Large Nc QCD & the Hagedorn Spectrum

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TDC &Vojtech Krejcirik in preparation

- Perhaps the most basic questions in "excited qcd" is the density of hadronic states as one goes up in the spectrum: how many hadrons are there in a mass range from m to $m+\Delta m$?

- A useful way to parameterize this information is to look at its integral, $N(m)$, the number of hadrons with a mass less than m.

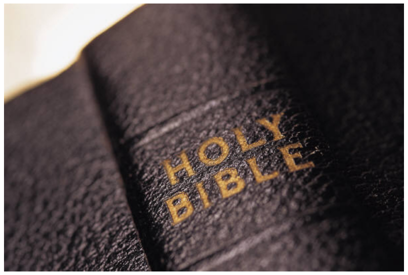
The Hagedorn Spectrum

- The Hagedorn spectrum refers to a hadronic spectrum in which the number of hadrons with mass less then *m* grows exponentially with *m*:

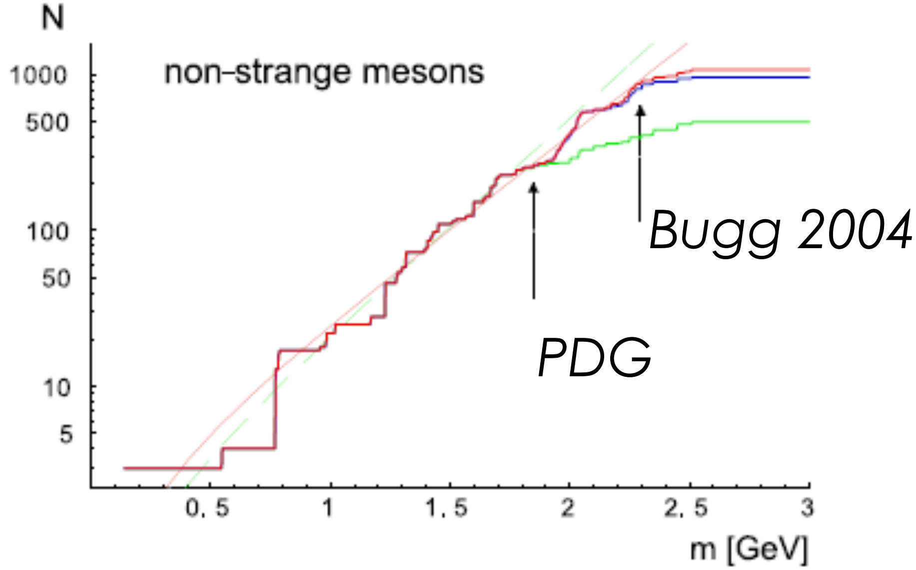
  $N(m) \sim m^{-2b} \exp(m/T_H)$, where T_H, the Hagedorn temperature is a mass parameter.
- Hagedorn proposed this spectrum in the mid-sixties based on data on high energy collisions and a "fireball" picture combined with statistical analysis.
- This idea has had a wild intellectual ride over the decades. It has played a central role in fields from string theory to the equation of state of QCD.

There is some phenomenological support for of this

Get data from the Particle Data Book



Integrated # of mesons with mass less then m



From Broniowski, Florkowski and Glozman 2004

Some data should be viewed as being from the apocrypha

This apparent exponential growth in the empirical hadron spectrum may be misleading for reasons which may become clear later in the talk

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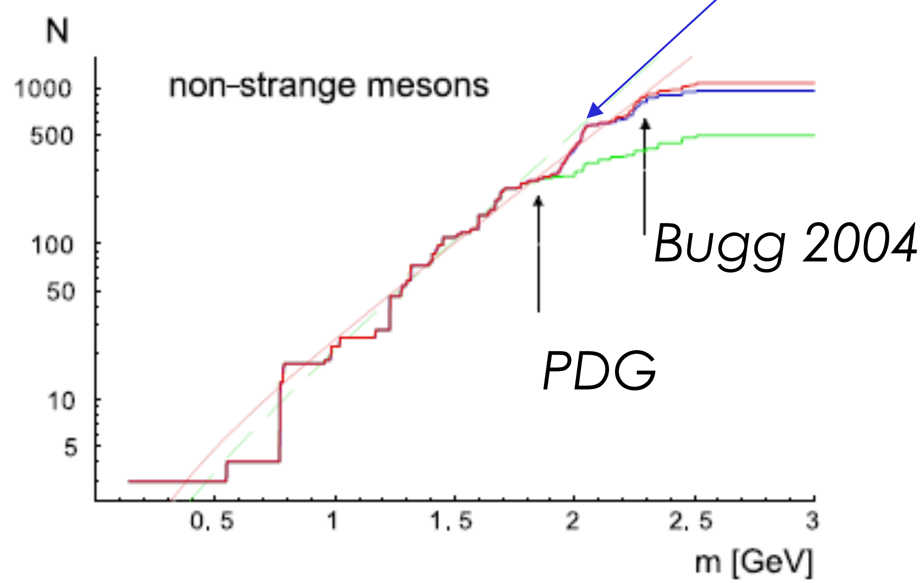

