The QCD phase diagram (in chiral fluid dynamics)

Marlene Nahrgang

with Marcus Bleicher and Stefan Leupold (Uppsala)

Excited QCD 2011, Les Houches

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

MainCampus
Stipendiatenwerk der Stiftung Polytechnische Gesellschaft Frankfurt am Main

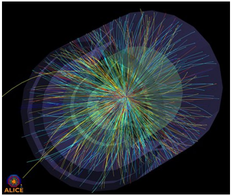
How to study the QCD phase diagram...

... be brave and solve

$$Z(T, \mu_B) = \int \mathcal{D}(A, q, q^\dagger) \mathrm{e}^{-S^E_{\rm QCD}}$$

ab initio and nonperturbatively,

... be strong and collide heavy ions at ultra-relativistic energies,



... be creative and study effective models of QCD.

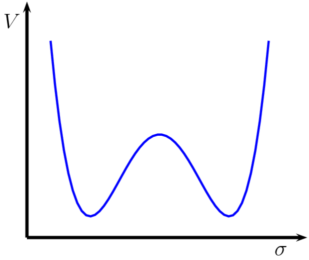
$$\mathcal{L}_{\rm eff}$$

Phase transitions - thermodynamically

first order phase transition

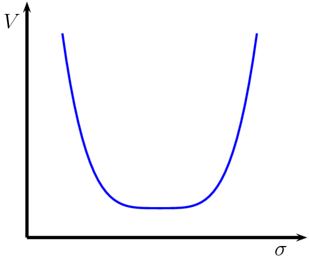
- two degenerate minima separated by a barrier
- nucleation
- spinodal decomposition

(I.N.Mishustin, Phys. Rev. Lett. 82 (4779) 1999; Ph.Chomaz, M.Colonna, J.Randrup, Physics Reports 389 (2004) 263)



critical point

- $m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \rightarrow 0$
- correlation length diverges $\xi = \frac{1}{m_\sigma} \rightarrow \infty$
- universality classes (for QCD: 3d Ising model) $\Rightarrow \langle \sigma^2 \rangle \propto \xi^2$
- critical opalescence

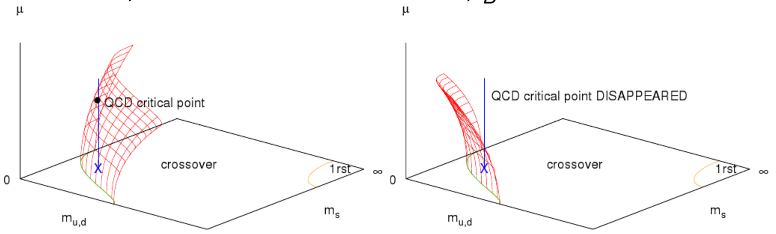


Being brave

The critical point in lattice QCD

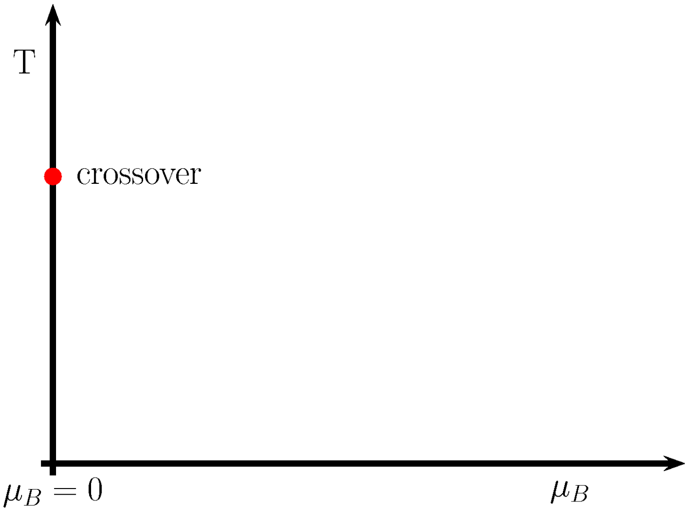
strictly valid only for $\mu_B = 0$
methods to explore the $T - \mu_B$–plane

