + \sum_i n_i \frac{x_i}{R} - \frac{\alpha}{4R} \lambda_1^a \lambda_2^a \vec{S}_1 \cdot \vec{S}_2$$

$$M^2 = E^2 - \sum_i n_i \left(\frac{x_i}{R} \right)^2$$

$$\alpha = 0.5 \quad B = (280 \text{ MeV})^4$$

J. F. Donoghue, *Phys. Rev. D* 29 (1984) 2559

Gluon mass $740 \pm 100 \text{ MeV}$ in the bag model



COULOMB GAUGE APPROACH TO GLUEBALLS

SZCZEPANIAK, SWANSON, JI, AND COTANCH [Phys. Rev. Lett. **76**, 2011 (1996)]

Transverse gluons with **2 helicities** $\{-1, +1\}$
Gluon mass given by gap equation

$$\omega^2(k) = k^2 + m^2 e^{-k/\kappa}$$

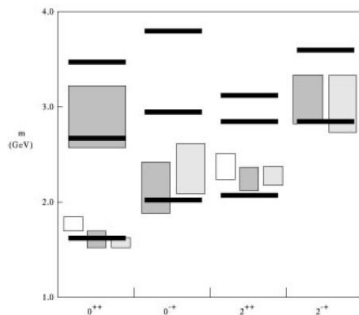
Relativistic Hamiltonian with

$$V(r) = \frac{9\sigma}{4} r (1 - e^{-\Lambda r}) - \frac{3\alpha_s}{r}$$

No vector state (Yang's theorem)

Agreement with (preliminary) lattice results
without OGE

Small gap, 250 MeV, between 0^{++} and 0^{-+}

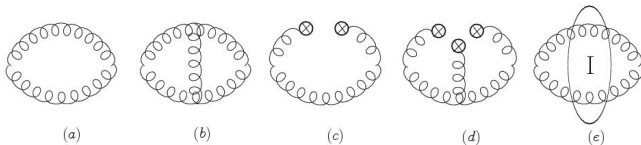


Gluonic currents: $J_S(x) = \alpha_S \text{Tr} G_{\mu\nu} G^{\mu\nu}$ $J_P(x) = \alpha_S \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$

$$\Pi(Q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T J_G(x) J_G(0) | 0 \rangle = \frac{1}{\pi} \int_0^\infty \frac{\text{Im}\Pi(s)}{s + Q^2} ds$$

Theoretical side (OPE):

$$J_G(x) J_G(0) = C_{(a)+(b)+(e)} \mathbf{1} + C_{(c)} G_{\mu\nu}^a G_a^{\mu\nu} + C_{(d)} f_{abc} G_{\alpha\beta}^a G_{\beta\gamma}^b G_{\gamma\alpha}^c + \dots$$



Confinement parameterized with condensates $\langle 0 | \alpha_S G_{\mu\nu}^a G_a^{\mu\nu} | 0 \rangle, \dots$

Phenomenological side:

$$\text{Im}\Pi(s) = \sum_i \pi f_{G_i}^2 m_{G_i}^4 \delta(s - m_{G_i}^2) + \pi \theta(s - s_0) \text{Im}\Pi(s)^{\text{Cont}}$$

Glueonic currents: $J_S(x) = \alpha_S \text{Tr} G_{\mu\nu} G^{\mu\nu}$ $J_P(x) = \alpha_S \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$

$$\Pi_{\mathcal{O}}(0) = \lim_{q^2 \rightarrow 0} i \int d^4x e^{iq \cdot x} \langle 0 | T J_S(x) \mathcal{O}(0) | 0 \rangle = \frac{8\pi d_{\mathcal{O}}}{b_0} \langle 0 | \mathcal{O} | 0 \rangle$$

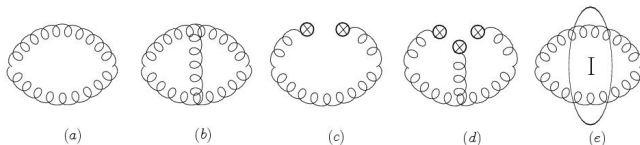
$$\Pi_S(Q^2 = 0) = \frac{32\pi}{b_0} \langle 0 | \alpha_S G_{\mu\nu}^a G_a^{\mu\nu} | 0 \rangle$$

$$\Pi_P(Q^2 = 0) = (8\pi)^2 \frac{m_u m_d}{m_u + m_d} \langle 0 | \bar{q}q | 0 \rangle$$

