

The Yang-Mills spectrum with arbitrary simple gauge algebras

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Prologue

Yang-Mills

$$\mathcal{L}_{YM} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \text{simple gauge algebra}$$

- Nonabelian
- Asymptotic freedom
- Glueballs
 - Lattice : $su(N)$
 - Effective approaches ?

Why other algebras?

Better understand confinement dynamics

- Check the robustness of standard frameworks
- Maybe see what makes $su(3)$ special

Example

- $A_r : su(r+1)$
 - QCD and large N limit
 - Center, Z_{r+1}
- $G_2 : \text{No center}$

Confinement ~center symmetry ?
~Polyakov loop

A dashed purple box containing the text 'Confinement ~center symmetry ? ~Polyakov loop' has two arrows pointing to the list items. One arrow points to the A_r entry, and the other points to the G_2 entry.

Outline

Quasiglasons

- Constituent picture
- Relevance

Low-lying Yang-Mills spectrum

- Gluelumps, glueballs

Static energy

- Two and three adjoint sources

Large N limit

- Comparison to lattice data

Conclusions

Quasiglçons

Constituent approaches

Relevance for quarks

- Heavy quarks: $1/m_q$ expansion
- Light quarks
 - Dynamically generated mass thanks to chiral symmetry breaking

What about gluons?

- Dynamically generated mass
 - Finite zero momentum limit of the gluon propagator
 - Not universally accepted
- Justification of constituent picture

Gluon mass gap (I)

Coulomb gauge QCD

- Instantaneous potential

$$V_{(r)}(z) = \frac{C_2^{(r)}}{C_2^{(adj)}} \left(\sigma_0 z - \frac{\alpha_0}{z} \right)$$

- One-gluon exchange
- Casimir scaling

- Gluon mass gap equation

$$\omega(q)^2 = q^2 + \int \frac{d^3k}{4(2\pi)^3} \tilde{V}_{(adj)}(|\vec{k} + \vec{q}|) [1 + (\hat{k} \cdot \hat{q})^2] \frac{\omega(k)^2 - \omega(q)^2}{\omega(k)}$$

A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D **65**, 025012 (2002)

A. P. Szczepaniak and E. S. Swanson, Phys. Rev. Lett. **87**, 072001 (2001)

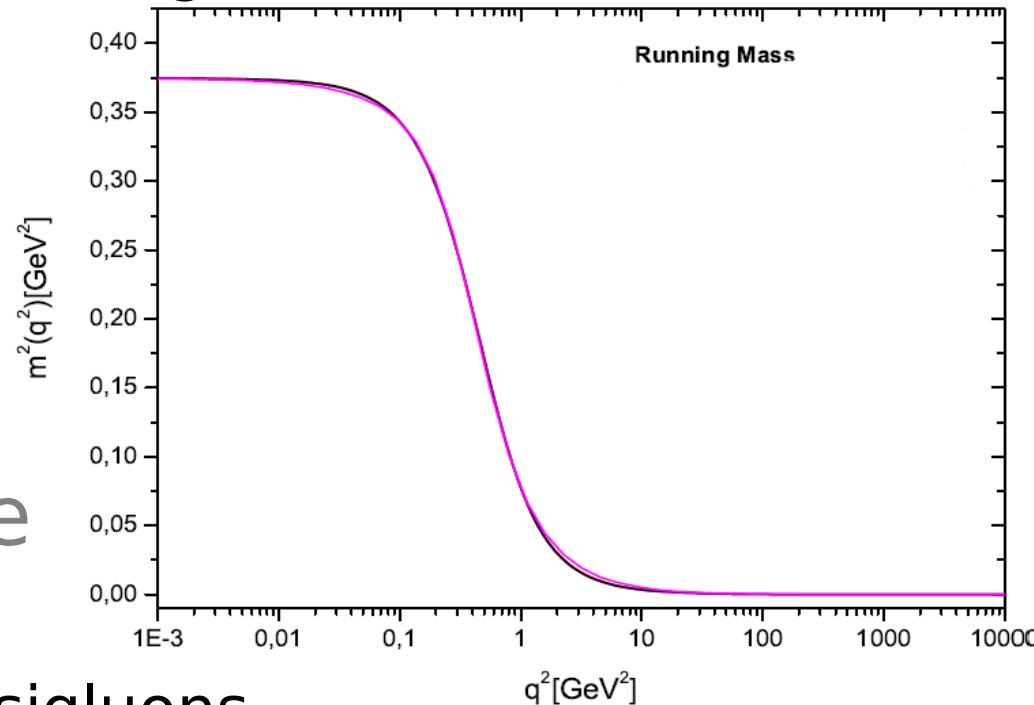
Gluon mass gap (II)

