

VISCOUS HYDRODYNAMICS FOR RELATIVISTIC HEAVY ION COLLISIONS

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Excited QCD 2011
22 February, 2011

1 INTRODUCTION

- Relativistic viscous hydrodynamics
- Application to heavy ion collisions

2 RESULTS

- Past results
- Current status/future prospects

OUTLINE

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IDEAL (RELATIVISTIC) HYDRODYNAMIC EQUATIONS

- Ideal hydro: isotropic energy-momentum tensor

$$T_{rest}^{0i} \equiv 0 \Rightarrow T_{0rest}^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$T^{\mu\nu} = T_0^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

- Conservation equations:

$$\partial_\mu T^{\mu\nu} = 0$$

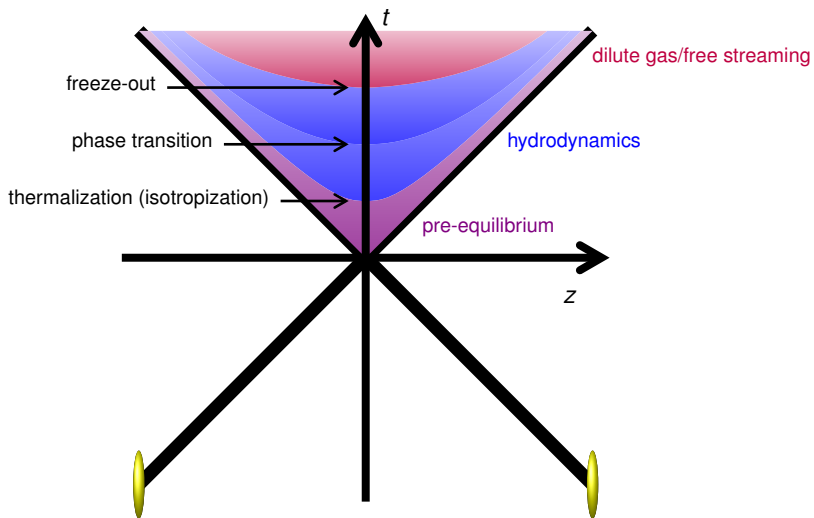
- Equation of State

$$p = p(\epsilon)$$

- Viscosity: gradient expansion

$$T^{\mu\nu} = T_0^{\mu\nu} + \eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha + \dots$$

HEAVY ION COLLISION TIMELINE

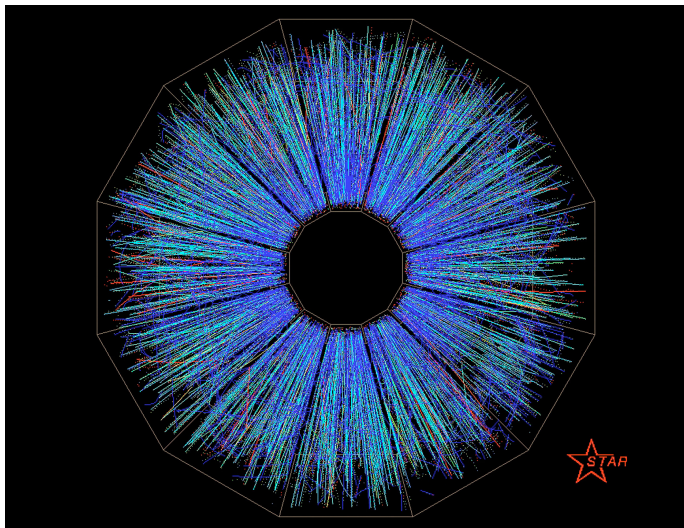


COMPLETE HYDRO MODEL

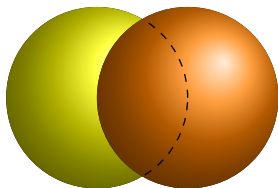
A complete model includes:

- Initial conditions:
Minimal standard — boost-invariant I.C.s, transverse ϵ profile from simple model, no initial flow, free parameters τ_0 , T_f
- Hydro parameters:
Minimal standard — constant η/s , EoS with crossover, no bulk viscosity
- Freeze out:
Minimal standard — Cooper-Frye freeze out with free parameter T_f

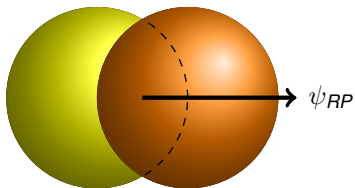
EXPERIMENTAL RESULT:



COLLECTIVE BEHAVIOR (“ELLIPTIC FLOW”)