Instantons contribution essential for LETs

Forkel, *Phys. Rev. D* **71** (2005) 054008

$$M(0^{++}) = 1.25 \pm 0.20 \text{ GeV} \quad M(0^{-+}) = 2.20 \pm 0.20 \text{ GeV}$$



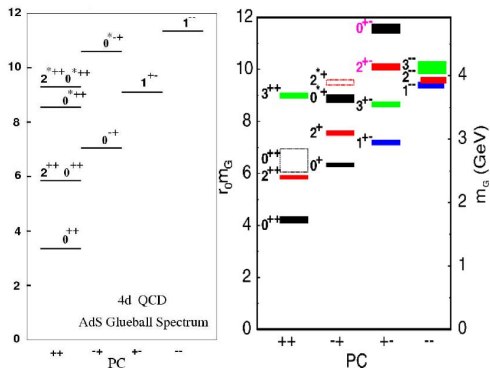
AdS/CFT correspondence:

Correspondance between conformal theories and string theories in AdS spacetime
 QCD not conformal \rightarrow breaking conformal invariance somehow

Introduction of a black hole in AdS to break conformal invariance

Parameter adjusted on 2^{++}

Same hierarchy but some states are missing (spin 3,...)



R. C. Brower *et al.*, Nucl. Phys. B587 (2000) 249

TWO MODELS

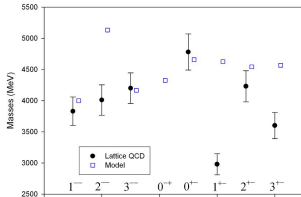
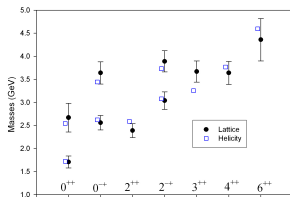
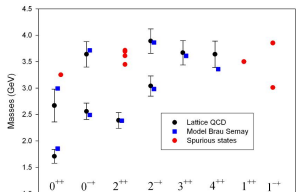
$$H_{gg} = 2\sqrt{\mathbf{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r} + V_{\text{OGE}}$$

Gluons spin-1 usual rules of spin couplings

Gluons transverse Gluons **helicity-1** particles

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \text{ with } S = 0, 1, 2$$

$$\mathbf{J} \neq \mathbf{L} + \mathbf{S}$$



- OGE no needed $V_{\text{OGE}} = 0$
- No vector states
- 3 d.o.f. not compatible with J^{+-}
- Instanton contribution

GLUEBALLS IN THE REAL WORLD

Crede and Meyer, The Experimental Status of Glueballs,
Prog. Part. Nucl. Phys. **63**, 74 (2009)

Production in **gluon rich processes** (OZI forbidden,...)

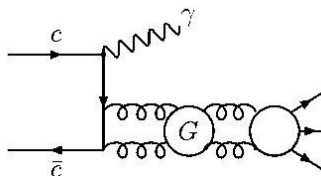
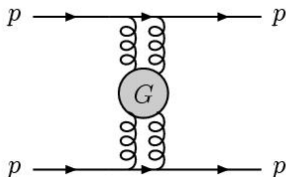
Closely linked to the **Pomeron**: $J = 0.25M^2 + 1.08$

Mixing between glueball 0^{++} and light mesons

Scalar Candidates: $f_0(1370)$ $f_0(1500)$ $f_0(1710)$ $f_0(1810)$...

Pseudoscalar Candidates: $\eta(1295)$ $\eta(1405)$ $\eta(1475)$ $\eta(1760)$...

0^{++} and 0^{-+} glueballs shared between those states



Three light quarks $\rightarrow 3 \times 3 = 9$ (pseudo)scalar mesons

A **10th** light mesons would be the realization of the **glueball**

CHIRAL SUPPRESSION

Chanowitz, *Phys. Rev. Lett.* **95** (1999) 172001

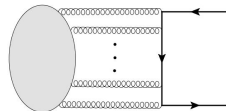
$$A(0^{++} \rightarrow \bar{q}q) \propto m_q$$

Decay to $K\bar{K}$ favoured over $\pi\pi$

Controversy about m_q (current or constituent mass ?)

