Heavy-quark Langevin dynamics and single-electron spectra in AA collisions Andrea Beraudo

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Work done and in progress in Torino in collaboration with
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^aPresented at "Hot Quarks 2010" (arXiv:1007.4170 [hep-ph]), "Jets in protonproton and heavy-ion collisions" (arXiv:1009.2434 [hep-ph]), CERN Th. Institute "The first heavy ion collisions at the LHC" and "Hard Probes 2010" (arXiv:1011.0400 [hep-ph]).

Outline

- Heavy quarks as *hard* probes of the Quark Gluon Plasma
- Theoretical framework:
 - the relativistic Langevin equation in an expanding medium
 - evaluation of the transport coefficients
- Numerical results (for RHIC and LHC): from the initial $Q\overline{Q}$ production to the final *e*-spectra
 - Invariant yields $E(dN/d^3p)$ (pp vs AA)
 - Nuclear modification factor $R_{AA}(p_T)$
 - Elliptic flow coefficient $v_2(p_T)$



Properties of the QGP @ RHIC $(T_{max} \leq 2T_c)$ arising from the experimental data:

• It behaves like a fluid $(\lambda_{mfp} \ll L)$: p_T -spectra are well described by hydrodynamics:

$$\underbrace{\partial_{\mu}T^{\mu\nu} = 0}_{\text{four-momentum}}, \quad \underbrace{\partial_{\mu}j_{B}^{\mu} = 0}_{\text{baryon number}} \quad \text{and} \quad \underbrace{p = p(\epsilon)}_{\text{EOS}}$$

Only the *conservation laws* matter; the properties of the medium are encoded into the *equation of state*!

• It is a very opaque medium: sizeable energy loss suffered by high- p_T particles.





Sizeable in-medium energy-loss suffered by high- p_T partons!

- hard partons from the surface pointing outwards reach the detectors;
- energy of partons crossing the plasma gets dissipated (scattering in the medium, with radiation of soft gluons).

Heavy-quark dynamics in the Quark Gluon Plasma

Heavy quarks as probes of the QGP

• Relaxation to thermal equilibrium (cumulated effect of many random collisions):

$$\frac{\langle p^2 \rangle}{2M} = \frac{3}{2}T \implies \bar{p}_{\text{heavy}} = \sqrt{3MT} \quad (\bar{p}_{\text{light}} \sim T)$$
$$\langle p_{\text{heavy}}^2 \rangle \sim \frac{M}{T} \langle p_{\text{light}}^2 \rangle \implies \tau_{\text{heavy}} \sim \frac{M}{T} \tau_{\text{light}}$$

In an expanding fireball with a finite life-time one does not expect the heavy quarks to reach full equilibrium with the medium.

- One would also expect less quenching of the p_T -spectra wrt light hadrons
 - suppression of collinear gluon radiation for $\theta \leq M/E$ (dead-cone effect): collisional energy-loss should play an important role!
 - Casimir factor C_F entails weaker coupling with the medium wrt gluons (C_A) , hence smaller amount of energy-loss.

The relativistic Langevin equation...

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{i} + \underbrace{\xi^i(t)}_{i},$$

determ. stochastic

with the properties of the noise encoded in

$$\langle \xi^{i}(\boldsymbol{p}_{t})\xi^{j}(\boldsymbol{p}_{t'})\rangle = \boldsymbol{b}^{ij}(\boldsymbol{p}_{t})\frac{\delta_{tt'}}{\Delta t} \qquad \boldsymbol{b}^{ij}(\boldsymbol{p}) \equiv \kappa_{\parallel}(p)\hat{p}^{i}\hat{p}^{j} + \kappa_{\perp}(p)(\delta^{ij}-\hat{p}^{i}\hat{p}^{j})$$

Transport coefficients to calculate:

- Momentum diffusion $\kappa_{\perp} \equiv \frac{1}{2} \frac{\langle \Delta p_{\perp}^2 \rangle}{\Delta t}$ and $\kappa_{\parallel} \equiv \frac{\langle \Delta p_{\parallel}^2 \rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D^{\text{Ito}}(p) = \frac{\kappa_{\parallel}(p)}{2TE_p} - \frac{1}{E_p^2} \left[(1 - v^2) \frac{\partial \kappa_{\parallel}(p)}{\partial v^2} + \frac{d - 1}{2} \frac{\kappa_{\parallel}(p) - \kappa_{\perp}(p)}{v^2} \right]$$

fixed in order to insure the approach to equilibrium (Einstein relation): Langevin eq. \Leftrightarrow Fokker Planck eq. with steady solution $\exp(-E_p/T)$