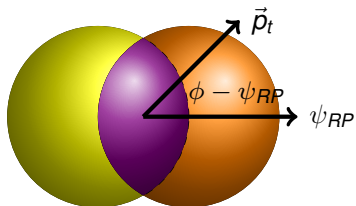
COLLECTIVE BEHAVIOR (“ELLIPTIC FLOW”)



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The distribution of emitted particles :

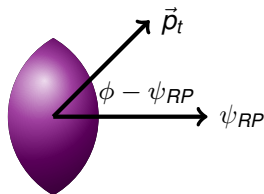
$$\frac{dN}{dY d^2p_T} \propto 1 + 2v_2 \cos 2(\phi - \psi_{RP}) + 2v_4 \cos 4(\phi - \psi_{RP}) + \dots$$



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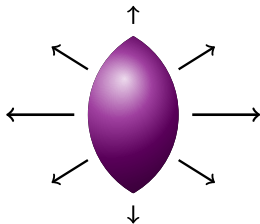


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Elliptic flow: $v_2 \equiv \langle \cos 2(\phi - \psi_{RP}) \rangle$



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HYDRO WORKS!

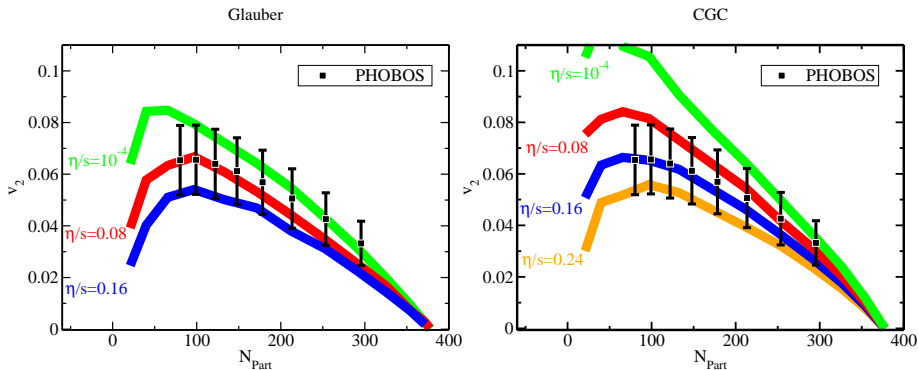
- Ideal hydrodynamic models fit RHIC data surprisingly well (Kolb *et al.*, Teaney *et al.*, Huovinen *et al.*, etc.):

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

- Adding shear viscosity to the models shows that the collision medium is close to conjectured lower bound $\frac{\eta}{s} \geq \frac{1}{4\pi} \simeq 0.08$:

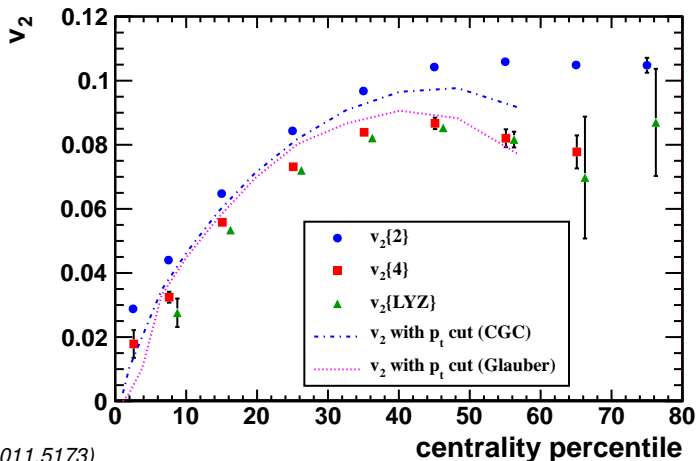
RHIC RESULTS: MOMENTUM INTEGRATED v_2 

(arXiv:0804.4015)

Even with significant uncertainties in the models, they provide strong evidence of a low viscosity/strongly interacting fluid

FLOW IN LHC Pb+Pb COLLISIONS

v_2 from ALICE is just as expected from viscous hydro:



(arXiv:1011.5173)

WHAT ARE HYDRO PEOPLE WORKING ON NOW?