$$R = \frac{A(0^{++} \rightarrow \bar{K}K)}{A(0^{++} \rightarrow \pi\pi)} > 1$$

$f_0(1710)$ good glueball candidate



RESULTS from Crystal Barrel ($p\bar{p}$), OBELIX ($p\bar{p}$), WA102 (pp), BES (J/ψ)

Name	Masse (MeV)	Width (MeV)	Decays	Production
$f_0(1370)$	1200 – 1500	200 – 500	$\pi\pi, K\bar{K}, \eta\eta$	$p\bar{p} \rightarrow PPP, pp \rightarrow pp(PP)$ weak signal in $J/\psi \rightarrow \gamma(PP)$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi, K\bar{K}, \eta\eta$	$J/\psi \rightarrow \gamma(PP), pp \rightarrow pp(PP)$ $p\bar{p} \rightarrow PPP$
$f_0(1710)$	1720 ± 6	135 ± 8	$\pi\pi, K\bar{K}, \eta\eta$	$J/\psi \rightarrow \gamma(PP), pp \rightarrow pp(PP)$ not seen in $p\bar{p}$

Belle and BaBar puzzle : $f_0(1500)$ strong coupling to $K\bar{K}$ and weak to $\pi\pi$

PHYSICAL STATES

Pure states: $|gg\rangle$, $|n\bar{n}\rangle$, $|s\bar{s}\rangle$

$$|G\rangle = |gg\rangle + \frac{\langle n\bar{n}|gg\rangle}{M_{gg} - M_{n\bar{n}}} |n\bar{n}\rangle + \frac{\langle s\bar{s}|gg\rangle}{M_{gg} - M_{s\bar{s}}} |s\bar{s}\rangle$$

Analysis of

Production: $J/\psi \rightarrow \gamma f_0, \omega f_0, \phi f_0$

Decay: $f_0 \rightarrow \pi\pi, K\bar{K}, \eta\eta$

Two mixing schemes

Cheng *et al*, Phys. Rev. D**74** (2006) 094005

Close and Kirk, PLB**483** (2000) 345

$f_0(1370)$

$f_0(1500)$

$f_0(1710)$



■ $u\bar{u} + d\bar{d}$



■ $s\bar{s}$



■ gg

$f_0(1370)$

$f_0(1500)$

$f_0(1710)$



■ $u\bar{u} + d\bar{d}$



■ $s\bar{s}$

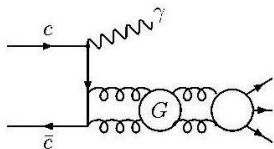


■ gg

$$M_{n\bar{n}} < M_{s\bar{s}} < M_{gg}$$

$$M_{n\bar{n}} < M_{gg} < M_{s\bar{s}}$$

Mark III : observation of **two** pseudoscalar in
 $J/\psi \rightarrow \gamma PPP$
 $\gamma\gamma$ fusion : $\eta(1475)$ seen but not $\eta(1405)$



RESULTS from Crystal Barrel ($p\bar{p}$), OBELIX ($p\bar{p}$)

Name	Masse (MeV)	Width (MeV)	Decays	Production
$\eta(1295)$	1294 ± 4	55 ± 5	$\gamma\gamma, K\bar{K}\pi, a_0\pi$	not seen in $p\bar{p}$
$\eta(1405)$	1409.8 ± 2.5	51.1 ± 3.4	$K\bar{K}\pi, a_0\pi, \eta\pi\pi$	not seen in $\gamma\gamma$
$\eta(1475)$	1476 ± 4	87 ± 9	$\gamma\gamma, K\bar{K}^*, K\bar{K}\pi, a_0\pi$	

$\eta(1205)$ and $\eta(1475)$ radial excitations of η and η' .
 $\eta(1405)$ **glueball** candidate

Possible **glue** content in η'

$$\frac{\Gamma(J/\psi \rightarrow \eta'\gamma)}{\Gamma(J/\psi \rightarrow \eta\gamma)} = \left(\frac{\langle 0|G\tilde{G}|\eta'\rangle}{\langle 0|G\tilde{G}|\eta\rangle} \right)^2 \left(\frac{M_{J/\psi}^2 - M_{\eta'}^2}{M_{J/\psi}^2 - M_{\eta}^2} \right)^3 = 4.81 \pm 0.77$$

Axial Anomaly \rightarrow mixing with η_1

$$\partial^\mu (\bar{q}\gamma_\mu\gamma_5 q) = 2im\bar{q}\gamma_5 q + \frac{\alpha_S}{4\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} \rightarrow \langle gg|\partial^\mu J_{\mu 5}^1|0\rangle \neq 0$$