...in an expanding fluid

The fields $u^{\mu}(x)$ and T(x) are taken from the output of two longitudinally boost-invariant ("Hubble-law" longitudinal expansion $v_z = z/t$)

$$x^{\mu} = (\tau \cosh \eta, \boldsymbol{r}_{\perp}, \tau \sinh \eta) \quad \text{with} \quad \tau \equiv \sqrt{t^2 - z^2}$$
$$u^{\mu} = \bar{\gamma}_{\perp} (\cosh \eta, \bar{\boldsymbol{v}}_{\perp}, \sinh \eta) \quad \text{with} \quad \bar{\gamma} \equiv \frac{1}{\sqrt{1 - \bar{\boldsymbol{v}}_{\perp}^2}}$$

hydro codes^a.

NB Both codes employ the simplifying assumption $\eta \equiv \eta_s \equiv \eta_L$, where

$$\eta_S \equiv \frac{1}{2} \ln \frac{t+z}{t-z}$$
 and $\eta_L \equiv \frac{1}{2} \ln \frac{1+v_z}{1-v_z}$,

corresponding to a "Hubble-law" longitudinal expansion $v_z = z/t$.

^aP.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909 P. Romatschke and U.Romatschke, Phys. Rev. Lett. **99** (2007) 172301

Evaluation $\kappa_{\perp/\parallel}(p)$

Intermediate cutoff $|t|^* \sim m_D^2$ ^a separating the contributions of

- soft collisions $(|t| < |t|^*)$: Hard Thermal Loop approximation
- hard collisions $(|t| > |t|^*)$: kinetic pQCD calculation

^aSimilar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).



$$\kappa_{\perp/\parallel}(p): \text{ soft contribution}$$

$$\kappa_{\perp}^{\text{soft}} = \frac{1}{2} \frac{C_F g^2}{4\pi^2 v} \int_0^{|t|^*} d|t| \int_0^v dx \frac{|t|^{3/2}}{2(1-x^2)^{5/2}} \overline{\rho}(|t|,x) \left(1-\frac{x^2}{v^2}\right) \coth\left(\frac{x\sqrt{\frac{|t|}{1-x^2}}}{2T}\right)$$

$$\kappa_{\parallel}^{\text{soft}} = \frac{C_F g^2}{4\pi^2 v} \int_0^{|t|^*} d|t| \int_0^v dx \frac{|t|^{3/2}}{2(1-x^2)^{5/2}} \overline{\rho}(|t|,x) \frac{x^2}{v^2} \coth\left(\frac{x\sqrt{\frac{|t|}{1-x^2}}}{2T}\right)$$

where

$$\overline{\rho}(|t|,x) \equiv \rho_L(|t|,x) + (v^2 - x^2)\rho_T(|t|,x) \quad (|t| \equiv q^2 - \omega^2, \ x \equiv \omega/q)$$

The result is then expressed in terms of the *spectral functions of the resummed gluons* exchanged in the collisions with the plasma particles:

$$\rho_{L/T}(\omega, q) \equiv 2 \text{Im} \Delta_{L/T}(\omega + i\eta, q) \quad \text{where}$$

$$\Delta_L(z,q) = \frac{-1}{q^2 + \Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2 - q^2 - \Pi_T(z,q)}$$

NB: medium effects embedded in the HTL gluon self-energy!

The Hard Thermal Loop approximation

It is a one-loop gauge-invariant approximation allowing for the calculation of thermal corrections to vacuum propagators.

$$\Delta_L(q^0, q) = \frac{-1}{q^2 + \Pi_L(x)}, \quad \Delta_T(q^0, q) = \frac{-1}{(q^0)^2 - q^2 - \Pi_T(x)}$$

with $x \equiv q^0/q$ and

$$\Pi_L(x) = m_D^2 \left(1 - \frac{x}{2} \ln \frac{x+1}{x-1} \right),$$

$$\Pi_T(x) = \frac{m_D^2}{2} \left(x^2 + (1-x^2) \frac{x}{2} \ln \frac{x+1}{x-1} \right),$$

where $m_D \equiv gT \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$ is the Debye mass, responsible for the screening of electro-static color fields.