- reweighting (Fodor, Katz): $\mu_B^c = 360 \pm 40$ MeV
- radius of convergence of the Taylor expansion of the pressure (Gavai, Gupta, RBC-Bielefeld): 250 MeV $< \mu_B^c <$ 400 MeV
- imaginary μ_B (de Forcrand, Philipsen): $\mu_B^c >$ 500 MeV



(de Forcrand, Philipsen, hep-lat/0607017)

Being brave

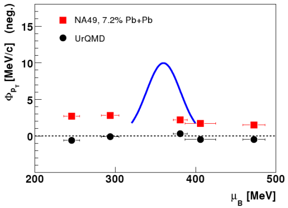


Being strong

The critical point in heavy ion collisions

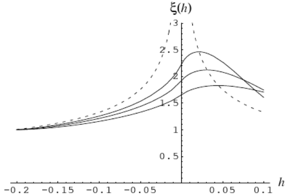
coupling to the order parameter of chiral symmetry
$\Rightarrow$ non-monotonic fluctuations in pion and proton multiplicities

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$



(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRD **60** (1999), NA49 collaboration J. Phys. G **35** (2008))

BUT: critical slowing down



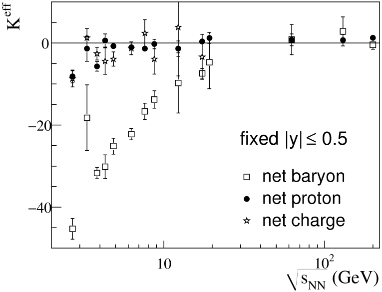
(B. Berdnikov and K. Rajagopal, PRD **61** (2000))

Being strong

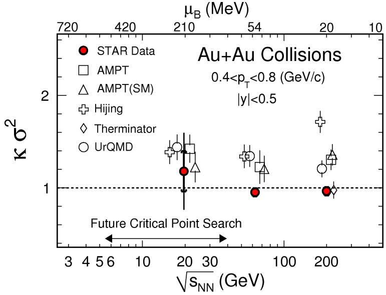
Higher moments and the kurtosis

the kurtosis: $K^{\mathrm{eff}} = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \propto \xi^7$

(M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009))

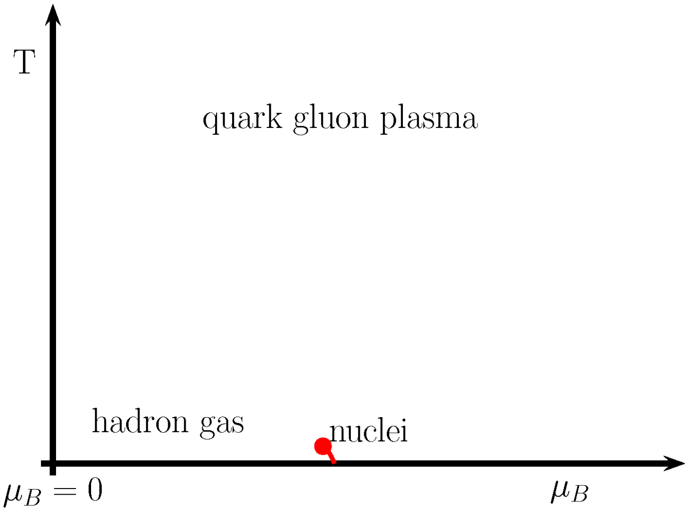


(MN, T.Schuster, M.Mitrovski, R.Stock and M.Bleicher, arXiv:0903.2911v2 [hep-ph], submitted to PLB)



(STAR collaboration, Phys. Rev. Lett. **105**, 022302 (2010))