Mixing Scheme with pseudoscalar glueball

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |\eta''\rangle \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_G & -\sin\phi_G \\ 0 & \sin\phi_G & \cos\phi_G \end{pmatrix} \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |gg\rangle \end{pmatrix}$$

Measurement of $Z_G^2 = \langle\eta'|gg\rangle^2$

$V \rightarrow P\gamma$ and $P \rightarrow V\gamma \rightarrow$	0.04 ± 0.09	Escribano and Nadal (2007)
$\frac{\text{Br}(\phi(1020) \rightarrow \eta'\gamma)}{\text{Br}(\phi(1020) \rightarrow \eta\gamma)} \rightarrow$	0.12 ± 0.04	KLOE collaboration (2008)
$J/\psi \rightarrow VP \rightarrow$	0.28 ± 0.21	Escribano (2008)

Where is the **physical pseudoscalar glueball** $|\eta''\rangle$?

$$\eta(1295), \quad \eta(1405), \quad \eta(1475), \quad \dots$$

Cheng *et al.*, *Phys. Rev D* **79** (2008) 014024

CHIRAL SYMMETRY

QCD = gauge theory with the **color group** $SU(3)$

$$\begin{aligned}\mathcal{L}_{QCD} &= -\frac{1}{4}\text{Tr} G_{\mu\nu}G^{\mu\nu} + \sum \bar{q}(\gamma^\mu D_\mu - m)q \\ G_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]\end{aligned}$$

Degrees of freedom at **High Energies**: Quarks q and Gluons A_μ

Degrees of freedom at **Low Energies**: Pions, Kaons,...

Goldstone bosons of **Chiral Symmetry** Breaking

(Global) Chiral Symmetry: $U(3)_V \otimes U(3)_A$

$$U(3)_V : \quad q \rightarrow \exp(i\theta_a \lambda^a)q$$

$$U(3)_A : \quad q \rightarrow \exp(i\gamma_5 \theta_{5a} \lambda^a)q$$

$U(3)_A$ broken **spontaneously** by quark condensate $\langle 0|\bar{q}q|0\rangle \neq 0$

$U(3)_A$ broken **explicitly** by quark masses m

→ 9 Goldstone pseudoscalar bosons with a small mass $\propto m\langle 0|\bar{q}q|0\rangle$

3π , $4K$ and 2η

$U(1)_A$ is not a symmetry of QCD

→ η' is **NOT a Goldstone boson** $M_{\eta'} \sim 958 \text{ MeV} > M_\eta \sim 548 \text{ MeV}$

But Anomaly vanishes for **large N** , for a gauge group $SU(N, \square \rightarrow \infty)$

MASS MATRIX AT LEADING ORDER

Chiral Lagrangian at leading order (in p^2 and $1/N$)

$$\mathcal{L}(p^2) = \frac{f^2}{8} \left\langle \partial_\mu U^\dagger \partial^\mu U + B(mU^\dagger + Um^\dagger) \right\rangle - \frac{1}{2} \alpha_0 \eta_0^2$$

with $U = \exp(i\sqrt{2}\pi/f)$

Isospin Symmetry $m = \text{diag}(\tilde{m}, \tilde{m}, \tilde{m}_s) \rightarrow m_\pi^2 = B\tilde{m}$ and $m_K^2 = B(\tilde{m} + m_s)/2$

Mass matrix in the flavor basis ($\eta_q - \eta_s$)

$$\mathcal{M}_{qs}^2 = \begin{pmatrix} m_\pi^2 + 2\alpha_0 & \sqrt{2}\alpha_0 \\ \sqrt{2}\alpha_0 & 2m_K^2 - m_\pi^2 + \alpha_0 \end{pmatrix}$$

Anomaly only source of mixing

Rotation to **Physical States**

$$R^\dagger(\phi) \mathcal{M}_{qs}^2 R(\phi) = \begin{pmatrix} m_\eta^2 & 0 \\ 0 & m_{\eta'}^2 \end{pmatrix}$$

ϕ determines **Decay Properties**

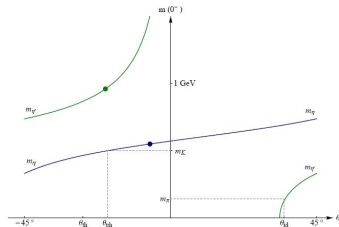
$\eta - \eta'$ MIXING AT LEADING ORDER

Only 1 parameter α_0 but 2 states (or 2 invariants)

Trace $\alpha_0 = (m_\eta^2 + m_{\eta'}^2 - 2m_K^2)/3$

Determinant $\alpha_0 = \frac{m_\eta^2 m_{\eta'}^2 - m_\pi^2 (2m_K^2 - m_\pi^2)}{4m_K^2 - m_\pi^2}$

! Not Equal !



