Signatures of the chiral critical endpoint of QCD: the role of finite-size effects

Letícia F. Palhares
IPhT, CEA-Saclay
Instituto de Física, Universidade Federal do Rio de Janeiro

In collaboration with:
E. S. Fraga (IF-UFRJ)
T. Kodama (IF-UFRJ)
P. Sorensen (BNL)

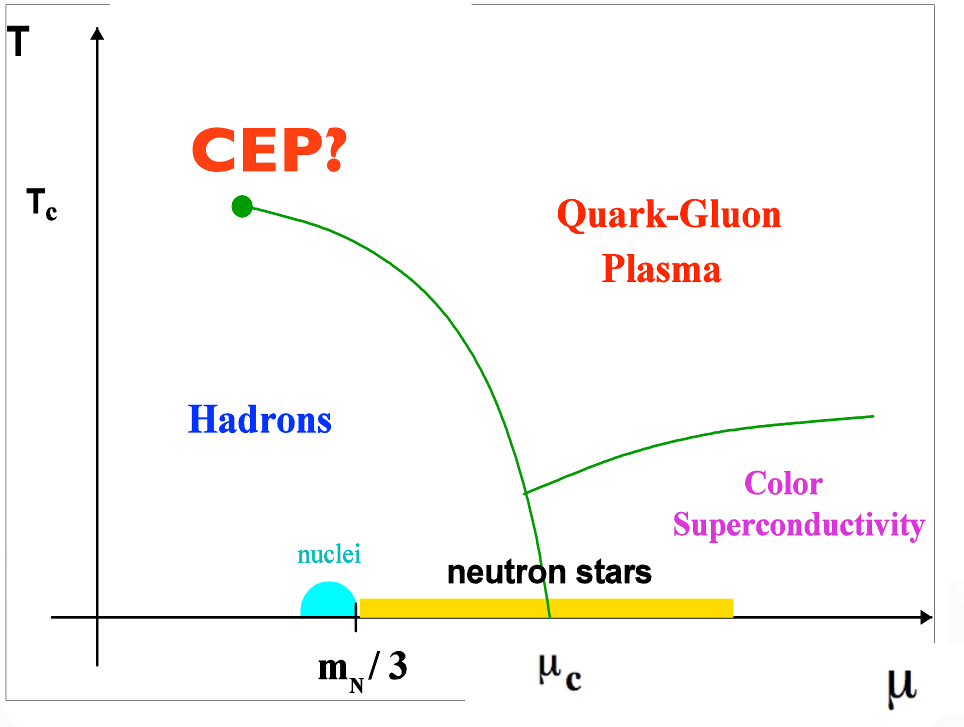
CAPES CNPq FAPERJ cea

Outline

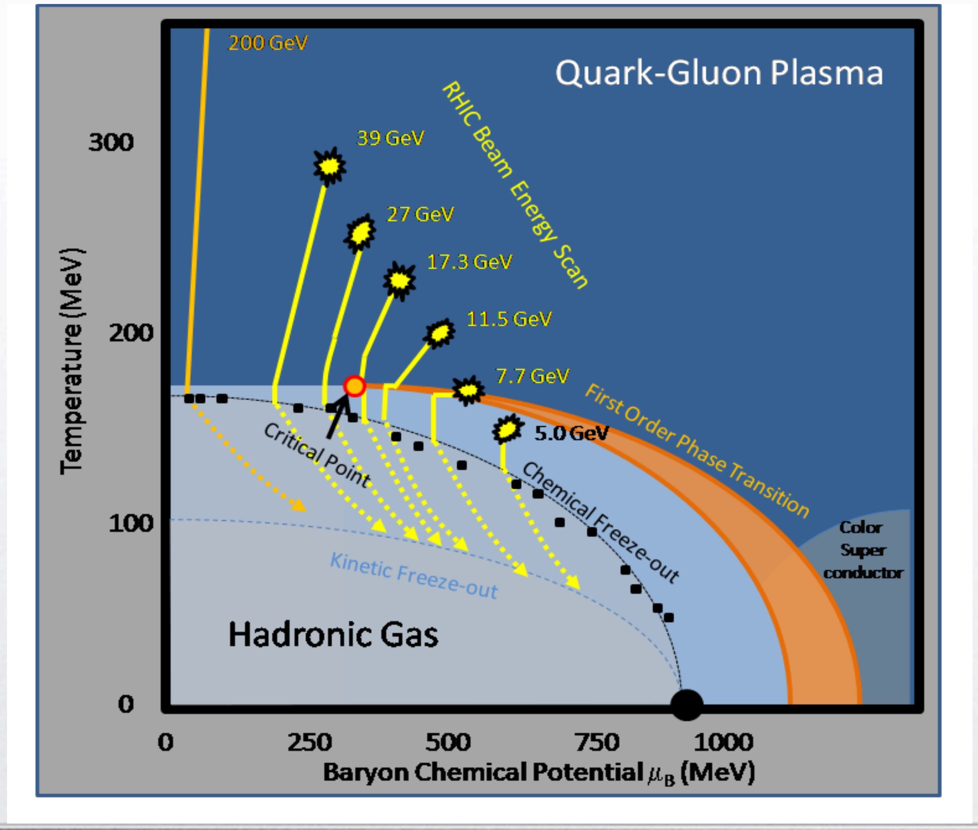
- Introduction: the CEP *in heavy-ion collisions*
- Finite-size effects on signatures of the chiral CEP
- Final remarks

Introduction

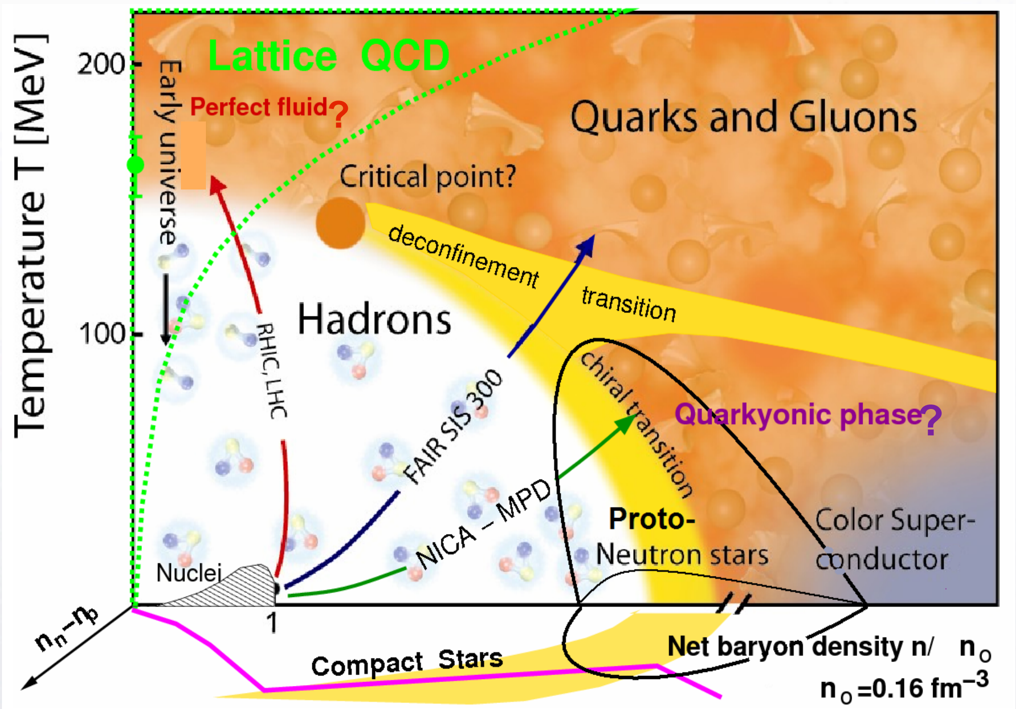
- Concrete possibility of locating the **first experimental point in the QCD phase diagram at high energies**.