Being strong



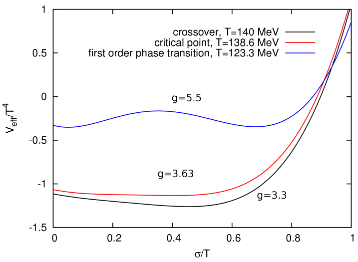
Being creative

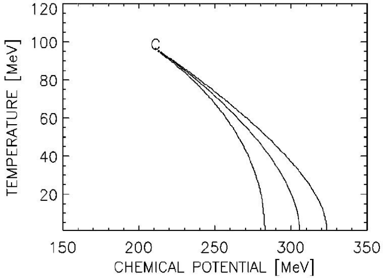
The linear sigma model with constituent quarks

$$\mathcal{L} = \overline{q}\left[i\gamma^\mu\partial_\mu - g\left(\sigma + i\gamma_5\tau\vec{\pi}\right)\right]q + 1/2\left(\partial_\mu\sigma\right)^2 + 1/2\left(\partial_\mu\vec{\pi}\right)^2 - U\left(\sigma,\vec{\pi}\right)$$

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - h_q\sigma - U_0$$

(M.Gell-Mann, M.Levy, Nuovo Cim. 16, 705,1960)

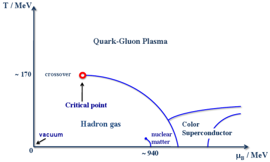


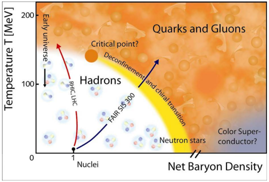


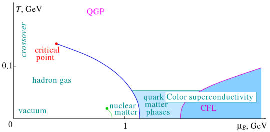
the effective potential at $\mu_B = 0$

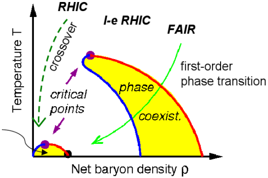
(O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, Phys. Rev. C64, 045202,2001)

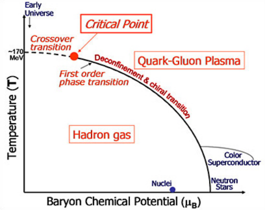
Being creative

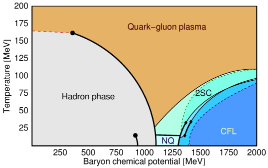


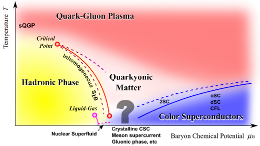


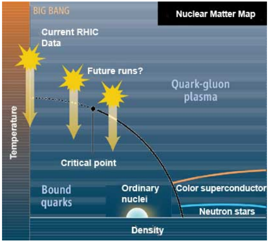


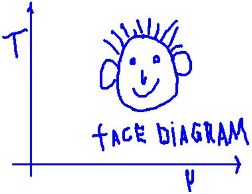




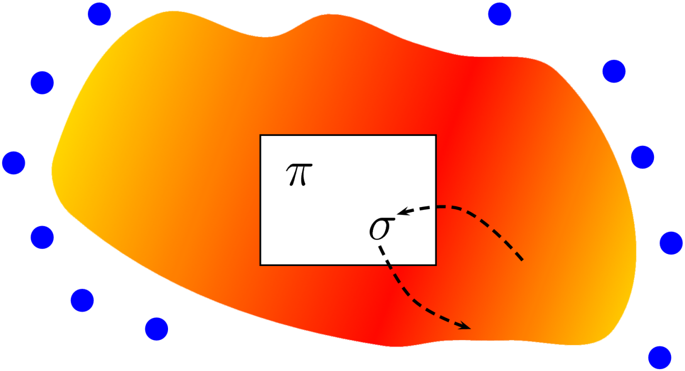








Chiral fluid dynamics



fluid dynamics + phase transition model + dissipation and noise

(I. N. Mishustin and O. Scavenius, PRL **83** (1999); K. Paech, H. Stoecker and A. Dumitru, PRC **68** (2003); MN, M. Bleicher, arXiv:1011.5379 [nucl-th])