2 photons decays : $\eta \rightarrow \gamma\gamma$ and $\eta' \rightarrow \gamma\gamma$

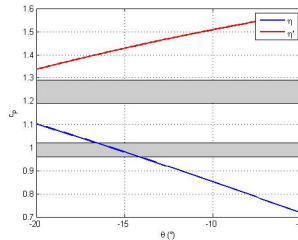
Degrande and Gérard, **JHEP 0905 (2009) 043**

$\theta \sim -(20 - 15)^\circ$ in the $U(3)$ basis ($\eta_8 - \eta_0$)
 $\phi \sim (40 - 45)^\circ$ in the flavor basis ($\eta_q - \eta_s$)

$$\theta = \phi - \theta_i$$

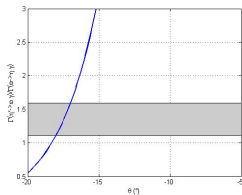
with the ideal mixing angle

$$\theta_i = \arccos(1/\sqrt{3}) \sim 54.7^\circ$$

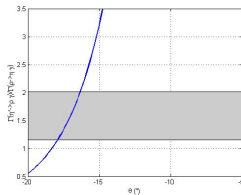


DECAYS WITH RESPECT TO θ

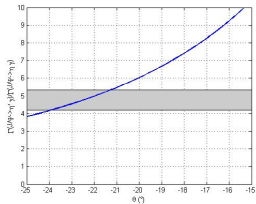
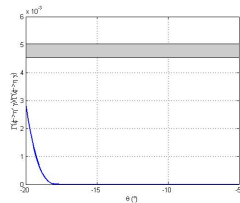
$$\frac{\Gamma(\eta' \rightarrow \omega\gamma)}{\Gamma(\omega \rightarrow \eta\gamma)}$$



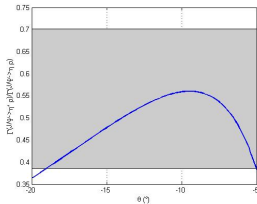
$$\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\rho \rightarrow \eta\gamma)}$$



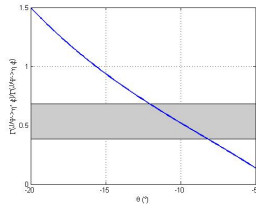
$$\frac{\Gamma(\phi \rightarrow \eta'\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \eta'\gamma)}{\Gamma(J/\Psi \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \rho\eta')}{\Gamma(J/\Psi \rightarrow \rho\eta)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \phi\eta')}{\Gamma(J/\Psi \rightarrow \phi\eta)}$$

THIRD GLUONIC STATE

$\hat{\theta}G_{\mu\nu}\tilde{G}^{\mu\nu} \rightarrow$ **Glueball** \sim **massive axion** in the chiral Lagrangian

$$\mathcal{L}^{(p^2)} = \frac{f^2}{8} \left\langle \partial_\mu U^\dagger \partial^\mu U + B(mU^\dagger + Um^\dagger) \right\rangle - \frac{\alpha}{2}(\eta_0 + \theta)^2 - \frac{1}{2}m_\theta^2\theta^2 + \frac{1}{2}\partial_\mu\theta\partial^\mu\theta$$

Inclusion of a gluonic state *via* the **θ -term and the anomaly**

$$\mathcal{M}_{qsg}^2 = \begin{pmatrix} m_\pi^2 + 2\alpha & \sqrt{2}\alpha & \sqrt{2}\beta \\ \sqrt{2}\alpha & 2m_K^2 - m_\pi^2 + \alpha & \beta \\ \sqrt{2}\beta & \beta & \gamma \end{pmatrix}$$

Rosenzweig, Salomone, and Schechter, *Phys. Rev. D***24** (1981) 2545

Degrande, V.M., Gérard, *in preparation*

What are the **theoretical constraints** on R ?

