Large N_c behavior of hadronic models at nonzero temperature

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Outline

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Two limits of QCD

$$\mathscr{L}_{QCD} = \bar{\Psi}\left(\imath\not\partial - g_{QCD}\not A_a T_a - m_{quark}\right)\Psi - \frac{1}{4}F_a^{\mu\nu}F_{a\ \mu\nu}$$

two limits of QCD

$$m_{\text{quark}} \to 0$$

invariant under chiral transformation
chiral phase transition (chiral condensate is exact order parameter)

$$m_{\text{quark}} \to \infty$$

pure gluedynamic
deconfinemant phase transition (Polyakov loop is exact order parameter)

in nature neither $m_{quark} = 0$ then $m_{quark} = \infty$

QCD at large N_c

$$N_c \to \infty, \qquad g_{QCD}^2 \ N_c \to \text{const.}$$

planar diagrams are at leading order, quark loops are suppressed
masses of mesons are not effected, but higher order interactions between mesons are suppressed:

1. three point meson interaction $\sim \frac{1}{\sqrt{N_c}}$
2. four point meson interaction $\sim \frac{1}{N_c}$
3. ...

G. 't Hooft, Nucl. Phys. **B72**, 461 (1974),

E. Witten, Nucl. Phys. **B160**, 57 (1979).

NJL model at large N_c

NJL effective model with quarks:

$$\mathscr{L}_{NJL} = \bar{\psi}(\imath\partial\!\!\!/ - m)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\imath\gamma_5\psi)^2\right]$$

$$m^* = m + m^* G(2N_c + \frac{1}{2}) \times \int_0^\Lambda \frac{dk\ k^2}{2\pi^2} \frac{1}{\sqrt{k^2 + m^{*2}}} \tanh\left(\frac{1}{2}\frac{\sqrt{k^2 + m^{*2}}}{T}\right)$$

$$T_c \simeq \Lambda\sqrt{\frac{3}{\pi^2}}\sqrt{1 - \frac{\pi^2}{2\Lambda^2 G N_c}} \sim \# N_c^0 + \# \frac{1}{N_c} \dots$$

S. P. Klevansky, Rev. Mod. Phys. **64**, 649-708 (1992)

Linear σ Model at large N_c

effective model with mesons
invariant under global $U(N_f)_R \times U(N_f)_L$ transformation

$$\mathscr{L}_\sigma = \frac{1}{2}(\partial_\nu \Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\Phi^4 + \epsilon\sigma, \qquad (\mu^2 > 0,\ \epsilon \to 0^+)$$

$$\Phi = \tau_0\sigma + \imath\tau_i\pi_i$$

due to spontaneous symmetry breaking there is a mass splitting:

$$\sigma \to \sigma + f_\pi \qquad m_\sigma^2 = 3\lambda f_\pi^2 - \mu^2$$

$$\pi_i \to \pi_i \qquad m_\pi^2 = 0$$

apply finite temperature schema (e.g. CJT formalism):

$$f_\pi \to \varphi(T), \text{ with } \varphi(T=0) = f_\pi$$

$$T_c = \sqrt{2} f_\pi$$

A. Bochkarev, J. I. Kapusta, Phys. Rev. **D54**, 4066-4079 (1996)

Linear σ Model at large N_c

implementing large N_c scaling

$$\mu \to \mu, \qquad \lambda \to \left(\frac{3}{N_c}\right)\lambda$$

$$\Rightarrow \mathscr{L}_\sigma(N_c) = \frac{1}{2}(\partial_\nu\Phi)^2 + \frac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4 + \epsilon\sigma, \qquad (\mu^2 > 0,\ \epsilon \to 0^+)$$

for $N_c \to \infty$ interactions vanish
system becomes a free hadron gas
chiral condensate now scales with N_c: $\varphi(N_c) = \sqrt{N_c}\varphi(T)$

$$T_c(N_c) = \sqrt{2}\varphi(N_c) \propto \sqrt{N_c}$$

since phase transition is driven by interaction it disappears for $N_c \to \infty$
such a behavior contradicts results from other effective theories
E. Megias, E. Ruiz Arriola, L. L. Salcedo, Phys. Rev. **D74**, 065005 (2006)

How to correct the large N_c behavior

one could include quark loops to solve the N_c problem
but we would like to have a pure hadronic model

alternative make the parameters μ and λ temperature dependent

- λ dimensionless parameter, for the sake of simplicity chosen to be T independent
- μ has dimension [energy]^2, $\mu^2 \to \mu(T)^2$

e.g.:

$$\mu(T)^2 = \mu^2 \left(1 - \frac{T^2}{T_d^2}\right), \qquad T_d \sim \Lambda_{QCD} \sim N_c^0$$

this leads to a modified large N_c dependency of T_c

$$\Rightarrow T_c = T_d \frac{1}{\sqrt{1 + 2\lambda \frac{T_d^2}{\mu^2} \frac{3}{N_c}}}, \qquad \lim_{N_c \to \infty} T_c = T_d \sim N_c^0$$

Linear σ model with correct large N_c behavior

$$\Rightarrow \mathscr{L}_\sigma(N_c) = \frac{1}{2}(\partial_\nu \Phi)^2 + \frac{1}{2}\mu(T)^2\Phi^2 - \frac{\lambda}{4}\frac{3}{N_c}\Phi^4 + \epsilon\sigma, \quad (\mu(0)^2 > 0,\ \epsilon \to 0^+)$$