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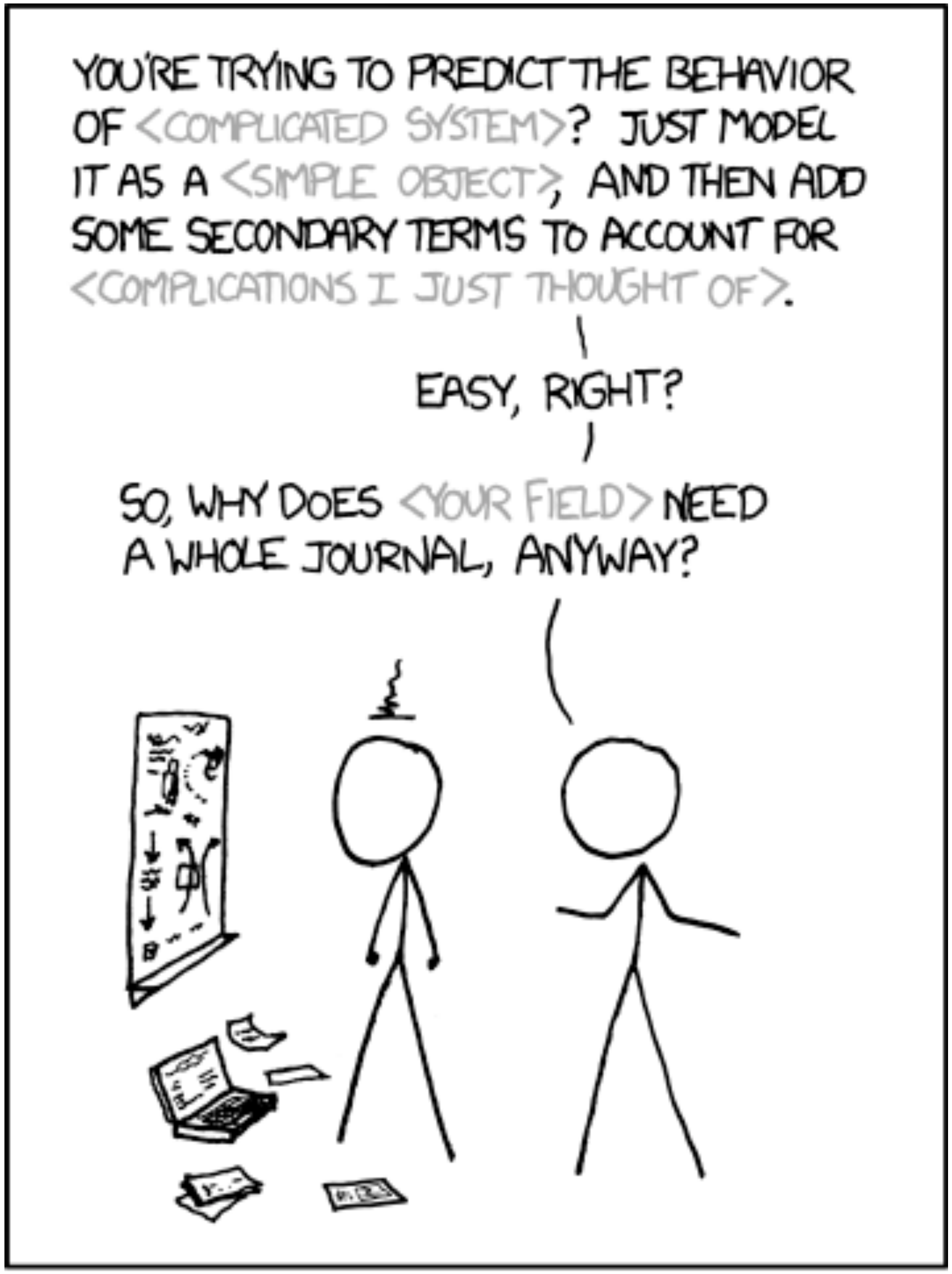
"The Holy Roman Empire is neither holy, nor Roman nor an Empire"----Voltaire

"The Particle Data Book is neither mostly about particles, nor mostly containing data nor mostly used in book form"---TDC

It is a web site, devoted almost entirely to the properties of hadronic resonances extracted in model-dependent ways from the data.