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \end{pmatrix} = \mathcal{R} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |gg\rangle \end{pmatrix}$$

Inclusion of a gluonic state *via* the **θ -term and the anomaly**

$$\mathcal{M}_{qsg}^2 = \begin{pmatrix} m_\pi^2 + 2\alpha & \sqrt{2}\alpha & \sqrt{2}\beta \\ \sqrt{2}\alpha & 2m_K^2 - m_\pi^2 + \alpha & \beta \\ \sqrt{2}\beta & \beta & \gamma \end{pmatrix}$$

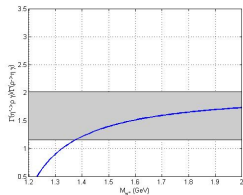
Diagonalization

$$\mathcal{R}^\dagger \mathcal{M}_{qsg}^2 \mathcal{R} = \mathcal{D}$$

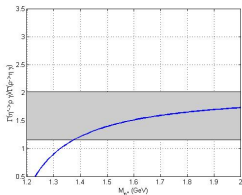
$\mathcal{D} = \text{diag} (M_\eta^2, M_{\eta'}^2, M_G^2)$ with M_G^2 **unknown**

DECAY WITH GLUEBALL FOR $M_\eta = 530$ MEV AND $M_{\eta'} = 1030$ MEV

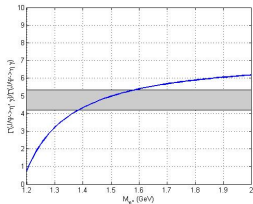
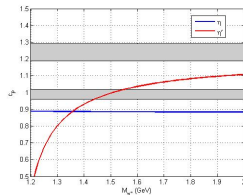
$$\frac{\Gamma(\eta' \rightarrow \omega\gamma)}{\Gamma(\omega \rightarrow \eta\gamma)}$$



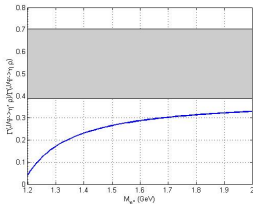
$$\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\rho \rightarrow \eta\gamma)}$$



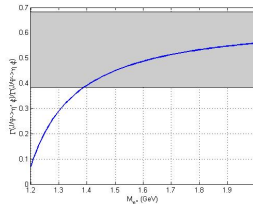
$$\frac{\Gamma(\eta(') \rightarrow \gamma\gamma)}{\Gamma(\pi \rightarrow \gamma\gamma)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \eta'\gamma)}{\Gamma(J/\Psi \rightarrow \eta\gamma)}$$



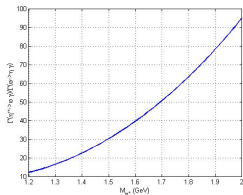
$$\frac{\Gamma(J/\Psi \rightarrow \rho\eta')}{\Gamma(J/\Psi \rightarrow \rho\eta)}$$



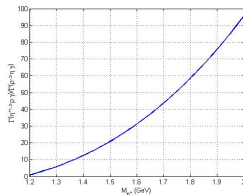
$$\frac{\Gamma(J/\Psi \rightarrow \phi\eta')}{\Gamma(J/\Psi \rightarrow \phi\eta)}$$

$\eta'' = \eta(1405)$? DECAYS FOR $M_\eta = 530$ MEV AND $M_{\eta'} = 1030$ MEV

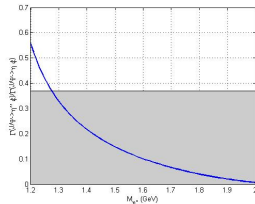
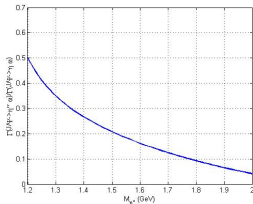
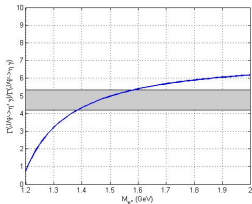
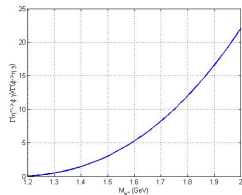
$$\frac{\Gamma(\eta'' \rightarrow \omega\gamma)}{\Gamma(\omega \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(\eta'' \rightarrow \rho\gamma)}{\Gamma(\rho \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(\eta'' \rightarrow \phi\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \eta''\gamma)}{\Gamma(J/\Psi \rightarrow \eta\gamma)}$$

$$\frac{\Gamma(J/\Psi \rightarrow \rho\eta'')}{\Gamma(J/\Psi \rightarrow \rho\eta)}$$

$$\frac{\Gamma(J/\Psi \rightarrow \phi\eta'')}{\Gamma(J/\Psi \rightarrow \phi\eta)}$$

