Chiral properties of the baryon ground state

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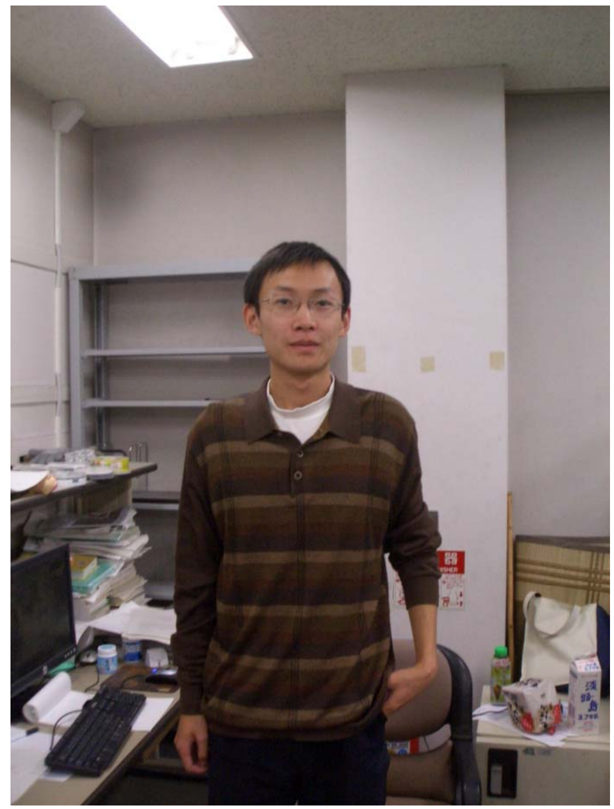

Outline

- **Intro: why chiral mixing?** the axial F,D values and the flavour-singlet axial coupling ("nucleon spin problem")
- **Method:** three-quark **baryon interpolating fields** and **chiral multiplets** $SU_L(3) \times SU_R(3)$ and $U_A(1)$ chiral (algebraic) properties of baryon fields.
- **Chiral interactions:** chiral $SU_L(3) \times SU_R(3)$ and $U_A(1)$ selection rules and their consequences: two permissible scenarios and predicted excited baryon masses.

Light quark chiral properties

$$\psi = \underbrace{\frac{1-\gamma_5}{2}\psi}_{\text{Left handed}} + \underbrace{\frac{1+\gamma_5}{2}\psi}_{\text{Right handed}}$$

$SU(3)_L \times SU(3)_R$ =	(3, 1)	(1, 3)	*Naïve*	g_A	= 1
	(1, 3)	(3, 1)	*Mirror*		– 1

Study chiral SU(3) multiplets of baryons:

M. Gell-Mann, Physics 1, 63 (‘64); I. Gerstein & B.W. Lee, PRL14,676 (’65), *ibid*. 16, 114,1060 (’66); H. Harari, PRL16, 964, *ibid*. 17, 56 (’66), Altarelli, Gatto, Maiani, Preparatta, PRL16,377, 918(’66); T. Cohen, X. Ji, PRD55, 6870 ('97); Chen, Dmitrasinovic, Hosaka, Nagata, Zhu, PRD78: 054021 (‘08); Chen, Dmitrasinovic, Hosaka, PRD81:054002 (‘10)

Baryon chiral multiplets

- Chiral multiplets of baryons that consist of three quarks are as follows

$$[(3,1)+(1,3)]^3 = (1,1)+[(\bar{3},3)+(3,\bar{3})]+[(8,1)+(1,8)]$$
$$+[(3,6)+(6,3)]+[(10,1)+(1,10)]$$

$[(\bar{3},3)+(3,\bar{3})]+[(8,1)+(1,8)]$: N

$[(3,6)+(6,3)]$: N, Δ

$[(10,1)+(1,10)]$: Δ

- "naïve" vs. "mirror", i.e. (1,8) vs. (8,1), in chiral multiplets is not known *a priori*
- Total spin/angular momentum of three quarks is left unspecified by this CG series.

Two chiral multiplet mixing

- (N_2,Δ)chiral multiplet: (6, 3) + (3, 6) -> $g_A(N_2)$= 5/3
- mix with N_1 in chiral multiplet [(3,3*)+(3*,3)] ("Harari"), or [(8,1)+(1,8)] ("Gerstein-Lee")-> $g_A(N_1)$=1
- Mixing leads to $g_A(N) = 1.267$
- This predicts the flavor-singlet and F,D axial couplings!