Beam Energy Scan, RHIC-BNL



FAIR-GSI, NICA-JINR



The chiral CEP

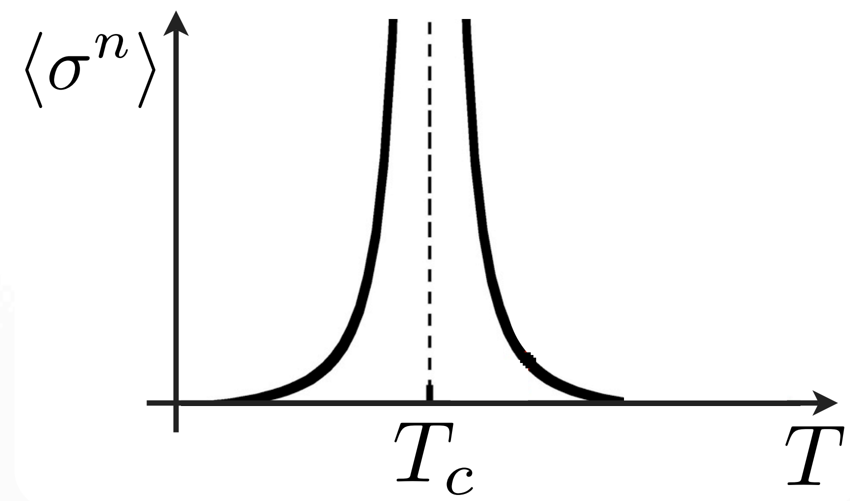
- <u>General features:</u>

Second order phase transition

⇒ Diverging correlation length
⇒ Conformal invariance at criticality
⇒ large fluctuations at all scales

Correlations of the chiral condensate:

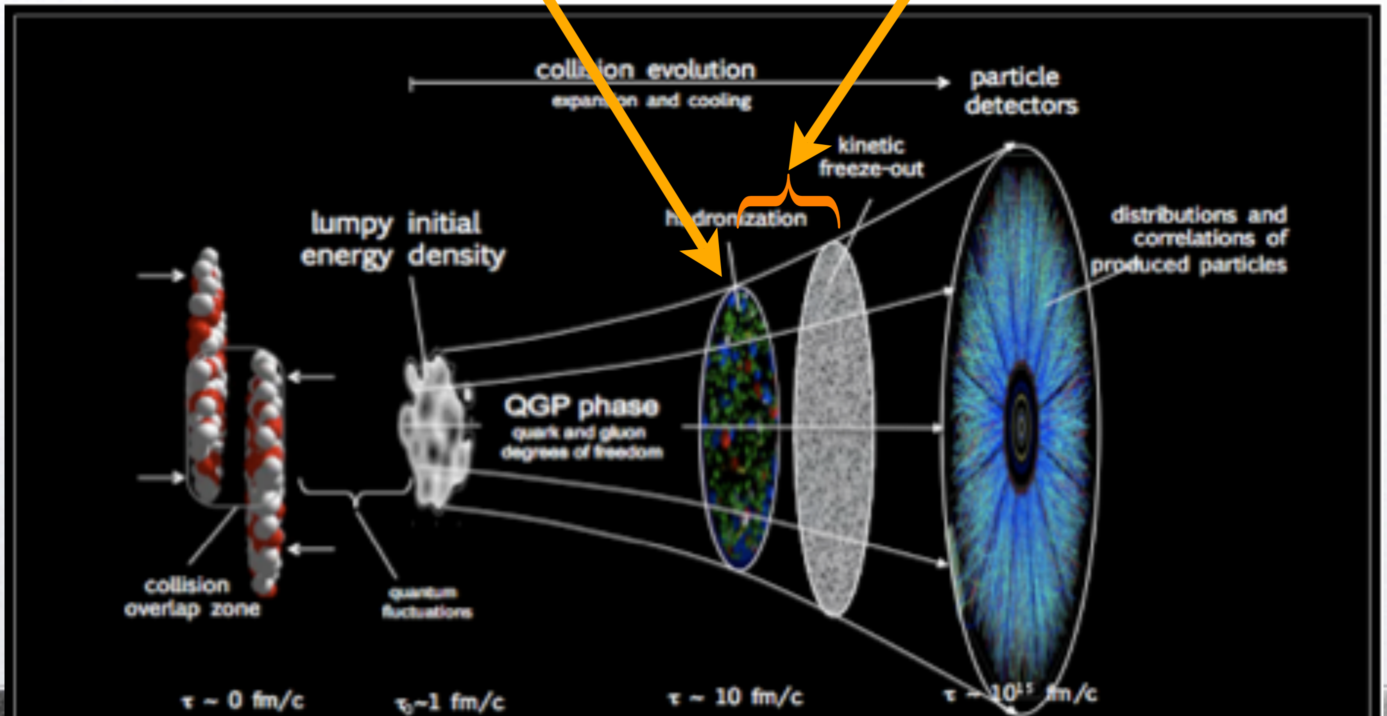
$$\langle \sigma^n \rangle \sim \xi^{p_n} \to \infty$$



- <u>In HICs:</u>

Chiral Ph. Trans.

Hadronic medium



Elliptic Flow: A Study of Space-Momentum Correlations In Relativistic Nuclear Collisions.
P. Sorensen, Quark Gluon Plasma, Vol. 4, World Scientific, arXiv:0905.0174 [nucl-ex]

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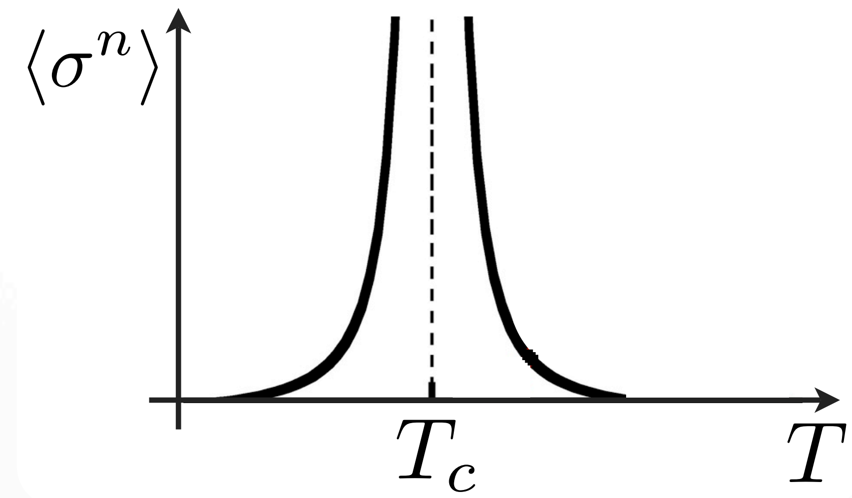

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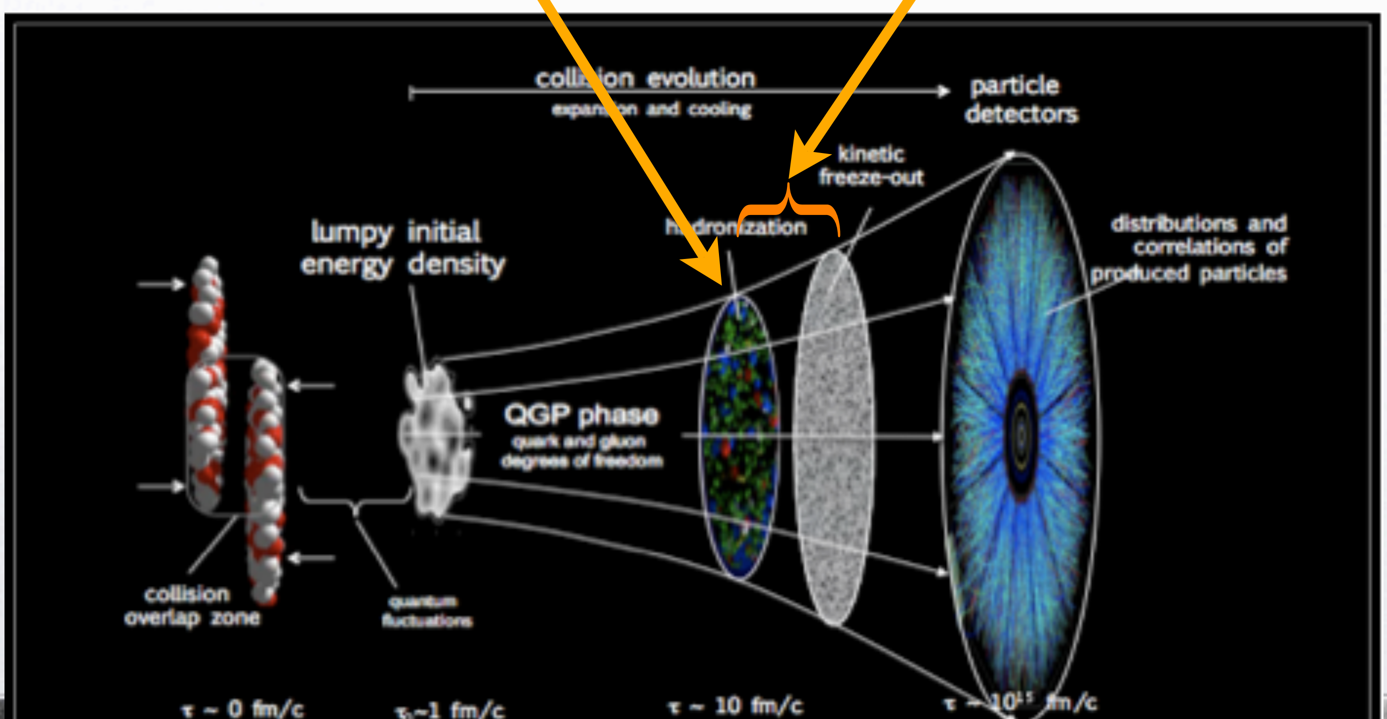