The two-particle irreducible effective action

- resummation of subdiagrams $\rightarrow$ full propagators
- restrict to the σ mean field and the quark propagators S^{ab}

$$\Gamma[\sigma, S] = S[\sigma] - i\mathrm{Tr}\ln S^{-1} - i\mathrm{Tr}S_0^{-1}S + \Gamma_2[\sigma, S]\,,$$

equation of motion for the σ mean field and the quark propagators S^{ab}

$$\frac{\delta\Gamma[\sigma, S]}{\delta\sigma^a} = 0 \quad \text{and} \quad \frac{\delta\Gamma[\sigma, S]}{\delta S^{ab}} = 0$$

and the proper self-energy

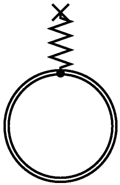
$$-i\Sigma^{ab}(x, y) = -\frac{\delta\Gamma_2[\sigma, S]}{\delta S^{ab}(x, y)}\,.$$

Dyson-Schwinger equation for S^{ab}

$$(i\partial\!\!\!/ - m_f)S^{ab}(x, y) - i\int_{\mathcal{C}} \mathrm{d}^4z\,\Sigma^{ac}(x, z)S^{cb}(z, y) = i\delta_{\mathcal{C}}^{ab}(x - y)$$

The two-particle irreducible effective action

$$\Gamma_2[\sigma, S] = g \int_{\mathcal{C}} \mathrm{d}^4 x \,\mathrm{tr}\left(S^{++}(x,x)\sigma^+(x) + S^{--}(x,x)\sigma^-(x)\right)$$

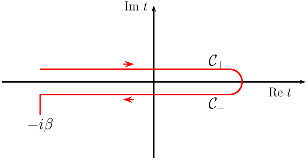


equation of motion for the σ mean field

$$-\frac{\delta S_{\mathrm{cl}}[\sigma]}{\delta \sigma^a} = \frac{\delta \Gamma_2[\sigma, S]}{\delta \sigma^a} = g\,\mathrm{tr} S^{aa}(x,x)$$

the effective action along the contour

$$\Gamma[\sigma, S] = g\,\mathrm{tr} S_{\mathrm{th}}^{++}(x,x)\Delta\sigma(x) - \frac{T}{V} \ln \mathcal{Z}_{\mathrm{th}}$$
$$+ \int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x)$$
$$+ \frac{i}{2} \int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x)\mathcal{I}[\bar{\sigma}](x,y)\Delta\sigma(y)$$



with $\Delta\sigma = \sigma^+ - \sigma^-$ and $\bar{\sigma} = 1/2(\sigma^+ + \sigma^-)$ on the contour.

MN, S.Leupold, M.Bleicher, in preparation

Classical equations of motion for the chiral field

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} - g \mathrm{tr} S_{\mathrm{th}}^{++}(x,x) + \eta \partial_t \sigma = \xi$$

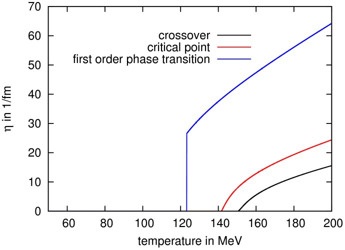
damping term η and noise ξ for $\mathbf{k} = 0$

$$\eta = g^2 \frac{d_q}{\pi} (1 - 2 n_{\mathrm{F}}(\frac{m_\sigma}{2})) \frac{(\frac{m_\sigma^2}{4} - m_q^2)^{\frac{3}{2}}}{m_\sigma^2}$$

$$\langle \xi(t) \xi(t') \rangle_\xi = \frac{1}{V} \delta(t - t') m_\sigma \eta \coth(\frac{m_\sigma}{2T})$$

large compared to $\eta = 2.2/\mathrm{fm}$

(T. S. Biro and C. Greiner, PRL **79** (1997))



simplified

$$\eta = \begin{cases} 20/\mathrm{fm} & \text{for} \quad m_\sigma > 2 m_q \\ 3/\mathrm{fm} & \text{for} \quad m_\sigma < 2 m_q \end{cases}$$