I. Gerstein & B.W. Lee, PRL14,676 ('65), *ibid*. 16, 114,1060 ('66)

$$
\begin{aligned}
1.267 &= g^{(1)}_{A\ (\frac{1}{2},0)} \cos^2\theta + g^{(1)}_{A\ (1,\frac{1}{2})} \sin^2\theta, \\
&= g^{(1)}_{A} \cos^2\theta + \frac{5}{3} \sin^2\theta
\end{aligned}
$$

Weinberg, PR 177 (1969) 2604

$$
\begin{aligned}
g^{(0)}_{A\ \mathrm{mix.}} &= g^{(0)}_{A\ (\frac{1}{2},0)} \cos^2\theta + g^{(0)}_{A\ (1,\frac{1}{2})} \sin^2\theta \\
&= g^{(0)}_{A} \cos^2\theta + \sin^2\theta,
\end{aligned}
$$

H. Harari, PRL16, 964, *ibid*. 17, 56 ('66); Altarelli, Gatto, Maiani, Preparatta, PRL16,918('66)

Chiral $U_A(1)$ transformations of baryons

$$q \to \exp(i\gamma_5 b)q$$

$$\left.\begin{aligned}\delta_5 N_1 &= ib\gamma_5(N_1 + 2N_2)\\ \delta_5 N_2 &= ib\gamma_5(N_2 + 2N_1)\end{aligned}\right\}\ \text{U(1)}_\text{A}\ \text{partners}$$

$$\delta_5(N_1 - N_2) = -ib\gamma_5(N_1 - N_2)$$

$$\delta_5(N_1 + N_2) = +3ib\gamma_5(N_1 + N_2).$$

$$\delta_5 N_3 = -ib\gamma_5 N_3$$

$$\delta_5 N_4 = -ib\gamma_5 N_4$$

$$\delta_5 N_5 = +3ib\gamma_5 N_5.$$

- $U_A(1)$ transformation "mixes" two kinds of baryons: N_1, N_2
- This "$U_A(1)$ symmetry doublet" reduces into two singlets $(N_1 \pm N_2)$ with different baryon axial "charges":-1/2, +3/2.
- Do these two kinds of baryons lead to "parity doublets"?
- Gilman and Kugler identified the $U_A(1)$ axial charge with the intrinsic quark spin S_z.

F.J. Gilman & M. Kugler, PRL30, 518 ('73)

Nucleon "spin-crisis"

- Nonrelativistic quark model predicts isoscalar ax. = 1:
- Whole of proton's spin is carried by constituent quarks!
- Basic problem for the NR quark model, but completely natural in relativistic chiral quark model.

8.1 Constituent quarks and $g_A^{(k)}$

First, consider the static quark model. The simple SU(6) proton wavefunction

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}|u\uparrow(ud)_{S=0}\rangle + \frac{1}{\sqrt{18}}|u\uparrow(ud)_{S=1}\rangle - \frac{1}{3}|u\downarrow(ud)_{S=1}\rangle - \frac{1}{3}|d\uparrow(uu)_{S=1}\rangle + \frac{\sqrt{2}}{3}|d\downarrow(uu)_{S=1}\rangle \qquad (8.1)$$

yields $g_A^{(3)} = \frac{5}{3}$ and $g_A^{(8)} = g_A^{(0)} = 1$.

Some implications of chiral mixing for the nucleon g_A

			$g_A^{(1)}$	$g_A^{(0)}$
N_1	LLL	$(\frac{1}{2},0)$	1	+ 3
N_2	RRL		1	− 1
N_3	LLR	$(1,\frac{1}{2})$	5/3	+1
N_{mirror}		$(0,\frac{1}{2})$	− 1	+ 1

Take a combination:

$$N = \cos\theta N_2 + \sin\theta N_3 \qquad g_A^{(1)} \sim 1.33,\ g_A^{(0)} \sim 0$$

$$\theta \sim 45^{\circ}$$

$$N^* = \cos\theta N_1 + \sin\theta N_{mirror} \qquad g_A^{(1)} \sim 0,\ g_A^{(0)} \sim 1$$

Two-field chiral mixing

TABLE II. The values of the baryon isoscalar axial coupling constant predicted from the naive mixing and $g_{A\text{expt.}}^{(1)} = 1.267$; compare with $g_{A\text{expt.}}^{(0)} = 0.33 \pm 0.03 \pm 0.05$, $F = 0.459 \pm 0.008$, and $D = 0.798 \pm 0.008$, leading to $F/D = 0.571 \pm 0.005$; see Ref. [2].