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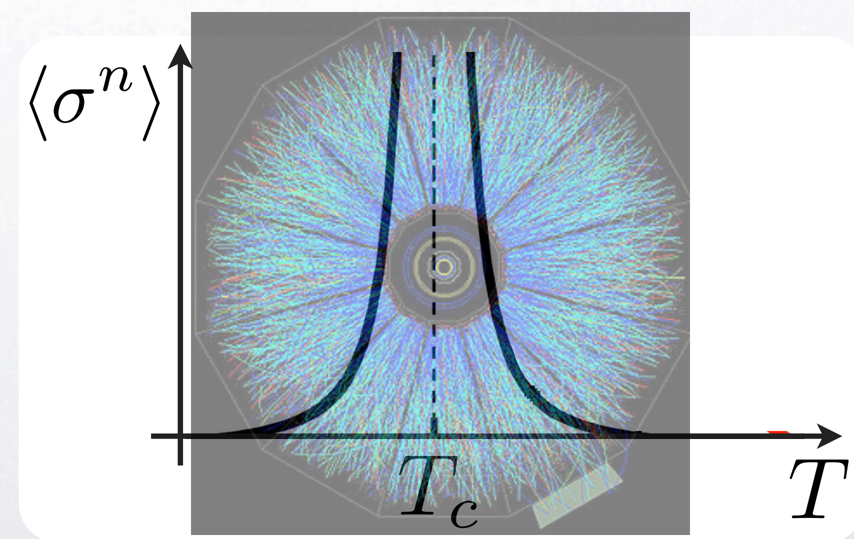
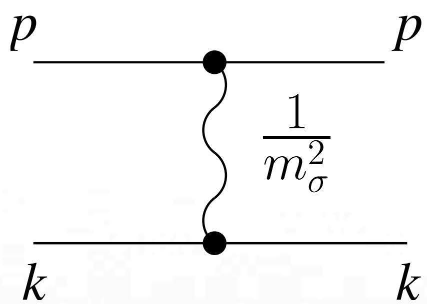


Signatures of the chiral CEP

- **<u>Proposal by Stephanov *et al*:</u>** critical correlations of the chiral condensate will be transmitted to particles coupled to the sigma field, e.g. pions ($G\sigma\pi\pi$) and nucleons ($g_N \sigma \bar{N} N$):



$$\langle \delta n_p \delta n_k \rangle_\sigma = \frac{1}{T} \frac{f_p(1+f_p)}{\omega_p} \frac{f_k(1+f_k)}{\omega_k} \frac{G^2}{m_\sigma^2} \sim \xi^2 \to \infty$$

- To be observable, the correlation signal must survive the hadronic phase.

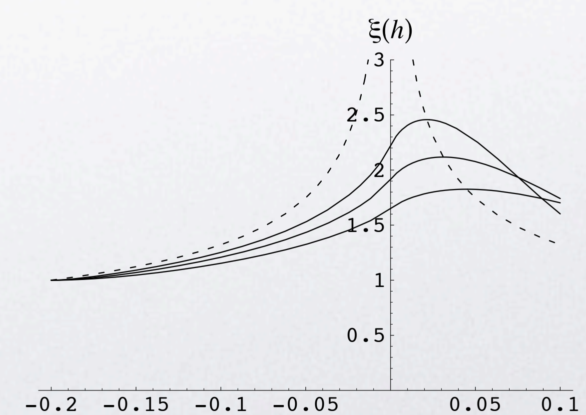
$$m_\sigma(T_{\text{CEP}}) \approx 0$$

⇒ *sigma decays into soft pions when it reaches the mass threshold, closer to freeze-out.*

⇒ *fluctuations from 1st order PhT will suffer earlier from the hadronic medium.*

- However, the growth of the correlation length is limited in HICs:

proximity to the critical point, finite lifetime, critical slowing down, finite size effects.



[Berdnikov & Rajagopal (2000)]

Signatures of the chiral CEP

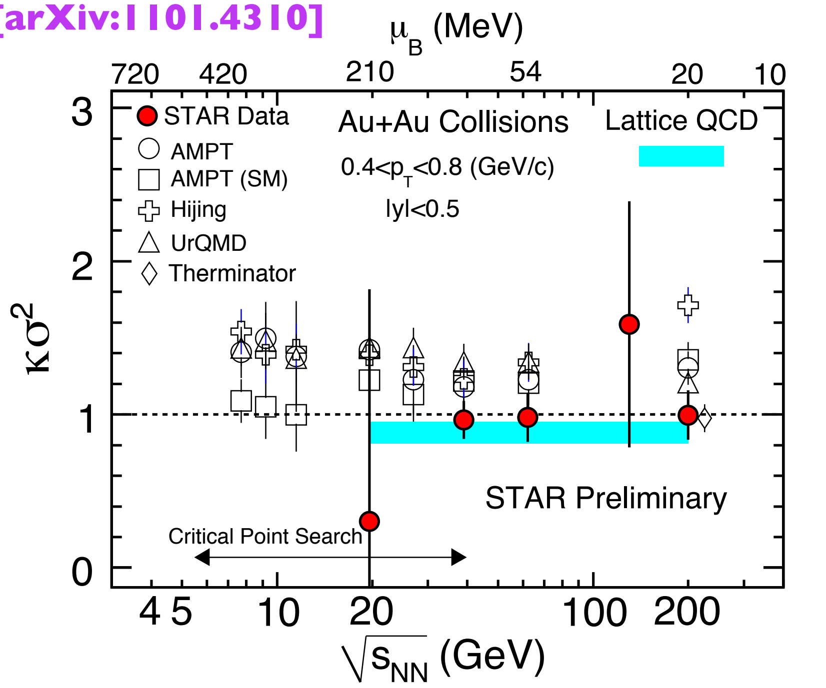
- **Improvements:** higher moments, ratios, etc **[Stephanov (2008); Athanasiou, Rajagopal & Stephanov (2010)]**

Skewness: $$\omega_3(N)_\sigma = \frac{2\lambda_3}{T}\frac{G^3}{m_\sigma^6}\left(\int_{\boldsymbol{p}} \frac{v_{\boldsymbol{p}}^2}{\omega_{\boldsymbol{p}}}\right)^3 \left(\int_{\boldsymbol{p}} \bar{n}_{\boldsymbol{p}}\right)^{-1} \sim \xi^{9/2}$$

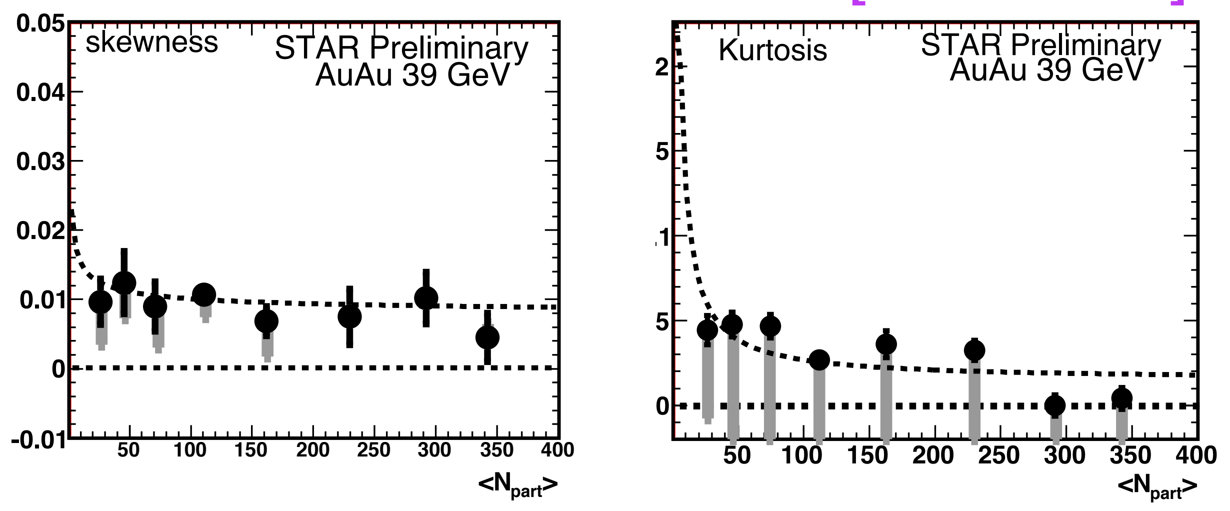
Kurtosis: $$\omega_4(N)_\sigma = \frac{6}{T}\left[2\frac{\lambda_3^2}{m_\sigma^2} - \lambda_4\right]\frac{G^4}{m_\sigma^8}\left(\int_{\boldsymbol{p}} \frac{v_{\boldsymbol{p}}^2}{\omega_{\boldsymbol{p}}}\right)^4 \left(\int_{\boldsymbol{p}} \bar{n}_{\boldsymbol{p}}\right)^{-1} \sim \xi^7$$