Fluid dynamics - the equation of state

pressure from the equilibrium $\Gamma_{\text{eq}}(\sigma, T)$ with $\Delta\sigma = 0$

$$p(\sigma, T) = -\Gamma_{\text{eq}}(\sigma, T)$$

energy density from thermodynamic consistency (guaranteed by the 2PIEA)

$$e(\sigma, T) = T\frac{\partial p(\sigma, T)}{\partial T} - p(\sigma, T)$$

Energy-momentum conservation

Energy-momentum tensor of the entire system is conserved:

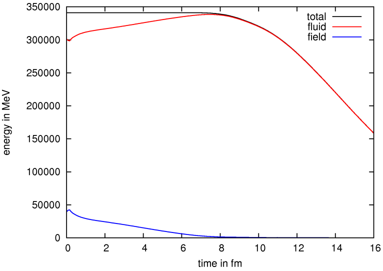
$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = g\mathrm{tr}S^{++}(x,x)$$
$$\partial_\mu T_{\sigma}^{\mu\nu} = -g\mathrm{tr}S^{++}(x,x)$$

then $\partial_\mu(T_{\mathrm{q}}^{\mu\nu} + T_{\sigma}^{\mu\nu}) = 0$ for the full propagator!
HERE, approximation of an ideal fluid

$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = g\mathrm{tr}S_{\mathrm{th}}^{++}(x,x) = 2d_q \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{E_p} f_{\mathrm{FD}}(E_p)$$

and a sourceterm

$$S^\nu = -\partial_\mu T_\sigma^{\mu\nu}$$



MN, S.Leupold, M.Bleicher, in preparation

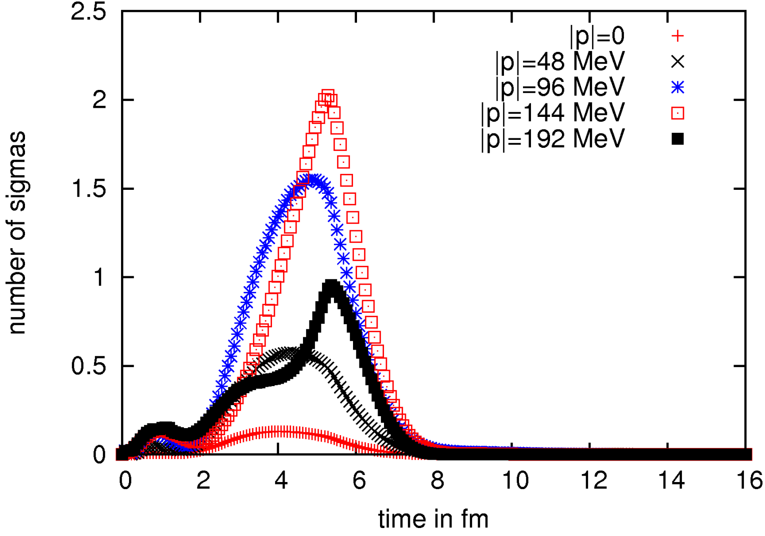
Intensity of sigma fluctuations

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3 k} = \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \frac{1}{(2\pi)^3 2\omega_k}(\omega_k^2 |\sigma_k|^2 + |\partial_t \sigma_k|^2)$$