$\kappa_{\perp/\parallel}(p)$: numerical results

Combining together the hard and soft contributions...



$\kappa_{\perp/\parallel}(p)$: hard and soft contributions

We let the intermediate cutoff $|t|^*$ vary...



We are ready to perform numerical simulations for a realistic case!

- Initial generation of $Q\overline{Q}$ pairs (POWHEG: pQCD@NLO);
- Langevin evolution in the QGP $(u^{\mu}(x) \text{ and } T(x) \text{ given by hydro});$
- At T_c HQs hadronize (fragmentation with PDG branching ratios)
- and decay into electrons (PYTHIA decayer with PDG decay tables).

NB One has first of all to check to be able to reproduce pp results!



- Single inclusive HQ spectra $E(dN/d^3p)$ generated with POWHEG (pQCD @ NLO) using the PDF set CTEQ6M (+EPS09 in AA)
- HQ distributed in the transverse plane according to the nuclear overlap $dN/dx_{\perp} \sim T_{AB}(x, y) \equiv T_A(x+b/2, y)T_B(x-b/2, y)$, with

$$T_{A/B}(\boldsymbol{x}_{\perp}) \equiv \int_{-\infty}^{+\infty} dz \, \rho_{A/B}(\boldsymbol{x}_{\perp}, z)$$

• Each quark is given a random k_{\perp} broadening extracted from a gaussian distribution, with $\langle k_{\perp}^2 \rangle = \langle k_{\perp}^2 \rangle_{pp} + \langle \delta k_{\perp}^2 \rangle_{AB}(\vec{b}, \vec{s}).$

So far the experimental observables are single-electron spectra from the decays of charm

 $D \to X \nu e$

and bottom hadrons:

 $\begin{array}{rccc} B & \to & D\nu e \\ B & \to & D\nu e \to X\nu e\nu e \\ B & \to & DY \to X\nu eY \end{array}$

We plot the electrons falling into the PHENIX/ALICE acceptance $(|\eta| < 0.35/0.9)$

$$\eta \equiv \frac{1}{2} \ln \frac{p + p_z}{p - p_z} = -\ln \tan \frac{\theta}{2}$$

pp collisions



PHENIX results in pp collisions at $\sqrt{s} = 200$ GeV are nicely reproduced!

AA collisions: Au-Au @ RHIC and Pb-Pb @ LHC

Initialization: hydro scenario $(0.1 < \tau_0 < 1 \text{ fm})$							
	$\eta/s = 0$			$\eta/s = 0.08$			
	$ au_0 ~({ m fm})$	$s_0 \; ({\rm fm}^{-3})$	$T_0 ({\rm MeV})$	$ au_0 ~({ m fm})$	$s_0 \; ({\rm fm}^{-3})$	$T_0 ({\rm MeV})$	
				0.1	8.4	666	
RHIC	0.6	110	357	0.6	140	387	
				1	84	333	
	0.1	2438	1000	0.1	1840	854	
LHC	0.45	271	482	1	184	420	

initial $Q\overline{Q}$ production (from POWHEG)

\sqrt{s}_{NN}	$\sigma^{pp}_{c\bar{c}} \ (mb)$	$\sigma^{AA}_{car{c}}~(mb)$	$\sigma^{pp}_{bar{b}}~(mb)$	$\sigma^{AA}_{b\bar{b}}~(mb)$
200 GeV	0.254	0.236	1.77×10^{-3}	2.03×10^{-3}
$5.5 { m TeV}$	3.015	2.288	0.187	0.169

NB huge *shadowing effects* for $c\bar{c}$ production in Pb-Pb @ LHC!





• Continuous curves: AA result after Langevin (viscous hydro, $\tau_0=1$ fm).