- **Experimental results:**

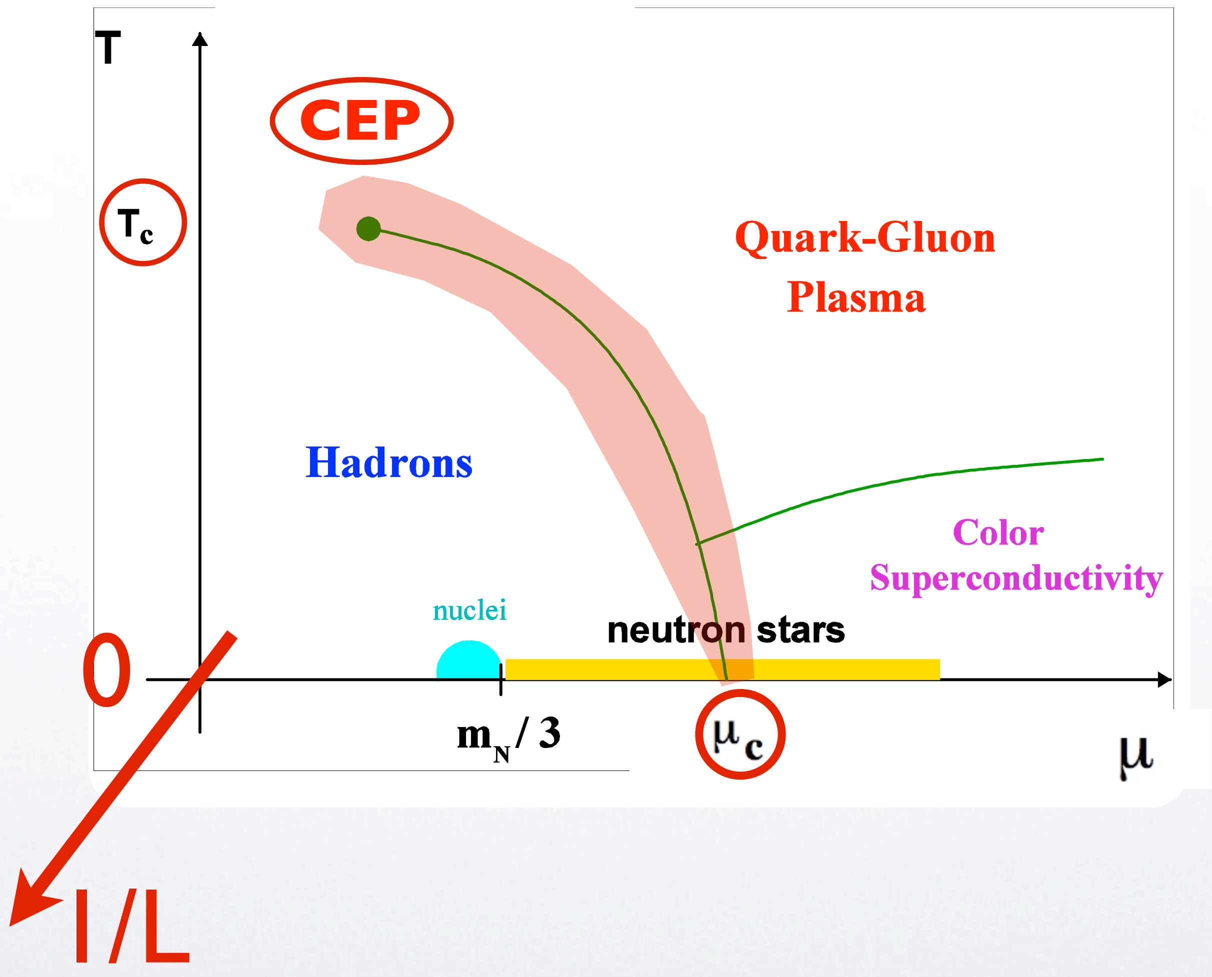
[arXiv:1101.4310]



[arXiv:1101.5125]

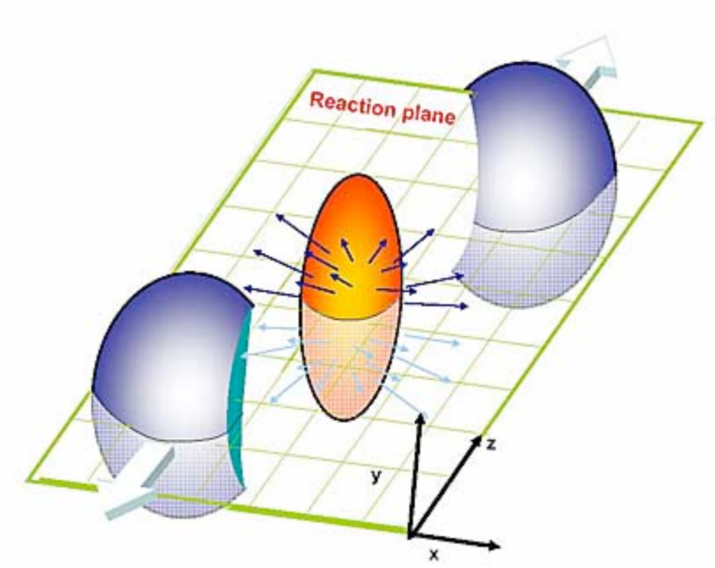


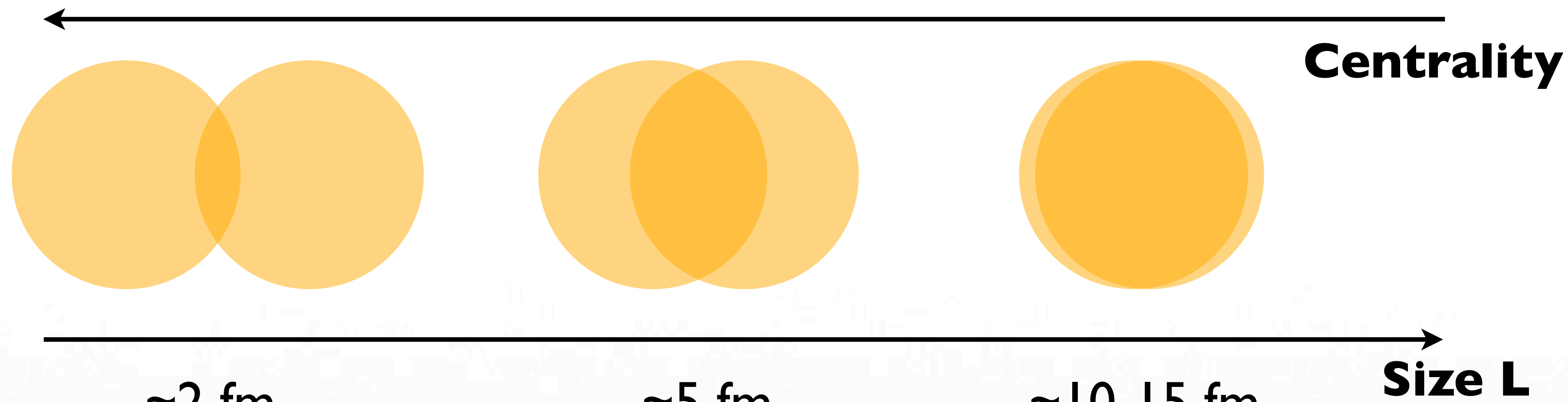
- In this talk:


T
CEP
T_c
Quark-Gluon
Plasma
Hadrons
Color
Superconductivity
nuclei
neutron stars
0
$m_N / 3$
μ_c
μ
1/L

[LFP, Fraga & Kodama (2009),(2010)]

The system created in HICs is <u>FINITE</u> and its size is <u>CENTRALITY-DEPENDENT</u>:





- How does the (pseudo)phase diagram of strong interactions differ from the expectation at L → ∞?