Progress is being made on several fronts:

- Improving freeze out prescription: A number of people are now using hybrid hydro/transport models (Hirano *et al.*, Petersen *et al.*, Heinz *et al.*, etc.).
- Improving hydro stage: Investigations are ongoing into the effect of bulk viscosity and temperature-dependent shear viscosity (Song *et al.*, Mota *et al.*, etc.)
- Improving initial conditions: c.f., J. Albacete's talk on CGC
- Investigating rapidity dependence (Schenke *et al.*, Werner *et al.*, Hama *et al.*, etc.)
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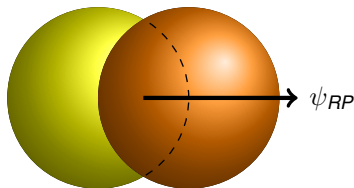
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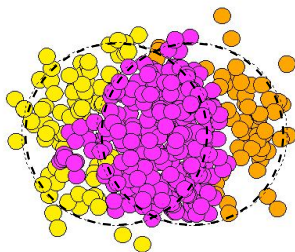
FLOW FLUCTUATIONS

$$\frac{dN}{dY d^2p_t} \propto 1 + 2v_2 \cos 2(\phi - \psi_{RP}) + 2v_4 \cos 4(\phi - \psi_{RP}) + \dots$$



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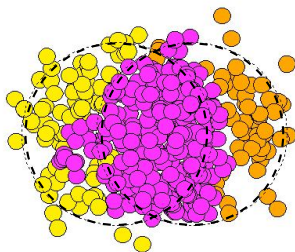
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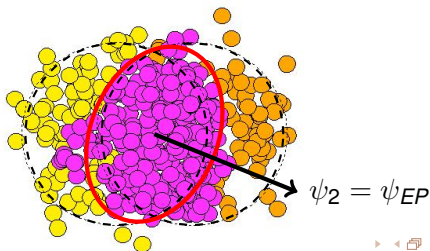
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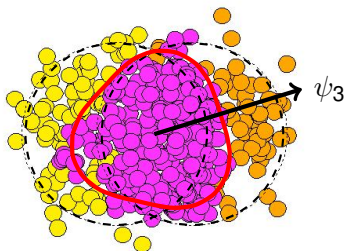
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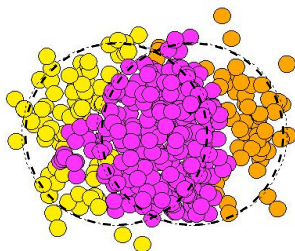


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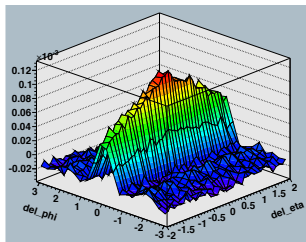
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$$\implies \langle e^{in(\phi_1 - \phi_2)} \rangle = \langle e^{in\phi_1} \rangle \langle e^{-in\phi_2} \rangle = v_n^{(1)} v_n^{(2)}$$

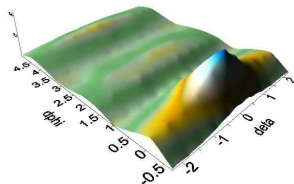


TWO-PARTICLE CORRELATION MEASUREMENTS

These flow fluctuations provide a natural explanation for “ridge” and “shoulder” phenomena in heavy ion collisions.



(arXiv:1004.0805)



(arXiv:1008.0139)

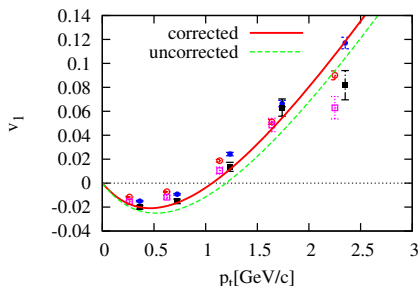
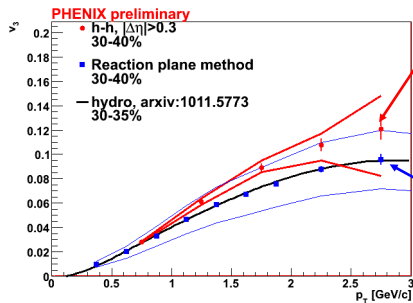
But they also imply new flow measurements that will constrain, e.g., the initial dynamics:

TRIANGULAR FLOW, DIRECTED FLOW, ETC.

From measurements of:

v_1 , $v_2\{2\}$, $v_2\{4\}$, $v_3\{2\}$, $v_3\{4\}$, $v_4\{\psi_2\}$, $v_4\{\psi_4\}$, ...

we will be able to significantly constrain both hydro parameters, and initial dynamics.



SUMMARY

- Viscous hydrodynamic models have been very successful at describing heavy ion collisions at RHIC, and now LHC
- In the future, look for more precision extraction of, e.g., η/s , as well as constraints on geometry and fluctuations of the early-time state.
- LHC will probe higher temperatures, but also provides better detection capabilities to measure all these flow observables (higher multiplicity, larger detector coverage, etc.)