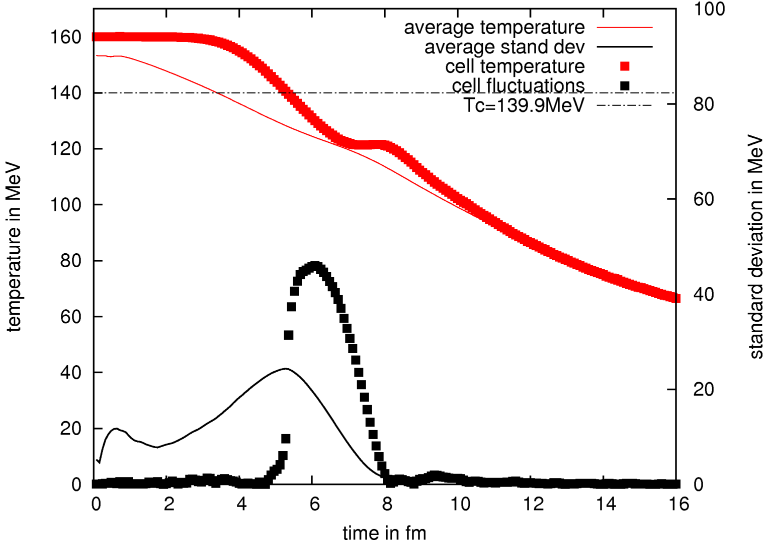
$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

$$m_\sigma = \sqrt{\frac{\partial^2 V_{\mathrm{eff}}}{\partial \sigma^2}|_{\sigma=\sigma_{\mathrm{eq}}}}$$

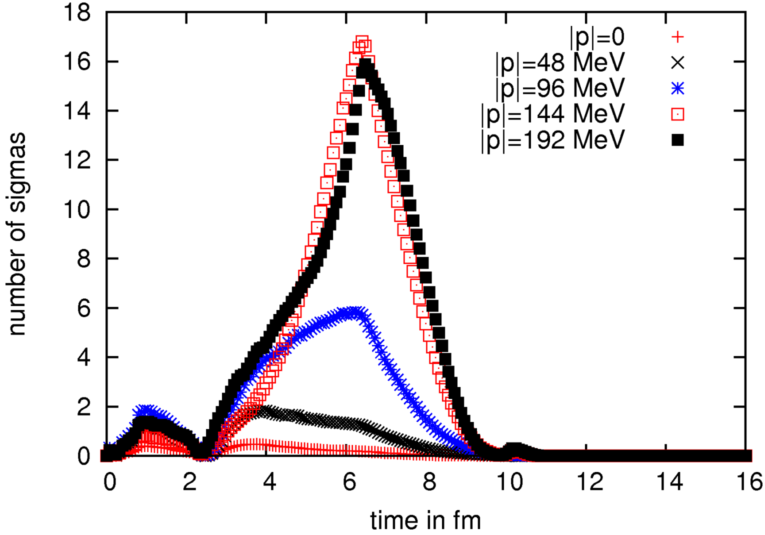
Intensity of sigma fluctuations - critical point



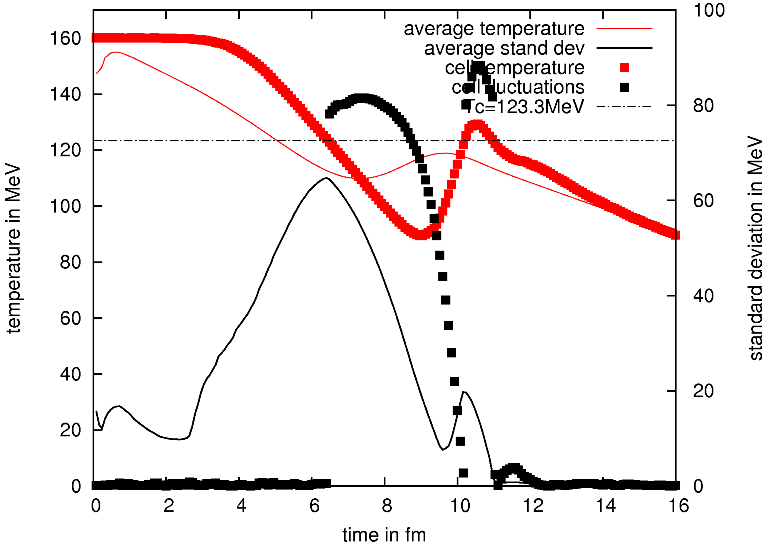
Fluctuations - critical point



Intensity of sigma fluctuations - first order PT



Fluctuations - first order PT



Summary & outlook