Viscous hydrodynamics for relativistic heavy ion collisions

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Excited QCD 2011
22 February, 2011
1 INTRODUCTION
- Relativistic viscous hydrodynamics
- Application to heavy ion collisions

2 RESULTS
- Past results
- Current status/future prospects
OUTLINE

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   - Past results
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Ideal (Relativistic) Hydrodynamic Equations

- Ideal hydro: isotropic energy-momentum tensor

\[ T^0_i \text{rest} \equiv 0 \Rightarrow T^\mu_\nu = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \]

\[ T^{\mu\nu} = T^0_0 = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu} \]

- Conservation equations:

\[ \partial_\mu T^{\mu\nu} = 0 \]

- Equation of State

\[ p = p(\epsilon) \]

- Viscosity: gradient expansion

\[ T^{\mu\nu} = T^0_0 + \eta \nabla \langle \mu u^\nu \rangle + \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha + \ldots \]
**Heavy Ion Collision Timeline**

- **Pre-equilibrium**
- **Phase transition**
- **Thermalization (isotropization)**
- **Freeze-out**
- **Dilute gas/free streaming**
- **Hydrodynamics**

*Timeline diagram showing the sequence of events in heavy ion collisions.*
Complete hydro model

A complete model includes:

- Initial conditions:
  Minimal standard — boost-invariant I.C.s, transverse $\epsilon$ profile from simple model, no initial flow, free parameters $\tau_0$, $T_f$

- Hydro parameters:
  Minimal standard — constant $\eta/s$, EoS with crossover, no bulk viscosity

- Freeze out:
  Minimal standard — Cooper-Frye freeze out with free parameter $T_f$
EXPERIMENTAL RESULT:
Collective behavior ("elliptic flow")
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$$dN/dY d^2 p \propto \left(1 + \frac{2}{v^2} \cos 2(\phi - \psi_{RP}) + \frac{2}{v^4} \cos 4(\phi - \psi_{RP}) + \ldots\right)$$

Elliptic flow:

$$v_2 \equiv \langle \cos 2(\phi - \psi_{RP}) \rangle$$
Collective behavior ("elliptic flow")

The distribution of emitted particles:

\[
\frac{dN}{dY \, d^2p_T} \propto 1 + 2v_2 \cos 2(\phi - \psi_{RP}) + 2v_4 \cos 4(\phi - \psi_{RP}) + \ldots
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**COLLECTIVE BEHAVIOR ("ELLiptic FLOW")**

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Elliptic flow: \( v_2 \equiv \langle \cos 2(\phi - \psi_{RP}) \rangle \)
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RESULTS

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Hydro works!

- Ideal hydrodynamic models fit RHIC data surprisingly well (Kolb et al., Teaney et al., Huovinen et al., etc.):

  \[
  \frac{\eta}{s} \geq \frac{1}{4\pi} \simeq 0.08
  \]

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

- Adding shear viscosity to the models shows that the collision medium is close to conjectured lower bound \( \frac{\eta}{s} \geq \frac{1}{4\pi} \simeq 0.08 \):
Even with significant uncertainties in the models, they provide strong evidence of a low viscosity/strongly interacting fluid
$v_2$ from ALICE is just as expected from viscous hydro:

\[(arXiv:1011.5173)\]
What are Hydro people working on now?

Progress is being made on several fronts:

- Improving freeze out prescription: A number of people are now using hybrid hydro/transport models (Hirano et al., Petersen et al., Heinz et al., etc.).
- Improving hydro stage: Investigations are ongoing into the effect of bulk viscosity and temperature-dependent shear viscosity (Song et al., Mota et al., etc.)
- Improving initial conditions: c.f., J. Albacete’s talk on CGC
- Investigating rapidity dependence (Schenke et al., Werner et al., Hama et al., etc.)
- Understanding flow fluctuations (Petersen et al., Schenke et al., Mota et al., Holopainen et al., Werner et al., etc.)
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Flow fluctuations

\[
\frac{dN}{dY \, d^2 p_t} \propto 1 + 2 v_2 \cos 2(\phi - \psi_{RP}) + 2 v_4 \cos 4(\phi - \psi_{RP}) + \ldots
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\[\psi_2 = \psi_{EP}\]
FLOW FLUCTUATIONS

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\]

\[\Rightarrow \langle e^{in(\phi_1 - \phi_2)} \rangle = \langle e^{in\phi_1} \rangle \langle e^{-in\phi_2} \rangle = v_n^{(1)} v_n^{(2)}\]
These flow fluctuations provide a natural explanation for “ridge” and “shoulder” phenomena in heavy ion collisions.

(arXiv:1004.0805)

But they also imply new flow measurements that will constrain, e.g., the initial dynamics:

(arXiv:1008.0139)
TRIANGULAR FLOW, DIRECTED FLOW, etc.

From measurements of: 
\( v_1, v_2\{2\}, v_2\{4\}, v_3\{2\}, v_3\{4\}, v_4\{\psi_2\}, v_4\{\psi_4\}, \ldots \)

we will be able to significantly constrain both hydro parameters, and initial dynamics.
SUMMARY

- Viscous hydrodynamic models have been very successful at describing heavy ion collisions at RHIC, and now LHC.
- In the future, look for more precision extraction of, e.g., $\eta/s$, as well as constraints on geometry and fluctuations of the early-time state.
- LHC will probe higher temperatures, but also provides better detection capabilities to measure all these flow observables (higher multiplicity, larger detector coverage, etc.).