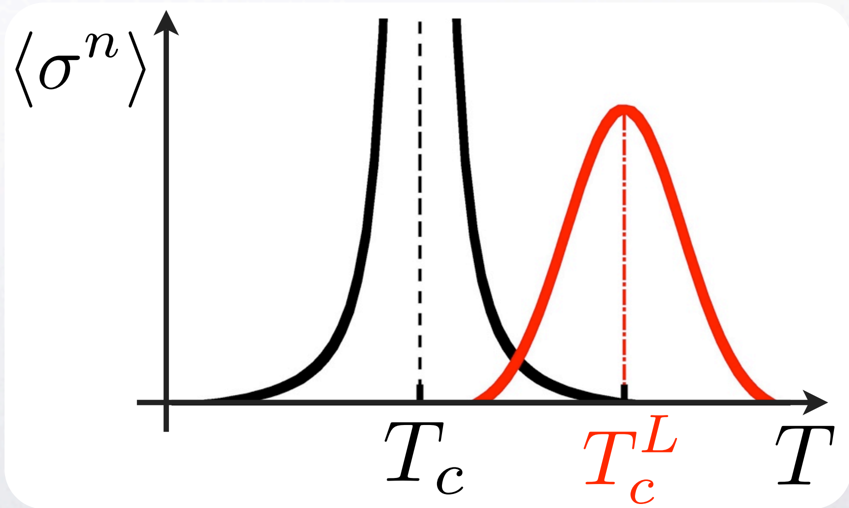
  ⇒ Investigate the importance of the shifts within a chiral model for typical HIC size scales.

- Consequences for signatures of the CEP at HICs.

  - most signatures will probe pseudo-critical behavior

  - HIC data as an ensemble of systems of different sizes:
  ⇒ finite-size scaling analysis

$$\langle \sigma^n \rangle \sim \xi^{p_n} f_n(\xi/L)$$



The Linear σ Model with quarks at finite volume

$$\mathcal{L} = \overline{\psi}_f \left[i\gamma^\mu \partial_\mu + \mu\gamma^0 - g\sigma \right] \psi_f + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \left[\frac{\lambda}{4}(\sigma^2 - v^2)^2 - h\sigma \right]$$

Models the Chiral Properties of QCD: spontaneous and small explicit breaking

Parameters fixed to reproduce observed properties of the QCD vacuum

Pions dropped for simplicity, since they do not affect much the phase structure

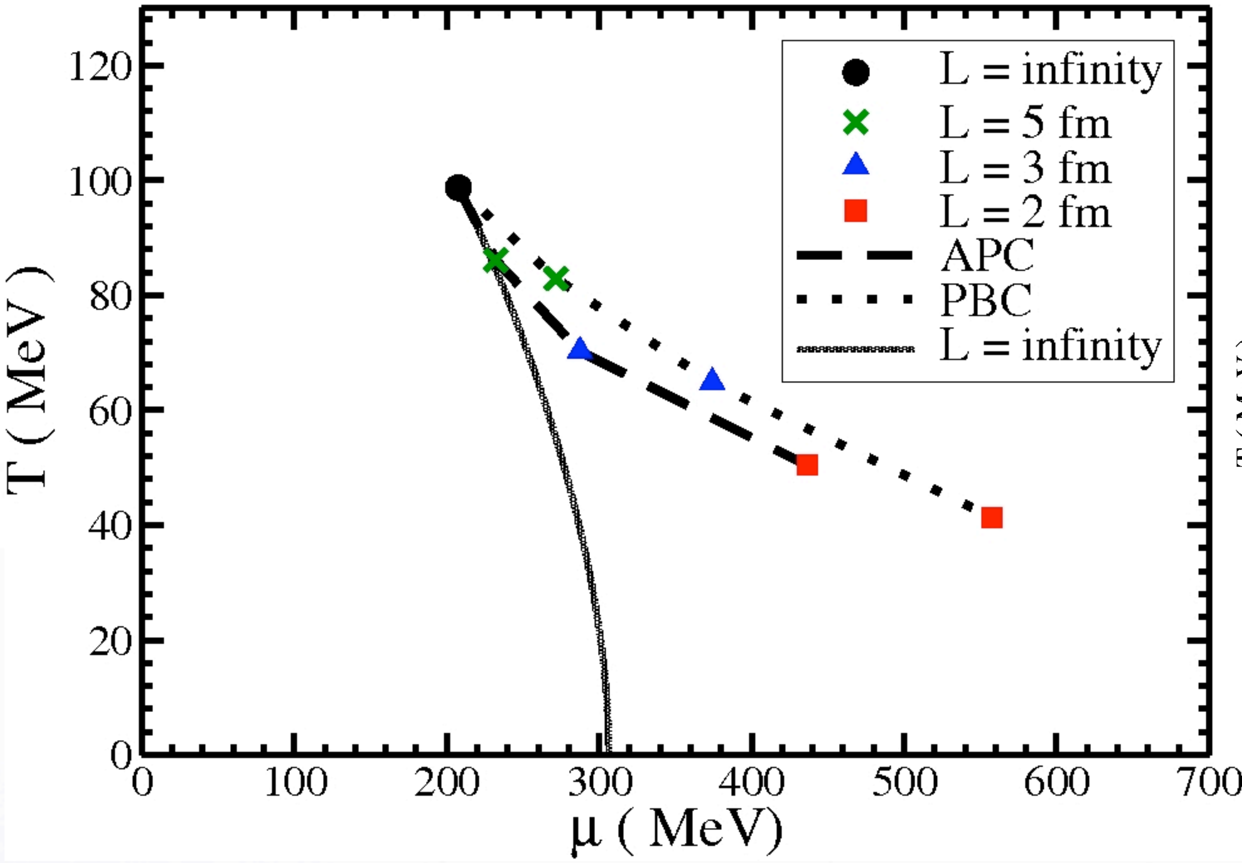
[Scavenius et al (2001)]

Finite Volume: $V \int \frac{d^3\vec{k}}{(2\pi)^3} f(\vec{k}) \mapsto \sum_n f\left(\vec{k}(n)\right)$ **Boundary Conds.:** $\vec{k}(n)$

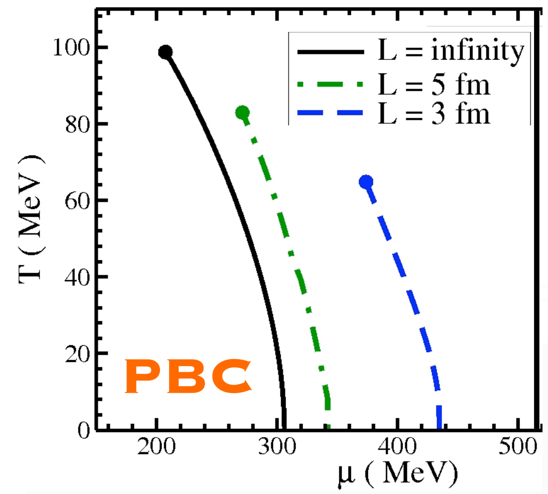
Main goal here: Estimate amplitude of shifts in the (pseudocritical) phase diagram of the chiral transition for system sizes typically encountered in HICs.