The fact that excited hadrons are resonances means that their masses are not strictly well-defined and thus neither is ***N(m)***. Thus, the question of whether QCD has a Hagedorn spectrum is not really well-posed. How can we proceed?



LIBERAL-ARTS MAJORS MAY BE ANNOYING SOMETIMES, BUT THERE'S *NOTHING* MORE OBNOXIOUS THAN A PHYSICIST FIRST ENCOUNTERING A NEW SUBJECT.

This effect is worse when the physicist in question is a theorist!

In this case replace the messy world of QCD, with the more orderly world of QCD in the limit of a large number of colors and assume that the large Nc world is similar to ours with relatively small 1/Nc corrections.

Meson and glueball masses become well defined as Nc$\rightarrow\infty$ (with widths scaling as 1/Nc and 1/Nc^2 respectively) and whether there is a Hagedorn spectrum in a large Nc world is a well-posed question.

Going to the large Nc limit also GREATLY simplifies many other aspects of the analysis!

The question of whether Large Nc QCD has a Hadgedorn Spectrum is related to a fundamental question

Does Large Nc QCD act like string theory (at least for sufficiently high states?

Schrödinger's cat?

Note that simple sting theories (with unbreakable strings like flux tubes at large Nc) automatically have Hagedorn spectra.

Does large Nc QCD?

Theoretical Argument: QCD **must*** have a Hagedorn Spectrum at Large Nc

- Theoretical inputs:
  - Confinement
    - Only uses confinement in the sense of only color neutral states in the physical space
    - Does not explicitly require unbroken Zn symmetry; linear rising potential
  - Asymptotic freedom.
  - Standard assumption about the onset of perturbation theory: perturbation theory is valid when perturbation theory tells you perturbative corrections are small
  - Plausible technical assumptions about the ordering of limits

*Not a rigorous theorem as it depends on the assumptions above

- The keys to approach:
  - The number of local single color trace operators of fixed quantum number grows exponentially with the mass dimension of the operator. While each operator does ***not*** create a distinct hadron, this exponential growth in number of operator is ultimately translated into an exponential growth with mass in the number of hadrons.
  - The regime of validity of perturbation theory for point-to-point correlators grows is independent of the dimension of the operator *at large Nc.* (Note this false for finite Nc).

- The critical thing is that the derivation does **NOT**
  - Require any explicit assumption that the dynamics of QCD is “string-like”---thus it acts as an independent check on the assumption that QCD becomes stringy
  - Exploit in any way the notion of confinement as an unbroken center symmetry
    - It does exploit the notion of confinement as the requirement that all physical states be color singlet.

What is confinement?

There are two distinct notions of confinement

The real thing

All physical asymptotic states ar color singlets

A cartoon

An unbroken Zn symmetry or area law Wilson loop.

Of interest only to theorists looking at simplified limits!

A quick sketch of the argument:

- *Focus on* *N(M)* *and* $W(M) \equiv \sum_{n=1}^{N(M)} m_n = M\,N(M) - \int_0^M d\mu\, N(\mu)$

  It is easy to that if W(M) grows exponentially with M, then N(M) does as well.

- The key ingredient: a matrix of correlators *in (Euclidean) configuration space* for a set of *single-color-trace* currents which (for simplicity) will be taken to scalars & pseudoscalars

$$\overleftrightarrow{\Pi}_{ab}(r) = \left\langle J_a^+(\vec{r}) J_b(0) \right\rangle$$

  - There is such a matrix associated with any set of currents: $S=\{ J_1, J_2, J_3, J_4, \ldots, J_n\}$

$$\overleftrightarrow{\Pi}_{ab}(\tau) = \int ds\, \rho_{ab}(s)\, \Delta(\tau,\sqrt{s})$$

$$= \sum_n c^*_{a,n}\, c_{b,n} \Delta(\tau, m_n)$$

$$\rho_{ab}(s) = \sum_n c^*_{a,n}\, c_{b,n} \delta(s - m_n^2)$$

Scalar propagator for a particle of mass m_n

Amplitude for J_b to create n^{th} hadron when acting on vaccum

Fact that currents only make single meson states follows from large Nc planarity + confinement (in the sense of only singlet states) for single color trace operators

A standard result commonly exploited in lattice QCD:

$$-\frac{d\log(\langle J(\tau)J(0)\rangle)}{d\tau} \to m_0 \text{ as } \tau \to \infty$$

$$-\frac{d\log(\langle J(\tau)J(0)\rangle)}{d\tau} > m_0 \text{ for all } \tau$$

The generalization of this to a set of currents at large Nc yields a useful lemma:

$$-\frac{d\,\mathrm{Tr}\left(\log\left(\overleftrightarrow{\Pi}(\tau)\right)\right)}{d\tau} \geq \sum_{n}^{\dim(\Pi)} m_n \equiv W\left(M_{\dim(\Pi)}\right)$$

Follows from the preceding form of the correlation matrix + basic properties of the scalar propagator, Δ, and the Log structure.