PRELIMINARY RESULTS

Degrande, V.M., Gérard, in preparation

Glueball \sim massive axion in the chiral Lagrangian

$$\mathcal{L}^{(p^2)} = \frac{f^2}{8} \left\langle \partial_\mu U^\dagger \partial^\mu U + B(mU^\dagger + Um^\dagger) \right\rangle - \frac{\alpha}{2} (\eta_0 + \theta)^2 - \frac{1}{2} m_\theta^2 \theta^2 + \frac{1}{2} \partial_\mu \theta \partial^\mu \theta$$

Decays in better agreement with data but very sensitive to M_η

$$\begin{aligned} \theta &= -11.4^\circ & \varphi_G &= (4.7 \pm 0.3)^\circ & \varphi &= (46.8 \pm 1.8)^\circ \\ M_\eta &= 530 \text{ MeV} & M_{\eta'} &= 1030 \text{ MeV} & M_G &= 1400 - 1500 \text{ MeV} \end{aligned}$$

$\eta(1405)$ would be the third state, mainly gluonic !

BUT only leading order

Gérard and Kou, *Phys. Rev. Lett.* **97** (2006) 261804

Other anomalous decays

$$\text{Br}(B^0 \rightarrow K^0 \eta') = 65 \cdot 10^{-6}, \quad \text{Br}(B^0 \rightarrow K^0 \eta) < 2 \cdot 10^{-6}, \quad \text{Br}(B^0 \rightarrow K^0 \pi^0) = 10 \cdot 10^{-6}.$$

$$\eta' : \eta : \pi = 3 : 0 : 1$$

via **Penguin** diagram

PURE YANG-MILLS

- Various models reproduce lattice spectrum
- Dynamical gluon mass generation **BUT only 2 d.o.f.** in wave function
- **Instanton effects** \rightarrow mass difference between 0^{++} and 0^{-+}

SCALAR MESONS

- Chiral suppression $A(0^{++} \rightarrow \bar{q}q) \propto m_q$
 \rightarrow **$f_0(1710)$** realization of the glueball ?
- Not without controversy
- A confirmation of **three** f_0 is required
- Accurate data about their **productions and decays** would improve the understanding of their structure.

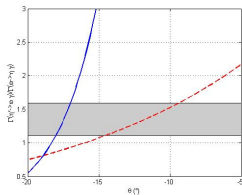
PSEUDOSCALAR MESONS

- Anomaly in pseudoscalar \rightarrow no ideal mixing for η and η'
- Chiral Lagrangian at LO not enough to describe η and η'
 \rightarrow **Inclusion of glueball** in chiral Lagrangian
- Preliminary results favor **$\eta(1405)$** realisation of the glueball
- Prediction for η'' decays \rightarrow need experimental confirmation !

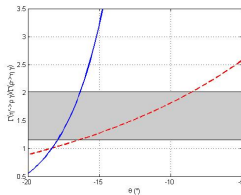
Backup Slides

DECAYS WITH RESPECT TO θ

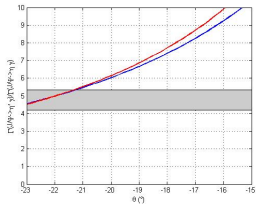
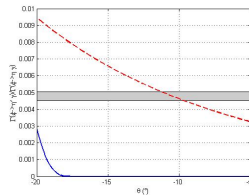
$$\frac{\Gamma(\eta' \rightarrow \omega\gamma)}{\Gamma(\omega \rightarrow \eta\gamma)}$$



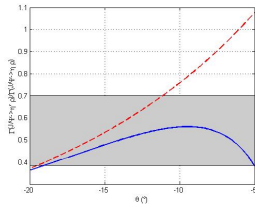
$$\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\rho \rightarrow \eta\gamma)}$$