Effects of the medium better dispayed through the nuclear modification factor $R_{AA}(p_T)$









Some general comments:

- mild dependence on the hydro scenario (ideal/viscous) and the thermalization time τ_0 (in RHIC plots $\tau_0 = 1$ fm), except for peripheral collisions;
- high- p_T reproduced better with $\mu \sim 1.5 \Rightarrow \alpha = 0.32$ at T = 300 MeV
- intermediate- p_T spectra could get increased by coalescence;
- peripheral collisions represent a puzzle (accomodated by a smaller τ_0 ?).





- plots done using the *integrated yields*;
- parameter set: $\mu = 3\pi T/2$ and viscous hydro with $\tau_0 = 1$ fm;
- similar general trend (medium softens the spectrum conserving N_e^{tot})
 - $-p_T > 0.3 \text{ GeV/c: flat } R_{AA} \sim 1 \ (R_{AA} \neq 1 \text{ at LHC due to nPDFs!})$
 - $-p_T > 4 \text{ GeV/c: suppression increases with centrality.}$

The minimum bias case: RHIC (left panel) vs LHC (right panel)

Our analysis for all centrality classes allows a more accurate estimate of the *minimum bias spectrum*. Employing in the simulations samples with the same number of events:







• Plot obtained setting $\lambda = 10$ in the transport coefficients

 $\kappa_T = \sqrt{\lambda} \pi T^3 \gamma^{1/2} \qquad \kappa_L = \sqrt{\lambda} \pi T^3 \gamma^{5/2}$

• A more systematic study could be of interest.



- v_2 with hot-QCD + fragmentation results a bit underestimated;
- Actually very good agreement with $\tau_0 = 0.1$ fm;
- v_2 could be increased by coalescence;
- Much larger flow of e_c @ LCH. Milder effect on $e_c + e_b$ due to role of b.

Conclusions and perspectives

- The relativistic Langevin equation is a powerful tool to study the HQ dynamics in the the QGP: it is an effective theory completely determined by the coefficients $\kappa_{T/L}(p)$ (no matter their microscopic origin!)
- $\kappa_{T/L}(p)$ have been evaluated considering only $2 \rightarrow 2$ collisions and distinguishing soft and hard scatterings
- For large p_T ($p_T \gtrsim 3 \text{ GeV/c}$) it is possible to accommodate RHIC data for the single-electron spectra
- Coalescence could further improve the description at lower p_T , raising R_{AA} and v_2
- Preliminary simulations for LHC were attempted. In order to provide predictions at the current $\sqrt{s_{NN}} = 2.76$ GeV we need a reliable hydrodynamical scenario....

Back-up slides

The easiest algorithm

Going to the fluid rest-frame:

$$\begin{split} \Delta \bar{\boldsymbol{p}}_{n}^{i} = -\eta_{D}(\bar{p}_{n})\bar{p}_{n}^{i}\Delta \bar{t} + \xi^{i}(\bar{t}_{n})\Delta \bar{t} \equiv -\eta_{D}(\bar{p}_{n})\bar{p}_{n}^{i}\Delta \bar{t} + g^{ij}(\bar{\boldsymbol{p}}_{n})\zeta^{i}(\bar{t}_{n})\sqrt{\Delta \bar{t}},\\ \Delta \bar{\boldsymbol{x}}_{n} = \bar{\boldsymbol{p}}_{n}/\bar{E}_{n}\Delta \bar{t} \end{split}$$

with $\Delta \bar{t} = 0.02 \text{ fm/c}$ (in the fluid rest-frame!) and

$$g^{ij}(\boldsymbol{p}) \equiv \sqrt{\kappa_{\parallel}(p)} \hat{p}^{i} \hat{p}^{j} + \sqrt{\kappa_{\perp}(p)} (\delta^{ij} - \hat{p}^{i} \hat{p}^{j}) \quad \text{and} \quad \langle \zeta_{n}^{i} \zeta_{n'}^{j} \rangle = \delta^{ij} \delta_{nn'}$$

Hence one needs simply to:

- extract three independent random numbers ζ^i from a gaussian distribution with $\sigma = 1$;
- update the momentum and position of the heavy quark;
- go back to the Lab-frame: x_{n+1} and p_{n+1} .



- HTL: Hard Thermal Loop approximation applied to any collision;
- Teaney: ~ shape given by pQCD with $\kappa(0)$ as a free parameter;
- Rapp: resonant scattering with formation of D-like mesons;
- Gossiaux: ~ pQCD with m_D used as a free parameter.