Case	$(g_A^{(1)}, g_A^{(0)})$	$g_{A\text{mix.}}^{(1)}$	θ_i	$g_{A\text{mix.}}^{(0)}$	$g_{A\text{mix.}}^{(0)}$	F	F/D
I	$(+1, -1)$	$\frac{1}{3}(4 - \cos 2\theta)$	39.3^o	$-\cos 2\theta$	-0.20	0.267	0.267
II	$(+1, +3)$	$\frac{1}{3}(4 - \cos 2\theta)$	39.3^o	$(2\cos 2\theta + 1)$	2.20	0.866	2.16
III	$(-1, +1)$	$\frac{1}{3}(1 - 4\cos 2\theta)$	67.2^o	1	1.00	0.567	0.81
IV	$(-1, -3)$	$\frac{1}{3}(1 - 4\cos 2\theta)$	67.2^o	$-(2\cos 2\theta + 1)$	0.40	0.417	0.491

- In two cases (I and IV) the predictions are in the same ballpark as experiment.
- Case I is the "Ioffe current", case IV is the "mirror" image of the orthogonal complement to Ioffe (seldom used).
- Can one relate mixing angles to baryon masses? Yes.
- Can one construct mirror fields? Yes.

Chen, Dmitrasinovic, Hosaka, PRD81:054002 (2010)

Experimental input: flavor singlet and F,D axial couplings

- DIS shows that only about 1/3 of the nucleon's helicity is carried by quarks. This can be expressed as flavor-singlet axial coupling ~1/3

$$g_A^{(0)} = 0.33 \pm 0.03(stat.) \pm 0.05(syst.)$$

Bass (2007)

$$g_A^{(0)} = 0.28 \pm 0.16$$

Fillipone, Ji (2001)

- The (semi-leptonic) weak decays of the hyperons yield F=0.459, D=0.798 subject to SU(3) symmetry breaking corrections.

F=0.477 D=0.835 with SU(3) symmetry breaking corrections, or F=0.459 D=0.798 without

Yamanishi, PRD76: 014006 (2007)

Summary of J=1/2 SU(3) baryons

TABLE I. The Abelian and the non-Abelian axial charges (+ sign indicates naive, − sign mirror transformation properties) and the non-Abelian chiral multiplets of $J^P = \frac{1}{2}$, Lorentz representation $(\frac{1}{2}, 0)$ nucleon and Δ fields; see Refs. [15–18].

Case	Field	$g_A^{(0)}$	$g_A^{(1)}$	F	D	$SU_L(3) \times SU_R(3)$
I	$N_1 - N_2$	-1	$+1$	0	$+1$	$(3, \bar{3}) \oplus (\bar{3}, 3)$
II	$N_1 + N_2$	$+3$	$+1$	$+1$	0	$(8, 1) \oplus (1, 8)$
III	$N_1' - N_2'$	$+1$	-1	0	-1	$(\bar{3}, 3) \oplus (3, \bar{3})$
IV	$N_1' + N_2'$	-3	-1	-1	0	$(1, 8) \oplus (8, 1)$
0	$\partial_\mu (N_3^\mu + \frac{1}{3} N_4^\mu)$	$+1$	$+\frac{5}{3}$	$+\frac{2}{3}$	$+1$	$(6, 3) \oplus (3, 6)$

Chen, Dmitrasinovic, Hosaka, Nagata, Zhu,PRD78:054021('08)
Chen, Dmitrasinovic, Hosaka, PRD81:054002 ('10)

$$g_A^{(0)} = 3F - D$$

Three-field chiral mixing

- With two mixing angles one can fit both axial couplings and predict F,D!
- But all three cases are identical! Why?
- The 3-quark fields satisfy

$$g_A^{(0)} = 3F - D$$

- The discrepancy between expt. and fit is due 5-quark contributions and/or SU(3) symmetry breaking.

$$\frac{5}{3}\sin^2\theta + \cos^2\theta\left(g_A^{(1)}\cos^2\varphi + g_A^{(1)\prime}\sin^2\varphi\right) = 1.267$$

$$\sin^2\theta + \cos^2\theta\left(g_A^{(0)}\cos^2\varphi + g_A^{(0)\prime}\sin^2\varphi\right) = 0.33$$

$$\cos^2\theta(F\cos^2\varphi + F'\sin^2\varphi) + \frac{2}{3}\sin^2\theta = F,$$

$$\cos^2\theta(D\cos^2\varphi + D'\sin^2\varphi) + \sin^2\theta = D.$$

TABLE III. The values of the mixing angles obtained from the simple fit to the baryon axial coupling constants and the predicted values of axial F and D couplings. The experimental values are $F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$, leading to $F/D = 0.571 \pm 0.005$; see Ref. [2].