Solution of type $\omega(q)^2 = q^2 + m_g(q)^2$

- $m_g(q) = \sqrt{\sigma_0} \bar{m}(q/\sqrt{\sigma_0}, \alpha_0)$
- Qualitatively valid for all algebras
- Quasigluons
 - Transverse
 - Dynamical mass

Quasigluon picture

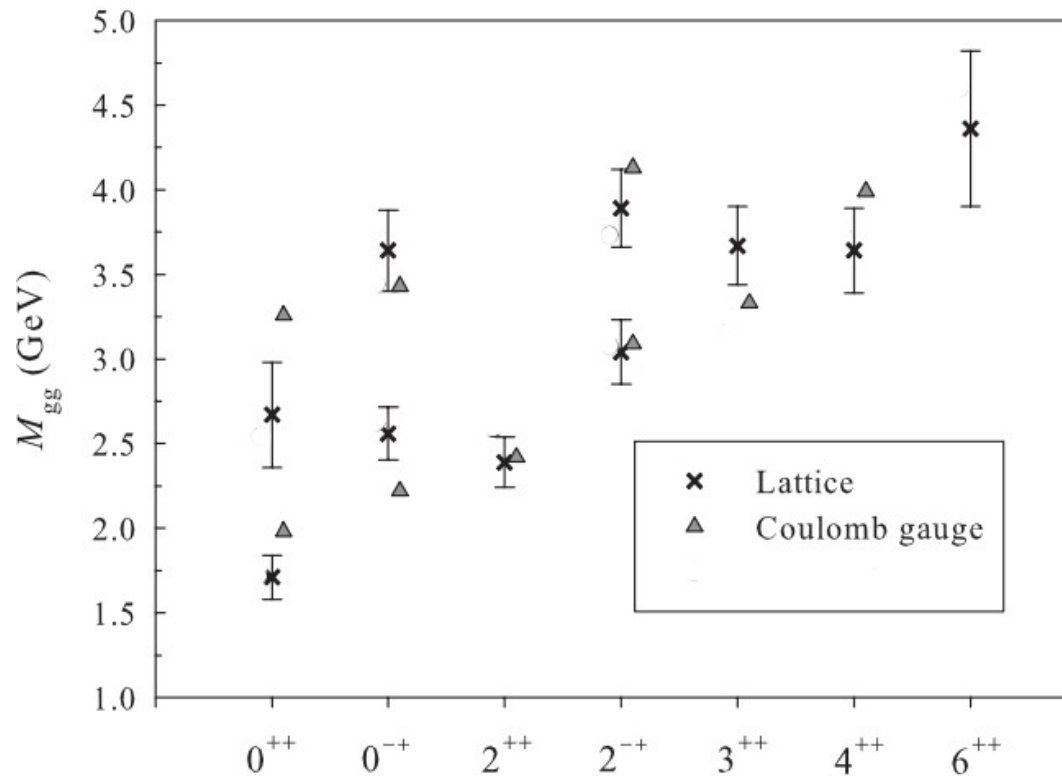
- Gluonic state
= bound state of quasigluons



Glueball spectrum

Coulomb gauge QCD

- Two quasigluons



A. P. Szczepaniak and E. S. Swanson, Phys. Lett. B **577**, 61 (2003)

Quasigluon's features

Adjoint representation

- Charge conjugation $\hat{C} A_\mu \hat{C}^{-1} = -A_\mu^T$

Transverse particles

- Helicity 1

Bound state of n_g quasigluons

- Jacob and Wick's helicity formalism

M. Jacob and G. C. Wick, Ann. Phys. **7**, 404 (1959).

$$|n_g; J^{PC}\rangle = |colour\rangle \otimes |spin - space\rangle \quad \curvearrowright \quad J^P$$