Glauber and k_{\perp} broadening

Each HQ is given a k_{\perp} -kick extracted from a gaussian distribution with

$$\langle k_{\perp}^{2} \rangle_{AB}(\vec{b}, \vec{s}) = \langle k_{\perp}^{2} \rangle_{pp} + \frac{a_{gN}}{2} \left[\frac{\int dz_{A} \, \rho_{A}(\vec{s}, z_{A}) \boldsymbol{l}_{A}(\vec{s}, z_{A})}{T_{A}(\vec{s})} + \frac{\int dz_{B} \, \rho_{B}(\vec{s} - \vec{b}, z_{B}) \boldsymbol{l}_{B}(\vec{s} - \vec{b}, z_{B})}{T_{B}(\vec{s} - \vec{b})} \right]$$

due to the length crossed by the incoming partons in nucleus A/B before the hard event:

$$l_A(\vec{s}, z_A) \equiv \int_{-\infty}^{z_A} dz \, \rho_A(\vec{s}, z) / \rho_0 \quad \text{and} \quad l_B(\vec{s} - \vec{b}, z_b) \equiv \int_{z_B}^{+\infty} dz \, \rho_B(\vec{s} - \vec{b}, z) / \rho_0$$

We choose

$a_{gN} \; ({\rm GeV}^2/{\rm fm})$	SPS	RHIC	LHC
С	0.072	0.092	0.153
b	0.197	0.252	0.420

Single-electron spectra: procedure

- As an outcome of the initial generation (+ Langevin evolution in AA) one has two samples $(90 \cdot 10^6)$ of c and b quarks;
- They are made fragment with Peterson FFs with the branching fractions given by the PDG;
- Each hadron is sent to the PYTHIA decayer (with updated PDG decay tables) till producing *at least* one electron in the final state (in case of no final electron the event is re-sampled);
- The two sources must be combined with their appropriate weight:

$$\left. \frac{dN}{dp_T} \right|_{e_c + e_b} \sim \sigma_{c\bar{c}} \sum_{h_c} \left. \frac{N_{h_c}^{\text{init}}}{N_{h_c}^{\text{sampl}}} \frac{dN}{dp_T} \right|_{e_c} + \sigma_{b\bar{b}} \sum_{h_b} \left. \frac{N_{h_b}^{\text{init}}}{N_{h_b}^{\text{sampl}}} \frac{dN}{dp_T} \right|_{e_b}$$

NB We plot the electrons falling into the PHENIX acceptance $(|\eta| < 0.35)$, where $\eta \equiv \frac{1}{2} \ln \frac{p+p_z}{p-p_z} = -\ln \tan \frac{\theta}{2}$

Centrality classes

$$f_{C_1-C_2} = \frac{\int_{b_1}^{b_2} db \, b[1 - \exp(\sigma_{\rm NN} T_{AB}(b)]]}{\int_0^\infty db \, b[1 - \exp(\sigma_{\rm NN} T_{AB}(b)]]},$$

where $T_{AB}(b) = \int ds T_A(s + b/2) T_B(s - b/2)$.

Au-Au ($\sqrt{s} = 200 \text{ GeV}$)			Pb-Pb ($\sqrt{s} = 5.5$ TeV)		
C_1 - C_2	$b~({\rm fm})$	$N_{\rm coll}$	C_1 - C_2	$b~({\rm fm})$	$N_{\rm coll}$
0-10%	3.27	963	0-10%	3.45	1698
10-20%	5.78	592	10-20%	6.11	1022
20-40%	8.12	282	20-40%	8.58	469
40-60%	10.51	81	40-60%	11.11	123
60-92%	12.80	10	60-90%	13.45	14
0-92%	8.44	247	0-90%	8.77	435





Fragmentation performed with Peterson FF tends to slightly suppress R_{AA}

- Mild dependence on the parameter ϵ
- $\epsilon = 0.04$ and 0.005 (for c and b) fixed in order to reproduce HQET FFs^a

Fragmentation fractions taken from DESY results and PDG_2009

^aE. Braaten, K. Cheung and T.C. Yuan, Phys. Rev. D 48, 5049 (1993)