The (pseudo)CEP: volume and BC dependence

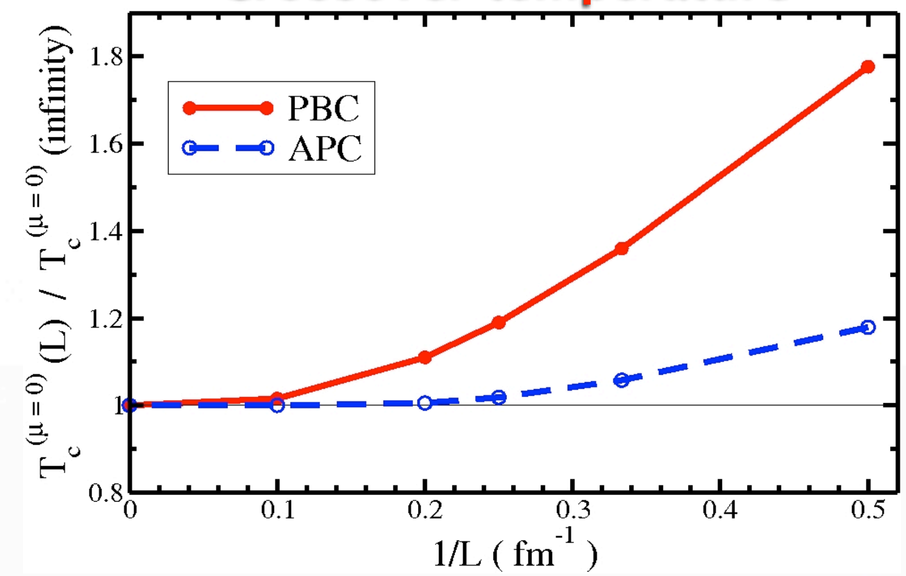
[LFP, Fraga & Kodama (2009),(2010)]



⇒ **Significant corrections for size scales probed in HICs.**

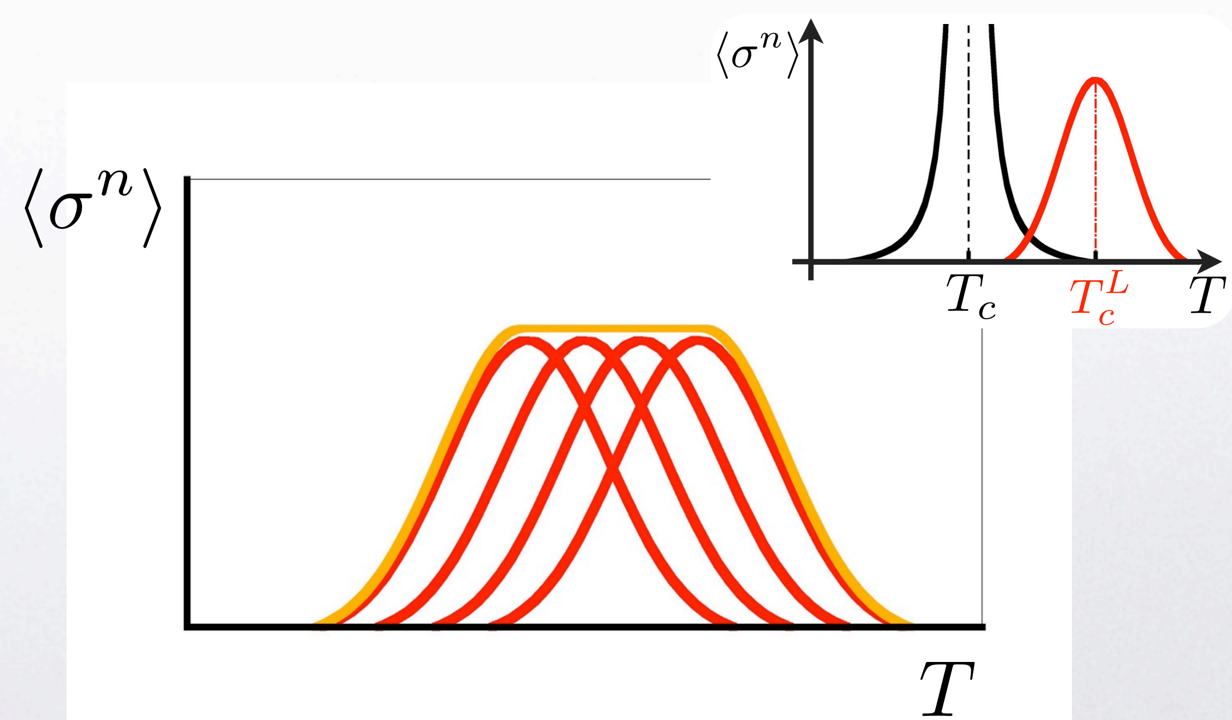


Crossover temperature



For signatures related to the nonmonotonic behaviour of the order parameter, correlations are averaged within a centrality window:

superposition of shifted peaks
⇒ broadening of signal



CEP ⇒ 2[ND] ORDER PHASE TRANSITION $\begin{cases}\text{divergent correlation length}\\ \text{scale invariance on the criticality}\end{cases}$

These features imply the existence of **Finite-size Scaling** *for finite systems in the vicinity of the CEP (rigorous proof through RG analysis):*

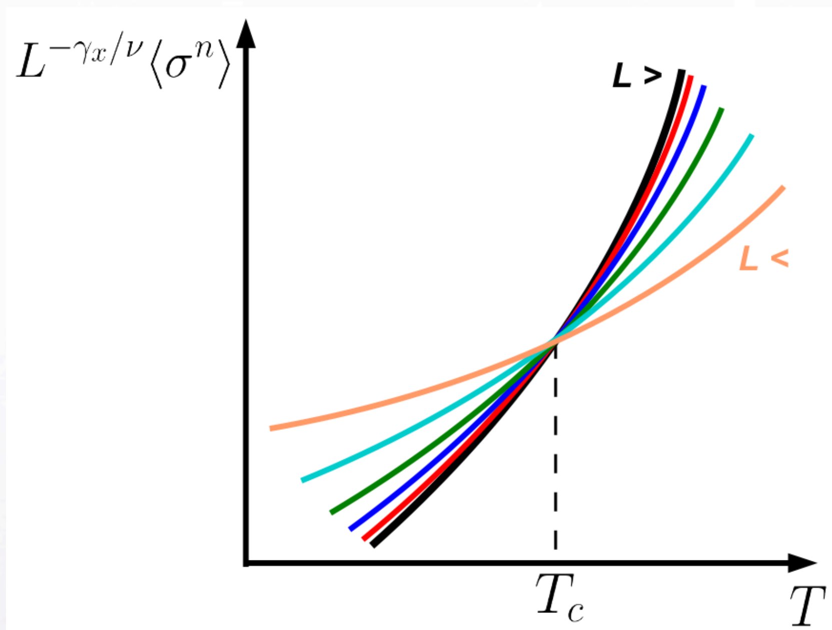
$$X(t, L) = L^{\gamma_x/\nu} f_x(tL^{1/\nu})$$

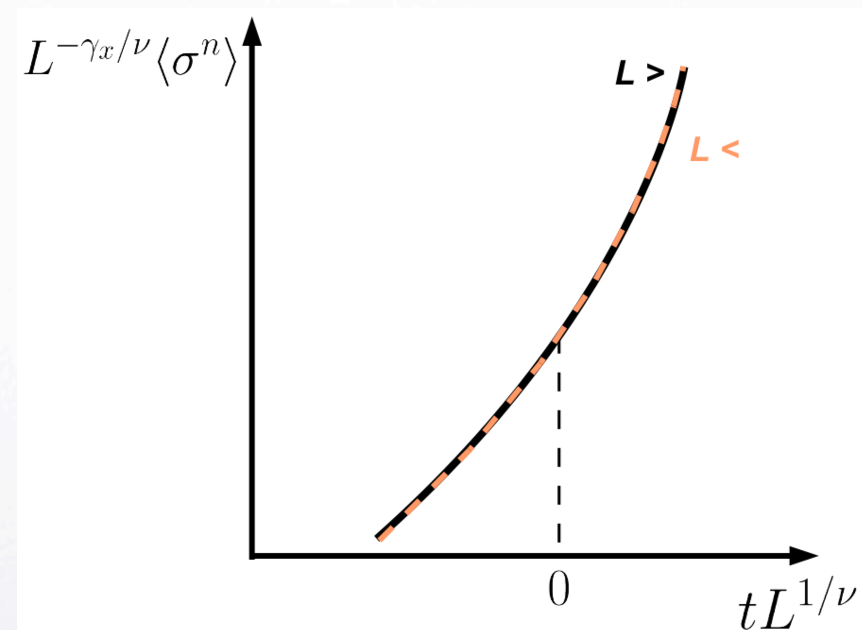
$t = (T - T_c)/T_c$ **(distance to the genuine CEP)**

X **⇒ (any) corr. function of the order parameter**

ν **⇒ universal critical exponent (div. of corr. length)**

SCALING PLOTS:





Finite-size Scaling as a tool for searching the CEP

CEP ⇒ 2ND ORDER PHASE TRANSITION { *divergent correlation length* / *scale invariance on the criticality* }