Case	$g^{(3)}_{A\text{expt.}}$	$g^{(0)}_{A\text{expt.}}$	θ	φ	F	D	F/D
I-II	1.267	0.33	39.3^o	$28.0^o \pm 2.3^o$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
I-III	1.267	0.33	$50.7^o \pm 1.8^o$	$23.9^o \pm 2.9^o$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
I-IV	1.267	0.33	$63.2^o \pm 4.0^o$	$54^o \pm 23^o$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04

Chen, Dmitrasinovic, Hosaka, PRD81:054002 (2010)

Can one reproduce this mixing dynamically i.e. from a model?

Baryon-meson chiral interactions

- Not all baryons' chiral multiplets allow chirally invariant interactions with the (σ,π) mesons: e.g.

$$\underbrace{[(8,1)+(1,8)]^2}_{N} \otimes [(\bar{3},3)+(3,\bar{3})] = [(1,1)+(8,8)]\otimes[(\bar{3},3)+(3,\bar{3})]$$

$$+[(8,1)+(1,8)]\otimes[(\bar{3},3)+(3,\bar{3})]+[(10,1)+(1,10)]\otimes[(\bar{3},3)+(3,\bar{3})]$$

$$+[(\overline{10},1)+(1,\overline{10})]\otimes[(\bar{3},3)+(3,\bar{3})]+[(27,1)+(1,27)]\otimes[(\bar{3},3)+(3,\bar{3})]$$

- Therefore [(1,8) + (8,1)] chiral multiplet may be discarded if stand-alone i.e. if no chiral mixing. Same with [(1,10) + (10,1)]!

First SU(3)xSU(3) chiral invariants constructed by J.A. Cronin, PR 161, 1483 ('67); B.W. Lee, PR170,1389 ('68); W. Bardeen, B.W.Lee, PR177,2389 ('69);

Baryon-meson chiral invariants I

$(SU_A(3),\, U_A(1))$	$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$[mir]	$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$[mir]	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$	$(\mathbf{1},\mathbf{10})\oplus(\mathbf{10},\mathbf{1})$[mir]
$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$[mir]	N/A	$(\surd,\, \surd)$	$(\surd,\, \surd)$	N/A
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$[mir]	$(\surd,\, \surd)$	$(\surd,\, \surd)$	$(\surd,\, \surd)$	N/A
$(\bar{\mathbf{6}},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\bar{\mathbf{6}})$	$(\surd,\, \surd)$	$(\surd,\, \surd)$	$(\surd,\, \surd)$	$(\surd,\, \surd)$
$(\mathbf{1},\overline{\mathbf{10}})\oplus(\overline{\mathbf{10}},\mathbf{1})$[mir]	N/A	N/A	$(\surd,\, \surd)$	N/A

$(SU_A(3),\, U_A(1))$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})$[mir]	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	N/A	$(\surd,\, \surd)$	$(\surd,\, \surd)$	N/A
$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$	$(\surd,\, \surd)$	$(\surd,\, \surd)$	$(\surd,\, \surd)$	N/A
$(\bar{\mathbf{3}},\bar{\mathbf{6}})\oplus(\bar{\mathbf{6}},\bar{\mathbf{3}})$[mir]	$(\surd,\, \surd)$	$(\surd,\, \surd)$	$(\surd,\, \surd)$	$(\surd,\, \surd)$
$(\overline{\mathbf{10}},\mathbf{1})\oplus(\mathbf{1},\overline{\mathbf{10}})$	N/A	N/A	$(\surd,\, \surd)$	N/A

$(SU_A(3),\, U_A(1))$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$[mir]
$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$	N/A	$(\surd,\, \times)$
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$[mir]	$(\surd,\, \times)$	N/A

- Chiral selection rules for baryons' chiral multiplets allow interactions with the (σ,π) mesons that are both SU(3)xSU(3) and U(1) chirally invariant, with only one off-diagonal exception.

Chen, Dmitrasinovic, Hosaka, PRD83:014015 (11);

Baryon-meson chiral invariants II

$(SU_A(3),\, U_A(1))$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})[\mathrm{mir}]$	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})[\mathrm{mir}]$	$(\surd,\, \surd)$	N/A	N/A	N/A
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})[\mathrm{mir}]$	N/A	$(\surd,\, \surd)$	N/A	N/A
$(\bar{\mathbf{6}},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\bar{\mathbf{6}})$	N/A	N/A	$(\surd,\, \surd)$	N/A
$(\mathbf{1},\overline{\mathbf{10}})\oplus(\overline{\mathbf{10}},\mathbf{1})[\mathrm{mir}]$	N/A	N/A	N/A	$(\surd,\, \surd)$

Chen, Dmitrasinovic, Hosaka, PRD83:014015 (11);

- Chiral selection rules for baryons' chiral multiplets allow interactions with the (σ,π) mesons that are both SU(3)xSU(3) and U(1) chirally invariant, with no exceptions.

