# Study of tetraquarks with a simple model

Marco Cardoso[1], Pedro Bicudo[1], Nuno Cardoso[1]

[1]CFTP, Instituto Superior Técnico, Lisboa

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# Contents

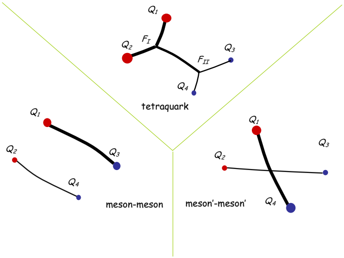
- Static tetraquarks on the lattice
- A Simplified potential to understand tetraquarks
  - Finite differences
  - Scattering theory

- Lattice studies[1] indicate that the $qq\bar{q}\bar{q}$ static potential is given by:

$$V_{FF} = \min(V^{Tetra}, V^{M_{13}M_{24}}, V^{M_{14}M_{23}})$$

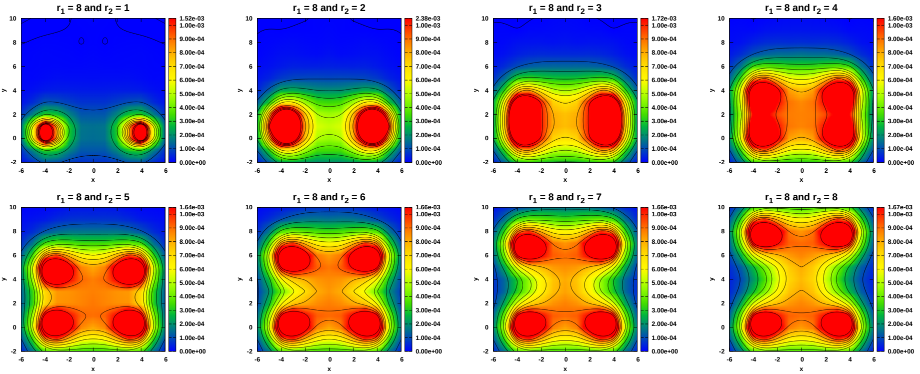
- Being $V_{Tetra}$ given by:

$$V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = C + \alpha_s \sum_{i<j} \frac{\lambda_i}{2} \cdot \frac{\lambda_j}{2} \frac{1}{r_{ij}} + \sigma L_{min}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)$$




[1] Okiharu, Suganuma and Takahashi, Phys. Rev. D 72, 014505 (2005)

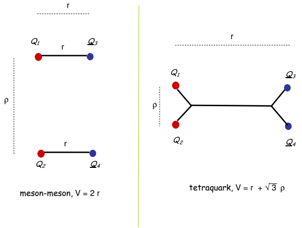
- Lagrangian density distribution for the tetraquark branch



- Now we will use a model, based on the tetraquark static potential
- The following simplifications are made:
  - No relativistic or spin effects
  - Consider all quarks with the same mass although distinguishable
  - No Coulomb terms
  - The number of variables is reduced by doing $\rho_{12} = \rho_{34}$

# Simplified Potential II

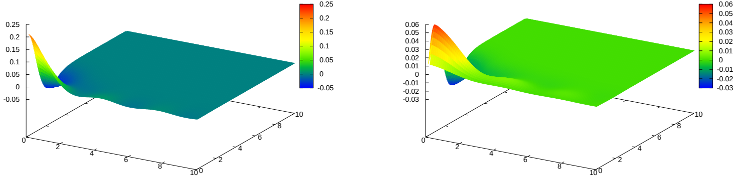
- A simplified potential is used $V_{FF} = \sigma \min(2r, \sqrt{3}\rho + r)$



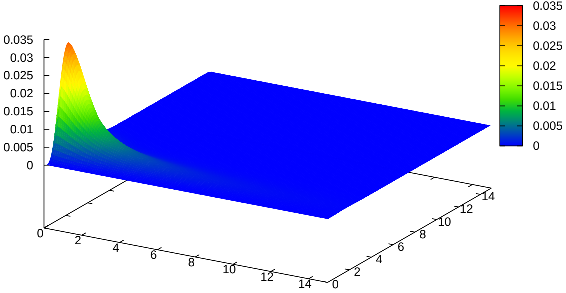

- Two approaches are used:
  - Finite differences : Usefull to test the existence of bound states
  - Scattering theory : Usefull to see resonances