$$\hookrightarrow C$$

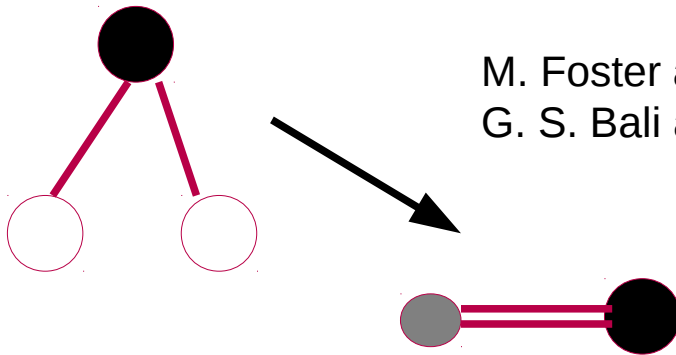
Low-lying Yang-Mills spectrum



Gluelumps (I)

Yang-Mills field + static adjoint source

- Static source \sim adjoint quark-antiquark pair
- Extreme simplification of heavy hybrids



M. Foster and C. Michael, Phys. Rev. D **59**, 094509 (1999)
G. S. Bali and A. Pineda, Phys. Rev. D **69**, 094001 (2004)

In general $adj \otimes adj = \bullet^S \oplus adj^A \oplus \dots$

- One quasigluon is enough
- Lightest states

Gluelumps (II)

Colour structure

- Negative C

$$\delta_{ab}\phi^a A_\mu^b \propto \text{Tr}(\phi A_\mu) = -\text{Tr}(\phi^c A_\mu^c)$$

Helicity states: $J > 0$

- Lowest spin

$$|g; 1^{+-}\rangle = \sqrt{\frac{2}{3}} |^3S_1\rangle + \sqrt{\frac{1}{3}} |^3D_1\rangle$$

$$|g; 1^{--}\rangle = -|^3P_1\rangle$$

Lightest, ok
with lattice

Glueballs (I)

Lightest : Two quasiguons

- $M_{gg} = O(2M_g)$
- Physical gluonic states

Colour structure $adj \otimes adj = \bullet^S \oplus adj^A \oplus \dots$

- Positive C

$$\delta_{ab} A_{\mu}^a A_{\nu}^b \propto \text{Tr}(A_{\mu} A_{\nu}) = \text{Tr}(A_{\mu}^C A_{\nu}^C)$$

Glueballs (II)

Helicity states + Pauli principle

- Color symmetric, spin-space symmetric
- No 1^{++} and 1^+ states
 - Yang's theorem, no $\rho \rightarrow \gamma\gamma$
 - Lattice, no light $J = 1$ glueball
- Examples

$$|0^{++}\rangle = \sqrt{\frac{2}{3}} |^1S_0\rangle + \sqrt{\frac{1}{3}} |^5D_0\rangle \quad \text{Lightest}$$

$$|0^{-+}\rangle = -|^3P_0\rangle$$

$$|2^{++}\rangle = \sqrt{\frac{2}{5}} |^5S_2\rangle + \sqrt{\frac{4}{7}} |^5D_2\rangle + \sqrt{\frac{1}{35}} |^5G_2\rangle$$

Glueballs (III)