[(6,3)+(3,6)] - [(3*,3)+(3,3*)] mixing

- Possible parity assignments and candidate states for two state mixing
- [(6,3)+(3,6)]-[(3*,3)+(3,3*)]-[(3,3*)+(3*,3)] ("Harari") mixing scenario Λ(1405) and Λ(1810) (*** PDG) are the best candidates.
- Predict Δ(2170)

$(N^{*P}, \Lambda^{P'}, \Delta^{P''})$	$(N,\ N^*)$	Λ (MeV)	$\Lambda_{\rm expt.}$ (MeV)	Δ (MeV)	$\Delta_{\rm expt.}$ (MeV)
$(-,+,+)$	N(940), R(1535)	2330	-	2330	1910
$(-,-,+)$	N(940), R(1535)	1140	1405	2330	1910
$(-,+,-)$	N(940), R(1535)	2330	-	1140	-
$(+,-,-)$	N(940), R(1440)	2030,2730	-	2030,2730	-
$(-,-,-)$	N(940), R(1535)	1140	1405	1140	-

Chen, Dmitrasinovic, Hosaka, PRD83:014015 (11);

No.	g_1	g_2	g_3	g_4	g_5	Λ_1^P (MeV)	Λ_2^P (MeV)	Δ^P (MeV)
1	-4.7	8.4	-3.4	2.9	9.8	1370^-	1850^+	2170^-
2	-7.2	4.6	7.9	9.1	-4.2	1940^+	2430^-	1200^-

H. Harari, PRL16, 964, *ibid*. 17, 56 ('66)

[(6,3)+(3,6)] - [(8,1)+(1,8)] mixing

- In general only possible with additional two-meson interaction
- Possible parity assignments and candidate states for two state mixing
- [(6,3)+(3,6)]-[(8,1)+(1,8)]-[(3*,3)+(3,3*)] ("Gerstein-Lee") mixing scenario Λ(1600)(*** PDG) and Λ(1800)(*** PDG) are the best candidates
- Predict Δ(2070) or Δ(2110)

$(N^{*P}, \Delta^{P'})$	(N, N^*)	Δ (MeV)	$\Delta_{\rm expt.}$ (MeV)
$(-,+)$	N(940), R(1535)	2330	1910
$(+,-)$	N(940), R(1440)	2030,2730	-
$(-,-)$	N(940), R(1535)	1140	-

Chen, Dmitrasinovic, Hosaka, PRD83:014015 (11);

No.	g_1	g_2	g_3	g_4	g_5	Λ^P (MeV)	Δ^P (MeV)
1	4.6	8.0	-1.8	-6.1	9.7	1580^+	2070^-
2	-8.4	4.3	7.1	10.6	-2.4	2750^-	1124^-
3	-1.3	10.2	2.1	-2.5	9.8	640^+	2660^-
4	-8.7	8.1	7.3	7.1	2.9	1850^-	2110^-

I. Gerstein & B.W. Lee, PRL14,676 ('65), *ibid*. 16, 114,1060 ('66)

Chiral mixing – Delta mass

- We have fitted two axial constants while fixing the masses of the two nucleon states -> thus we predicted the Delta mass.
- Odd (-) parity assignment allowed! The predicted masses of 2070, 2110, 2170 MeV are all reasonably close to the (*PDG) resonance S31(2150).
- This chiral partner of N(940) is much heavier!

$\Delta(1950)$	F_{37}	****
$\Delta(2000)$	F_{35}	**
$\Delta(2150)$	S_{31}	*
$\Delta(2200)$	G_{37}	*
$\Delta(2300)$	H_{39}	**
$\Delta(2350)$	D_{35}	*
$\Delta(2390)$	F_{37}	*
$\Delta(2400)$	G_{39}	**
$\Delta(2420)$	$H_{3,11}$	****
$\Delta(2750)$	$I_{3,13}$	**
$\Delta(2950)$	$K_{3,15}$	**

Summary

- We have used the chiral classification of qqq baryon interpolating fields. All bare axial couplings are pre-determined by the chiral multiplet.
- Mixing of two chiral configurations reveals new and simple explanations of the "spin content of the nucleon" and of the baryon's axial F,D values.
- We have constructed effective Lagrangians that reproduce this mixing and used them to fit observed baryons and to predict some high-lying members of baryons' chiral multiplets