$\sigma \to \sigma + \sqrt{N_c}\varphi(T)$, with $\varphi(T) \equiv$ chiral condensate
for $T > 0$ and $N_c \to \infty$ gap equations are exact solvable
(only mean field survives)

gap equation for the chiral condensate in the chiral limit

$$0 = \varphi(T)\left(\lambda\varphi(T)^2 - \mu(T)^2\right)$$

obtains two phases:

$T < T_c$:

$$\varphi(T) = \sqrt{\mu(T)^2/\lambda}$$
$$m_\pi^2 = 0$$
$$m_\sigma^2 = 2\mu(T)^2 > 0$$

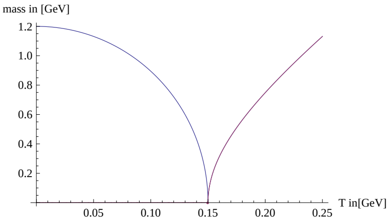
$T > T_c$:

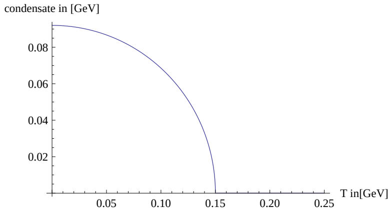
$$\varphi(T) = 0$$
$$m_\pi^2 = -\mu(T)^2 > 0$$
$$m_\sigma^2 = -\mu(T)^2 > 0$$

Chiral limit $\epsilon \to 0^+$

strict large N_c limit: $N_c \to \infty$

$$\mu(T)^2 = \mu^2 \left(1 - \frac{T^2}{T_d^2}\right), \qquad \text{temperature scale } T_d = 0.15 \text{ GeV} \sim \Lambda_{QCD}$$





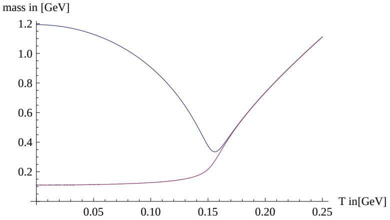
second order phase transition at $T_c = T_d = 0.15$ GeV

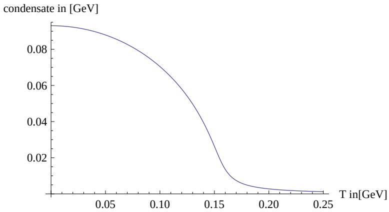
$M_\sigma = 1.2$ GeV, $M_\pi = 0$ GeV, and $\varphi(T=0) = 0.0924$ GeV

$\epsilon \neq 0$

strict large N_c limit: $N_c \to \infty$

$$\mu(T)^2 = \mu^2 \left(1 - \frac{T^2}{T_d^2}\right), \qquad \text{temperature scale } T_d = 0.15 \text{ GeV} \sim \Lambda_{QCD}$$





crossover phase transition at $T_c = T_d = 0.15$ GeV

$M_\sigma = 1.2$ GeV, $M_\pi = 0.13$ GeV, and $\varphi(T=0) = 0.0924$ GeV

Polyakov loop coupled to σ Model

How can we motivate this ansatz for $\mu(T)$? $\rightarrow$ Polyakov loop

$$\mathscr{L}_P(N_c) = \mathscr{L}_\sigma(N_c) + \frac{\alpha}{4\pi} N_c |\partial_\mu l|^2 T^2 - \mathscr{V}(l) - \frac{h^2}{2}\Phi^2 |l|^2 T^2$$

$$l(x) = \frac{1}{N_c} \mathrm{Tr}\left[\mathscr{P} \exp\left(\imath g \int_0^{1/T} A_0(\tau, x) d\tau\right)\right]$$

A. Dumitru, R. D. Pisarski, Phys. Lett. **B504**, 282-290 (2001),

D. H. Rischke, Prog. Part. Nucl. Phys. **52**, 197-296 (2004)

$$0 = \varphi(T)\left(\lambda\varphi(T)^2 - \mu^2 + h^2|l|^2 T^2 + 3\lambda \frac{3}{N_c} \int (G_\sigma + G_\pi)\right)$$

$$T_c = \frac{\mu}{\sqrt{h^2 |l(T_c)|^2 + 6\lambda \frac{1}{N_c}}}$$

Polyakov linear σ model

$$T_c = \frac{\mu}{\sqrt{h^2|l(T_c)|^2 + 6\lambda\frac{1}{N_c}}}$$

cannot be solved analytically, numerical calculations are in progress

but for an rough estimate for T_c: $|l(T_c)| \simeq \frac{1}{2}$

$$N_c \to \infty \Rightarrow T_c \simeq 2\frac{\mu}{h} \sim N_c^0$$

chiral phase transition is triggered by deconfinement phase transition

Conclusion and outlook

- pure hadron models fail at large N_c
- but: it is possible to correct their behavior
  $\rightarrow$ at least one temperature dependent coupling constant:
  $\mu^2 \rightarrow \mu(T)^2$
- in this way a hadron toy model has correct large N_c behavior
- chiral phase transition is triggered by deconfinement phase transition

- test the thermodynamics: pressure, speed of sound, ...
- go to nonzero T and $\mu \rightarrow$ critical point
- expand the model to $SU(3)$ (with all (axial-)vector mesons, glueballs,...)

Thank you