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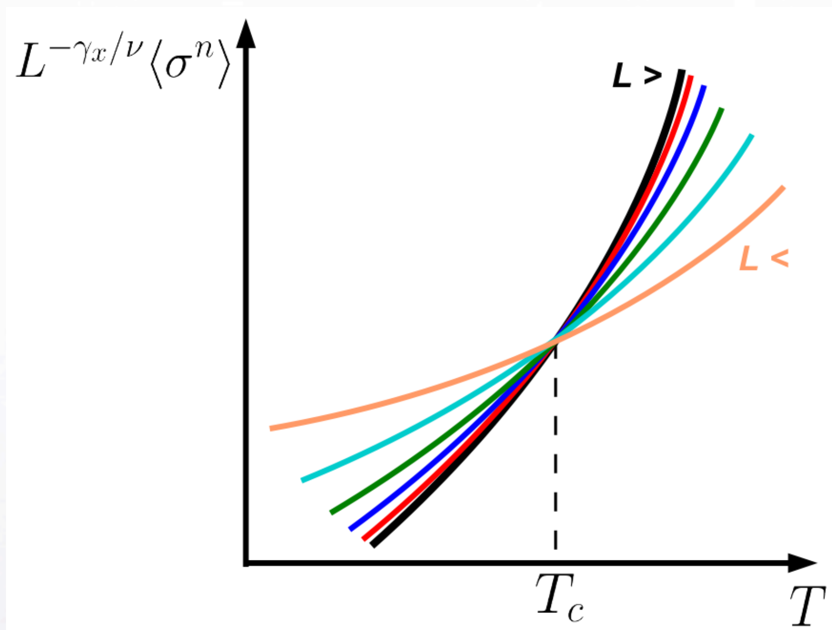
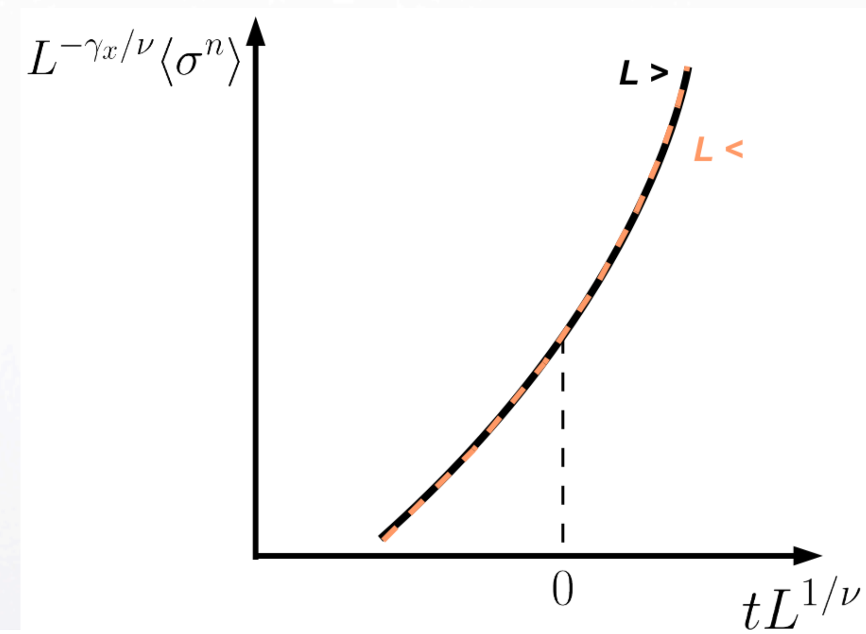




In HICs:

[LFP, Fraga & Kodama (2009)]

✓ *Size of the system (L):* from HBT analysis (interferometry).

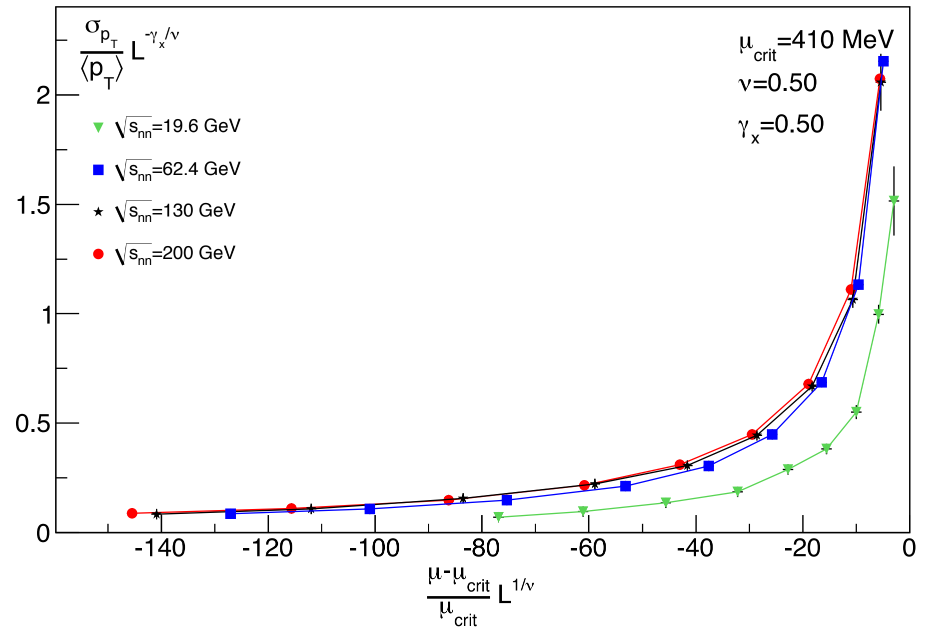
✓ *Distance to the CEP ("t"):* constrained by freeze-out curve, parametrized either by μ or by center-of-mass energy

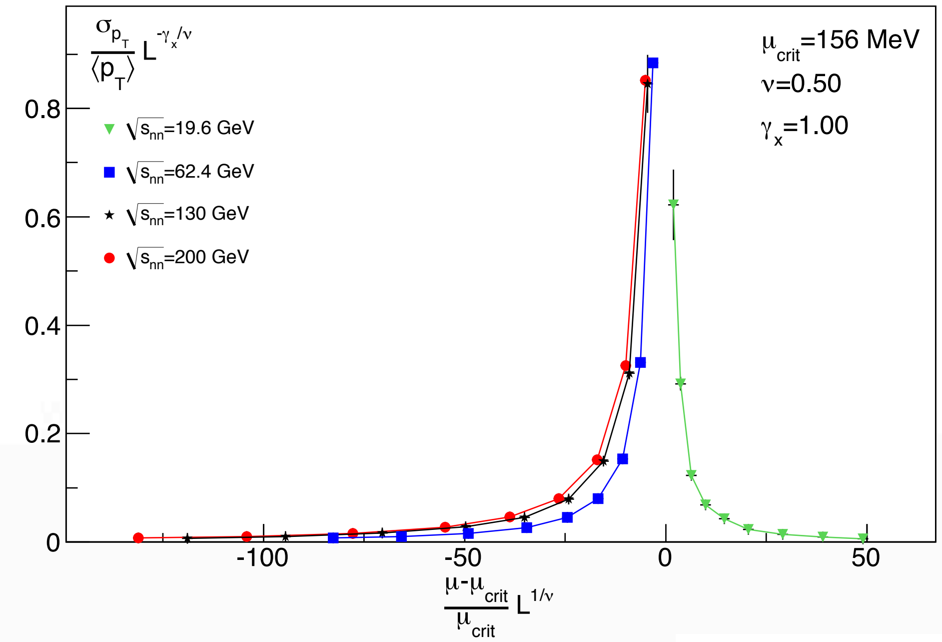
✓ *Observables (X):* transverse mom. fluct. or multiplicity fluct. of soft pions

FSS analysis of HIC data

[LFP, Fraga & Sorensen, in prep.]