- Write wavefunction as:
  - $\Psi(r,\rho) = \frac{u(r,\rho)}{r\,\rho} Y_{l_r m_r}(\theta_r, \varphi_r) Y_{l_\rho m_\rho}(\theta_\rho, \varphi_\rho)$
- $-\frac{\hbar^2}{2m}(\frac{d^2}{dr^2} + \frac{d^2}{d\rho^2})\Psi + V_{FF}(r,\rho)\Psi = E\Psi$
- Discretize differential operators:
  - $\frac{d^2 u}{dr^2} \longrightarrow \frac{u_{i+1,j} - 2u_{ij} + u_{i-1,j}}{a^2}$
  - $\frac{d^2 u}{d\rho^2} \longrightarrow \frac{u_{i,j+1} - 2u_{ij} + u_{i,j-1}}{b^2}$

- No complete bound state for $l_r < 3$ with $l_\rho = 0$
- However, we obtain some "semi-localized" states



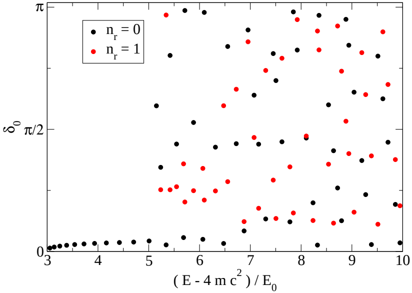
- We have a bound state for $l_r = 3$



- We tried to calculate the phase shifts with finite differences, by calculating:

$$\psi_i(\rho) = \int d^3\mathbf{r}\,\phi_i^*(\mathbf{r})\Psi(\mathbf{r},\rho)$$

- Where $\phi_i(\mathbf{r})$ are the eigenvalues of the confined hamiltonian $\hat{H} = -\frac{\hbar^2}{2m}\nabla_r^2 + 2\sigma r$
- Then calculating the large $\rho$limit $\psi_i \longrightarrow A_i \sin k_i r + \delta_i$




- Only works for one channel:

- Expand $\Psi(\mathbf{r},\rho) = \sum_i \psi_i(\rho)\phi_i(\mathbf{r})$ with

$$-\frac{\hbar^2}{2m}\nabla_r^2\phi_i + 2\sigma r\phi_i = \varepsilon_i\phi_i$$

- Coupled Channels Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla_\rho^2\psi_i + V_{ij}\psi_j = (E-\varepsilon_i)\psi_i$$

with $V_{ij}(\rho) = \int d^3\mathbf{r}\ \phi_i^*(\mathbf{r})(V_{FF}(r,\rho) - 2\sigma r)\phi_j(\mathbf{r})$

- Assymptoticaly $\Psi_i \to e^{ik_i\rho\cos\theta_\rho} + f_{ii}(\hat{\rho})\frac{e^{ik_i\rho}}{\rho}$

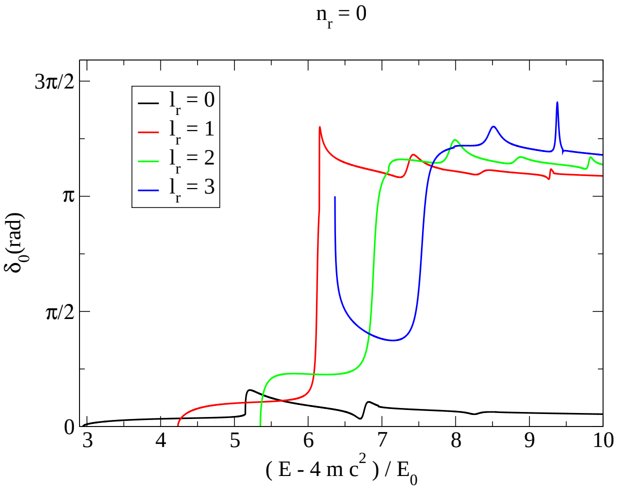
$$\Psi_j \to f_{ij}(\hat{\rho})\frac{e^{ik_j\rho}}{\rho}, \text{ for } i \neq j$$

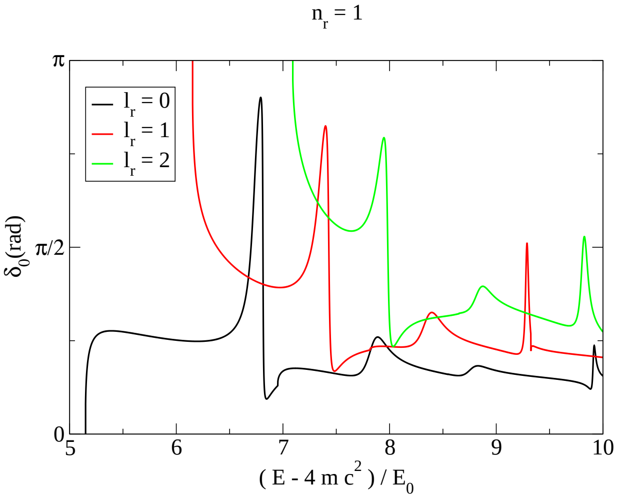
- $\psi_i(\rho) = e^{ik_h r}\delta_{ih} + \chi_i(\rho)$
- $\chi_i(\rho) = \frac{u_i^{l_\rho}(\rho)}{\rho} Y_{l_\rho m_\rho}(\theta_\rho, \varphi_\rho)$
- Solve the equation:

$$-\frac{\hbar^2}{2m}\frac{d^2 u_i^{l_\rho}}{dr^2} + V_{ij} u_j^{l_\rho} = (E - \varepsilon_i) u_i^{l_\rho} - V_{ih} j_{l_\rho}(k_h r) r$$