Form should be familiar to anybody trying to extract excited hadron masses from the lattice.

A useful set of currents for QCD with fundamental quarks:

Single color trace

$$S_n = \{J_1, J_2, J_3, \ldots, J_{2^n}\} \qquad J_a = \bar{q}\,\mathrm{O}_1\mathrm{O}_2\cdots\mathrm{O}_n q$$

where $\mathrm{O}_p = \mathrm{O}_+, \mathrm{O}_-$

$$\mathrm{O}_+ = \underbrace{\frac{1}{N_c^2} F_{\mu\nu} F^{\mu\nu}}_{\substack{\text{summed over} \\ \text{Lorentz indices} \\ \text{unsummed over} \\ \text{color}}} \qquad \mathrm{O}_- = \underbrace{\frac{1}{N_c^2} F_{\mu\nu} \tilde{F}^{\mu\nu}}_{\substack{\text{summed over} \\ \text{Lorentz indices} \\ \text{unsummed over} \\ \text{color}}}$$

Color “octet” (adjoint) at large Nc

$$\|S_n\| \equiv \dim(\overleftrightarrow{\Pi}) = 2^n$$

The number of independent operator with mass dimension d grows exponentially with d.

- As $\tau \to 0$ the correlation matrix becomes that of free field to Asymptotic Freedom.
- Large Nc forces the matrix to be diagonal for free fields

$$\text{Asymptically: } \Pi_{ab}(\tau) = \delta_{ab} \text{Const } \tau^{-(8n+6)} \quad \text{so} \quad -\frac{d\,\mathrm{Tr}\left(\log\left(\overleftrightarrow{\Pi}(\tau)\right)\right)}{d\tau} = \frac{2^n(8n+6)}{\tau}$$

If we ***knew*** that the matrix of correlators was effectively at its asymptotic value up to small corrections for $\tau < \tau_0$ with τ_0 some value independent of n, then our lemma implies

$$\frac{2^n(8n+6)}{\tau_0} \geq W\left(M_{2^n}\right) \quad \text{or} \quad \frac{(8\log_2(N(M))+6)}{\tau_0} \geq \frac{W\left(M\right)}{N(M)}$$

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Assume at large *M*, *N(M)* is bounded: $N(M)<e^{aM}$. The inequality becomes: $\frac{(8a\log_2(e)M)+6)}{\tau_0} \geq M - \frac{1}{a}$ and there is a contradiction as $M \rightarrow \infty$ unless $a \geq \frac{\tau_0}{8\log_2(e)}$ and the assumption that $N(M)<e^{aM}$ is false.

Consistency requires $N(M) \geq \exp\left(\frac{\tau_0}{8\log_2(e)} M\right)$

and we have a Hagedorn spectrum!!

True if the matrix of correlators is effectively asymptotic for $\tau<\tau_0$ for some τ_0 independent of n. Is it?

Yes, provided the standard assumptions about correlators are ture:

1. **That the leading corrections to correlators at short times are perturbative.**
2. **Perturbation theory is valid when PT says PT is valid---i.e. when perturbative corrections are small. That is PT announces its own doom!**

$$\frac{d\,Tr\log\left(\overleftrightarrow{\Pi}(\tau)\right)}{d\tau} = n\left(\frac{6\delta_{ab}}{\tau} + \frac{d}{d\tau}\sum_j c_j \alpha^j(\tau)\right) \times \left(1 + \vartheta\left(\frac{1}{n}\right)\right)$$

Free field result

Constants independent of n

Follows from a somewhat complicated argument which depends on the planarity of diagrams at large Nc, the logarithmic structure and the point-to-point nature of the correlator.

Key point: the perturbative corrections do NOT grow with n. Thus by going to short enough times so that the perturbative corrections are small (and the system is dominated by its asymptotic free value) at one *n* it remains true for all large *n*.

A Hagedorn spectrum has been demonstrated for Large Nc QCD!!

- Demonstration depends, on the applicability of perturbation theory for correlation functions at short distance. In particular it depends on the assumption that perturbation theory really is valid in the regime in which calculated pertubative corrections are small.
- This assumption is completely standard---however it is not mathematically rigorous. Note it was not invented here for the purpose of showing a Hagedorn spectrum.

- Note that this assumption was not invented for the purpose of showing a Hagedorn spectrum.
- No explicit assumption about stringy dynamics is made.
- Confinement is only assumed in the sense that all physical states are color singlets. **No explict assumptions made about area law of Wilson loops or unbroken center symmetry.**
- The analysis took the large Nc limit from the outset. What this tells us about Nc=3 is an open question.