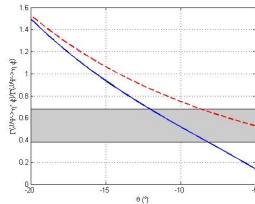
$$\frac{\Gamma(\phi \rightarrow \eta'\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \eta'\gamma)}{\Gamma(J/\Psi \rightarrow \eta\gamma)}$$

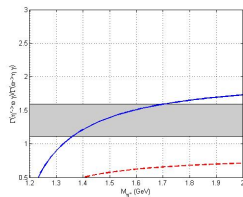


$$\frac{\Gamma(J/\Psi \rightarrow \rho\eta')}{\Gamma(J/\Psi \rightarrow \rho\eta)}$$

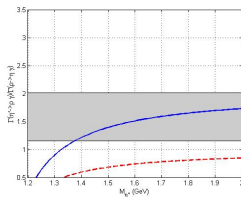


$$\frac{\Gamma(J/\Psi \rightarrow \phi\eta')}{\Gamma(J/\Psi \rightarrow \phi\eta)}$$

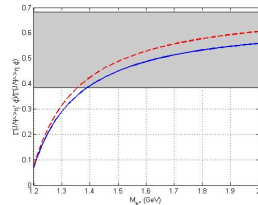
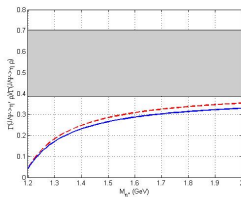
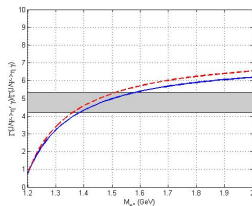
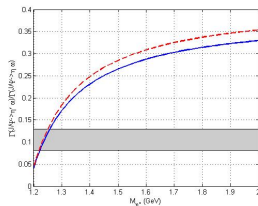
$$\frac{\Gamma(\eta' \rightarrow \omega\gamma)}{\Gamma(\omega \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(\eta' \rightarrow \rho\gamma)}{\Gamma(\rho \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \omega\eta')}{\Gamma(J/\Psi \rightarrow \omega\eta)}$$

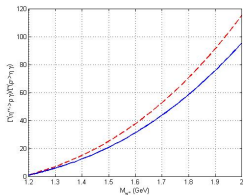


$$\frac{\Gamma(J/\Psi \rightarrow \eta'\gamma)}{\Gamma(J/\Psi \rightarrow \eta\gamma)}$$

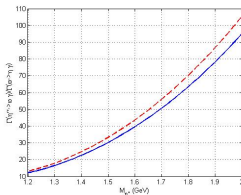
$$\frac{\Gamma(J/\Psi \rightarrow \rho\eta')}{\Gamma(J/\Psi \rightarrow \rho\eta)}$$

$$\frac{\Gamma(J/\Psi \rightarrow \phi\eta')}{\Gamma(J/\Psi \rightarrow \phi\eta)}$$

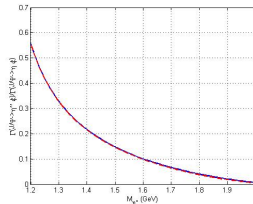
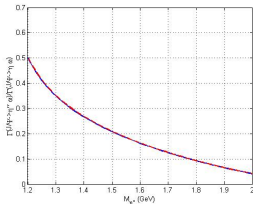
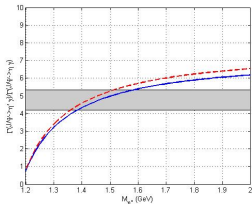
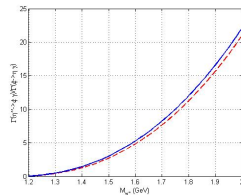
$$\frac{\Gamma(\eta'' \rightarrow \rho\gamma)}{\Gamma(\rho \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(\eta'' \rightarrow \omega\gamma)}{\Gamma(\omega \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(\eta'' \rightarrow \phi\gamma)}{\Gamma(\phi \rightarrow \eta\gamma)}$$



$$\frac{\Gamma(J/\Psi \rightarrow \eta''\gamma)}{\Gamma(J/\Psi \rightarrow \eta\gamma)}$$

$$\frac{\Gamma(J/\Psi \rightarrow \omega\eta'')}{\Gamma(J/\Psi \rightarrow \omega\eta)}$$

$$\frac{\Gamma(J/\Psi \rightarrow \phi\eta'')}{\Gamma(J/\Psi \rightarrow \phi\eta)}$$