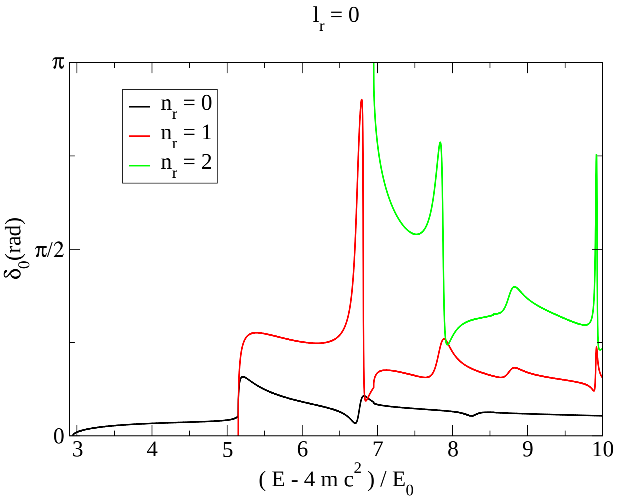
- Two conserved angular momenta: $\mathbf{L}_r = \mathbf{r} \times \mathbf{p}_r$ and $\mathbf{L}_\rho = \rho \times \mathbf{p}_\rho$
- Each assymptotic state could be described by $l_r$, $n_r$ and $l_\rho$
- Phase shifts could be calculated by the large distance behaviour of $\chi_i$

- Results for the phase shifts





n_r = 1
π
π/2
0
δ_0(rad)
l_r = 0
l_r = 1
l_r = 2
5
6
7
8
9
10
( E - 4 m c^2 ) / E_0


l_r = 0
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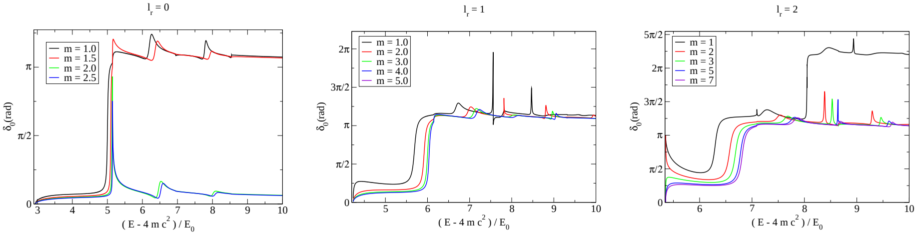
7
8
9
10
( E - 4 m c^2 ) / E_0

- Reduced mass correction: $\mu_i = m + \frac{\varepsilon_i}{4c^2}$



- The decay width is given by: $\frac{\Gamma}{2} = \left(\frac{d\delta}{dE}\right)^{-1}$ when$\delta = \frac{\pi}{2}$

| $l_r$ | $(E - 4mc^2)/E_0$ | $\Gamma/E_0$ |
|---|---|---|
| 1 | 6.116 | 0.037 |
| 2 | 6.855 | 0.131 |
| 3 | 7.462 | 0.352 |

- For light quarks $m \simeq \sqrt{\sigma} \simeq 0.4\, GeV$: $\Gamma_1 \simeq 15\, MeV$, $\Gamma_2 \simeq 52\, MeV$ and $\Gamma_3 \simeq 140\, MeV$
- For the charm $(m_c \simeq 1.5\, GeV)$: $\Gamma_1 \simeq 10\, MeV$, $\Gamma_2 \simeq 34\, MeV$ and $\Gamma_3 \simeq 90\, MeV$
- For the bottom $(m_b \simeq 5\, GeV)$: $\Gamma_1 \simeq 6\, MeV$, $\Gamma_2 \simeq 23\, MeV$ and $\Gamma_3 \simeq 60\, MeV$

- We have constructed a simplified model for the tetraquark, based on the Triple Flip-Flop Potential
- We have observed the formation of a resonance for $l_r > 0$
- A bound state for $l_r = 3$ was observed
- Ressonances are also formed for $l_r = 0$ for sufficiently large reduced masses
- The formation of tetraquarks with high angular orbital momentum seems to be plausible