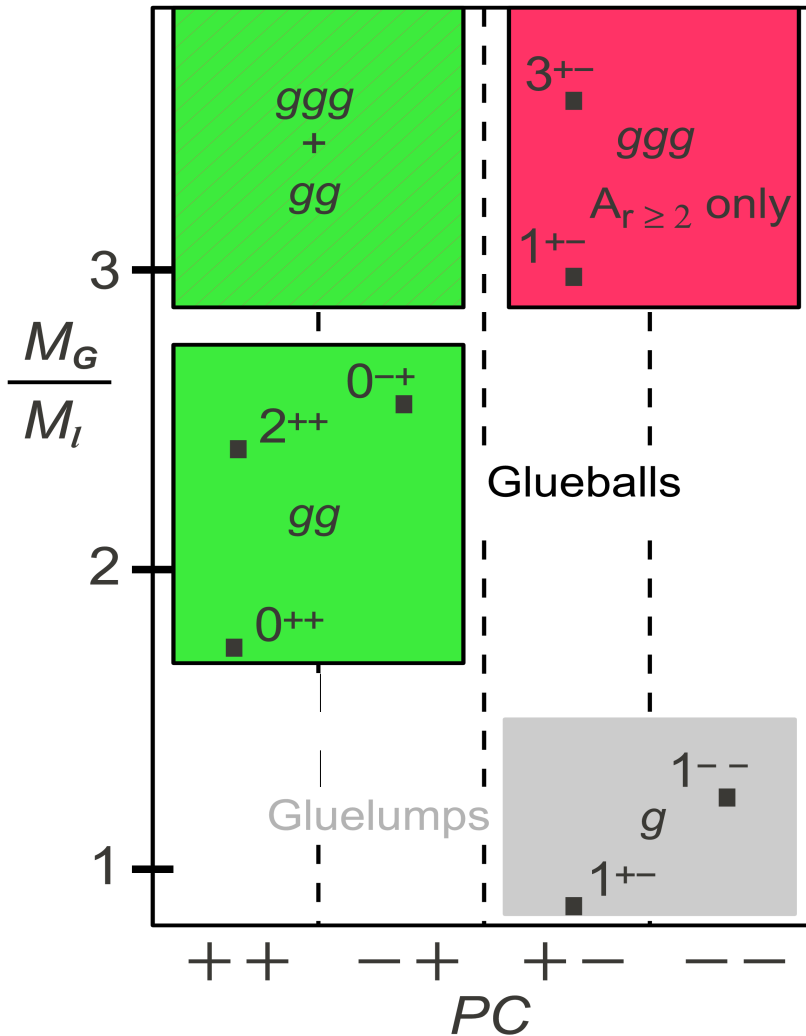
Three quasiguons

- $M_{gg} = O(3M_g)$

Colour structure

- Positive C $adj \otimes adj \otimes adj = \bullet^A \oplus \dots$
 - Always possible
- Negative C $adj \otimes adj \otimes adj = \bullet^A \oplus \bullet^S \oplus \dots$
 - Only for A_r gauge algebras with $r > 1$
- Example : C = - glueballs for su(3), not for su(2)

Partial summary



Ok with
current su(3)
lattice data

Static energy

Two adjoint sources

Funnel potential

- Standard form in QCD

$$V_{(adj)}(z) = \sigma_0 z - \frac{\alpha_0}{z}$$



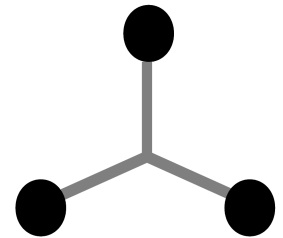
- Could be used to measure the parameter's values
- Casimir scaling assumed
 - Ok with lattice, also with G_2
G. S. Bali, Phys. Rev. D **62**, 114503 (2000)
A. Wipf, and C. Wozar, arXiv:1006.2305

Three adjoint sources (I)

Y-junction

- Three adjoint strings

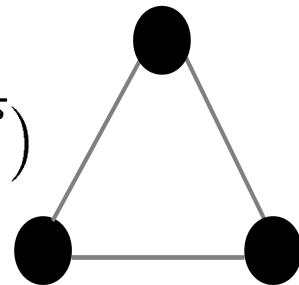
$$V_Y = \sum_i \sigma_0 |\vec{r}_i - \vec{Y}| - \frac{1}{2} \sum_{i < j} \frac{\alpha_0}{|\vec{r}_i - \vec{r}_j|}$$



Δ - shape

- Adjoint representation in $f \otimes f$ ($f \otimes \bar{f}$)
- Three fundamental strings

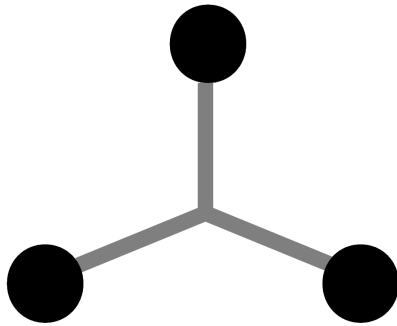
$$V_\Delta = \sum_{i < j} \left[\frac{C_2^{(f)}}{C_2^{(adj)}} \sigma_0 |\vec{r}_i - \vec{r}_j| - \frac{1}{2} \frac{\alpha_0}{|\vec{r}_i - \vec{r}_j|} \right].$$



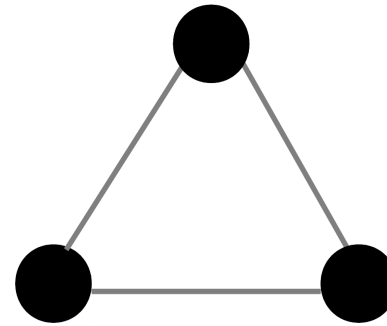
Three adjoint sources (II)

Equilateral triangle: $\frac{V_Y}{V_\Delta} = \frac{1}{\sqrt{3}} \frac{C_2^{(adj)}}{C_2^{(f)}}$

- Algebra-dependent



$B_3, D_4, E_6, E_7, E_8, F_4$



$A_r, B_{r \geq 4}, C_{r \geq 2}, D_{r \geq 5}, G_2$

Checked for $su(3)$

Large N limit

Two quasiglasons

Gauge algebra $A_{N-1} \cong su(N)$

- Fixed 't Hooft coupling $\lambda = N g^2$
- Potential and mass gap N-independent $\alpha_0, \sigma_0 \propto \lambda$

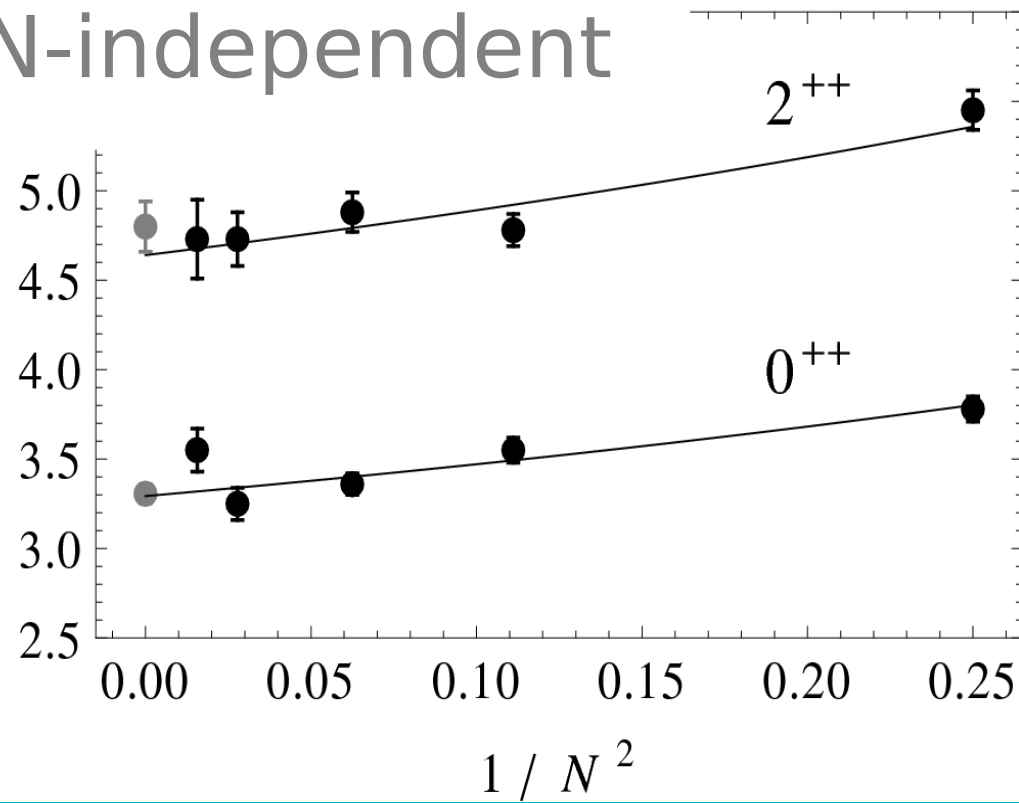
Glueball mass : N-independent

- OK with lattice

B. Lucini, A. Rago, and E. Rinaldi,
JHEP **1008**, 119 (2010)

$$\frac{M_{gg}}{\sqrt{\sigma(f)}} = \sqrt{\frac{2N^2}{N^2 - 1}} \theta_{gg}$$

B. Lucini, M. Teper, and U. Wenger,
JHEP **0406**, 012 (20004)



Three quasiglasons

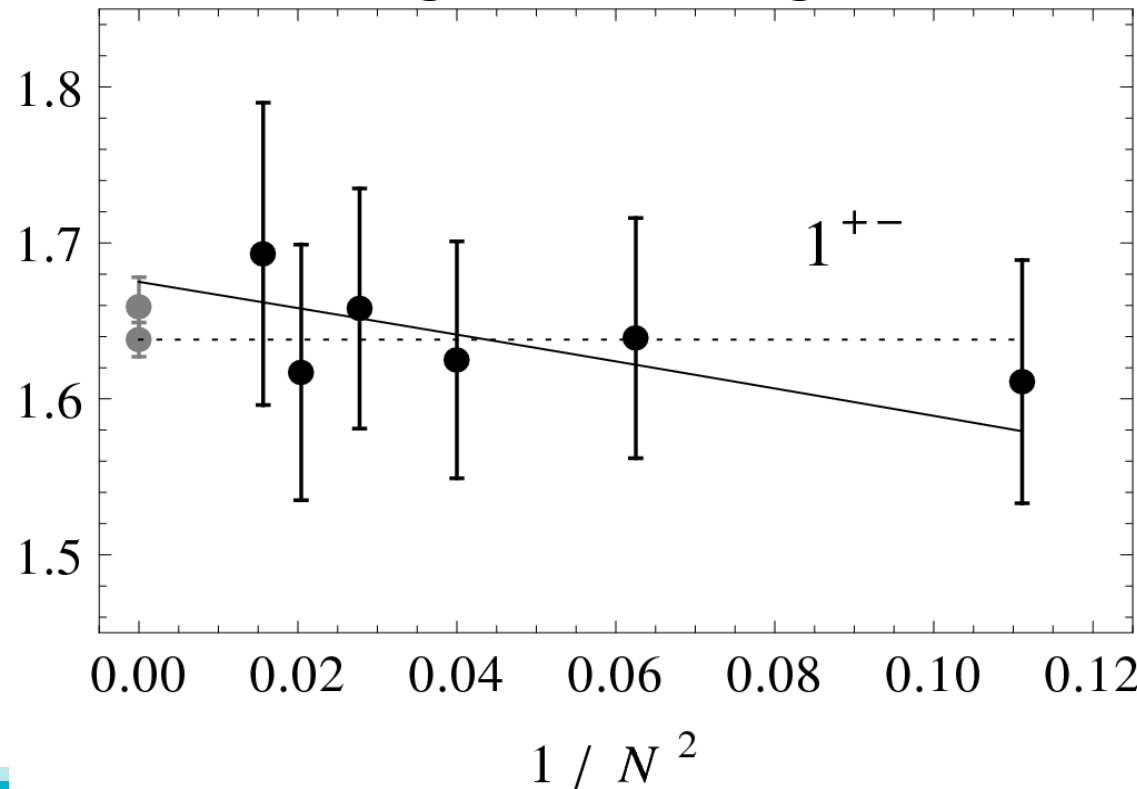
Potential

- Y- junction N-independent
- Δ shape N-dependent through the string tension

$$M_{ggg}^Y = \theta_{ggg}$$

$$M_{ggg}^\Delta = \sqrt{\frac{N^2 - 1}{2N^2}} \theta_{ggg}$$

B. Lucini, A. Rago, and E. Rinaldi,
JHEP **1008**, 119 (2010)



Conclusions

Summary

Yang-Mills theory

- Confining for any simple gauge algebra
- Glueballs: general feature

Quasigluon picture

- Bound states of transverse adjoint particles
- Global structure of the spectrum

Static energy

- Algebra-dependent for 3 adjoint sources

Large N limit

- Glueball masses versus N agree with lattice results

Confirmation of the present ideas

- Other theoretical frameworks
- Lattice computations ?

Very last slide

Full Yang-Mills

Constituent models

- Less fundamental
- Capture essential features
- Intuitive / analytical

Reliable approximation