Scaling plots for transverse momentum fluctuations from RHIC-BNL (not BES yet):

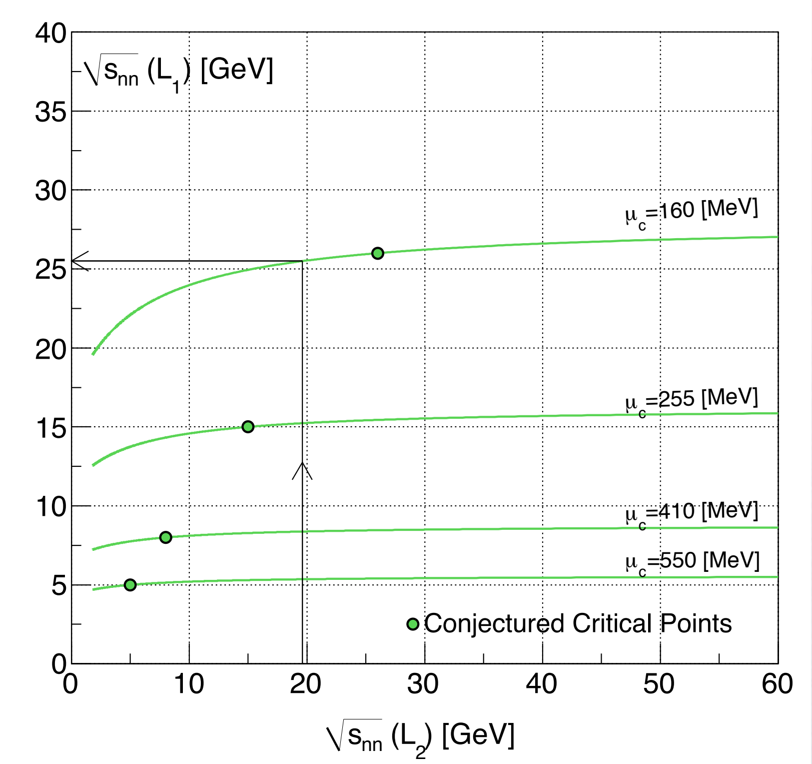




[data: Adams *et al* (2007)]

✓ Data in general not compatible with scaling, as expected.

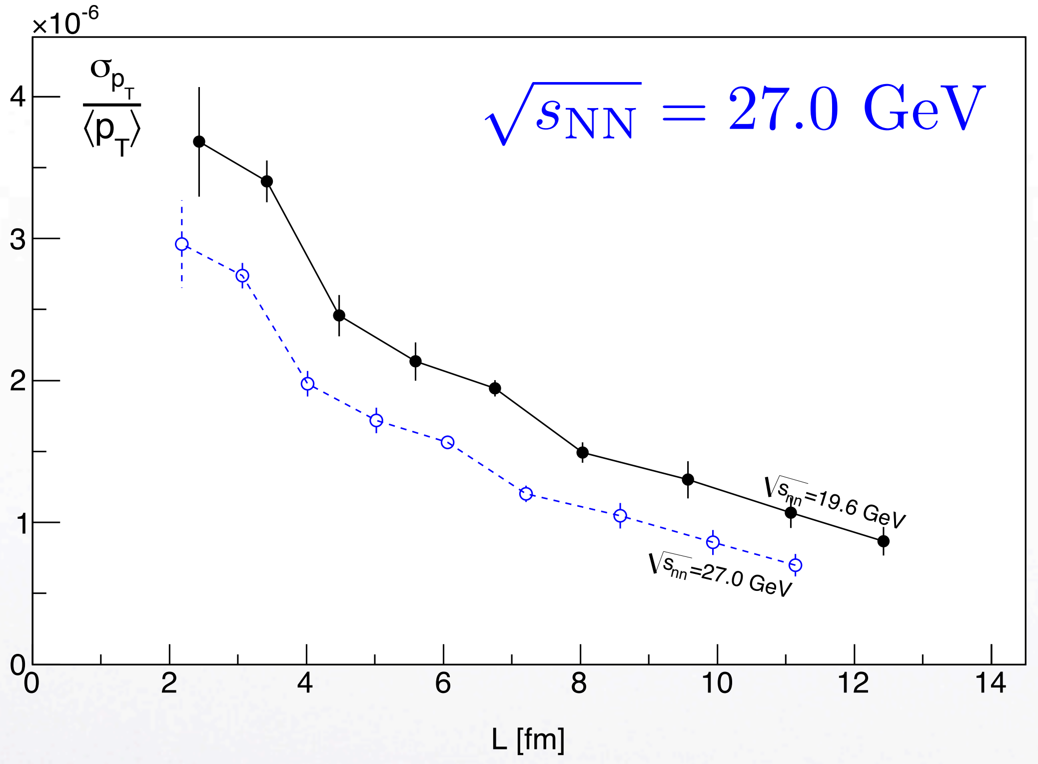
Limitation of the range of comparison: accessible system sizes determined by centrality dependence (up to a maximum factor 4)

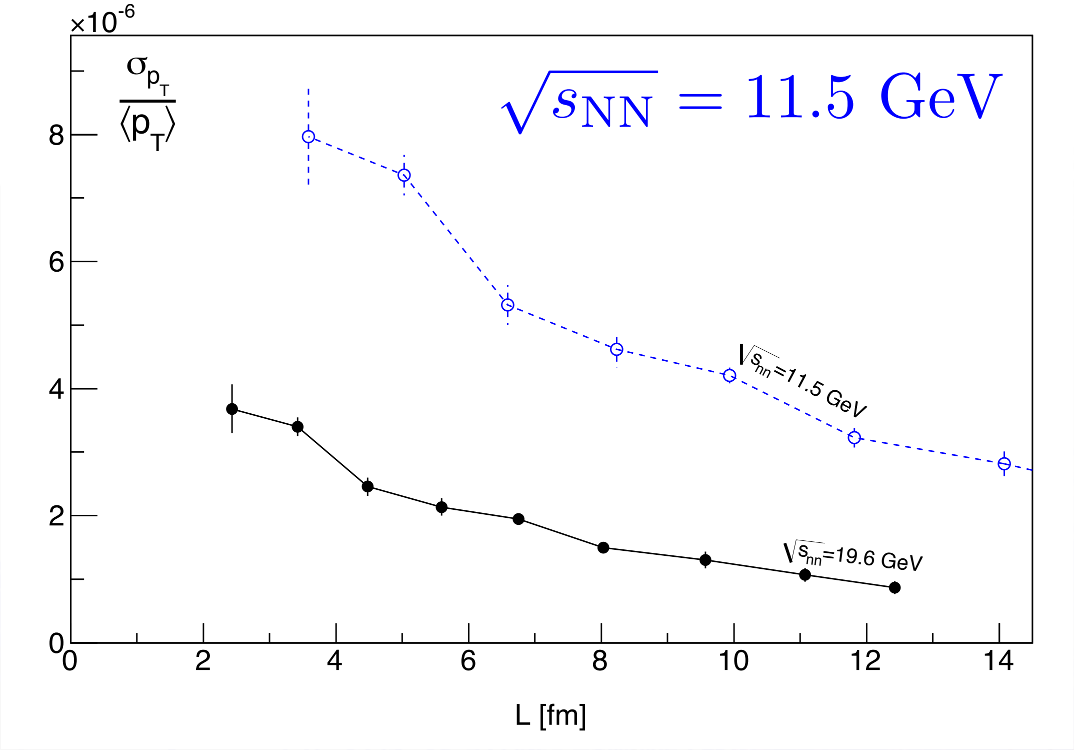


FSS analysis of HIC data

[LFP, Fraga & Sorensen, in prep.]

- FSS **predictions** for different energies based on STAR data for 19.6 GeV:





Final comments

- Finite size effects should be non negligible in heavy ion experiments

- Most thermodynamic quantities can be considerably shifted in the pseudocritical diagram that is actually probed ⇒ the phase diagram probed in HICs may be very different from the one in the thermodynamic limit.

- FSS techniques are simple and well defined in the case of heavy ion collisions. Even if it is hard to define the ideal scaling variable, it provides a pragmatic method to search for the critical endpoint and universality properties of QCD.

- Pragmatic methods are needed for the identification of the presumed critical point and 1st order line in the analysis of data coming from the Beam Energy Scan at RHIC-BNL and future data from ALICE-LHC.

- FSS may be a tool for analysis <u>and</u> predictions of the behavior of future data. If there is criticality, it <u>must</u> be there and can provide a clear connection to the results in the thermodynamic limit.

Thank you!

BACKUP SLIDES

BACKUP SLIDES

[Finite Size]

Big Bang vs. Little Bang

Using a simple approximation for the EoS, we can estimate the typical sizes:

$$3p \approx \epsilon \approx \frac{\pi^2}{30} N(T) T^4$$

Early universe (Big Bang):

The radius of the universe, as given by the particle horizon in a Robertson-Walker spacetime, where the scale factor grows as $a(t) \sim t^n$, is given by ($n=1/2$, $N \sim 50$ for QCD)

$$L_{univ}(T) \approx \frac{1}{4\pi} \left(\frac{1}{1-n} \right) \left(\frac{45}{\pi N(T)} \right)^{1/2} \frac{M_{Pl}}{T^2} = \frac{1.45 \times 10^{18}}{(T/\mathrm{GeV})^2 \sqrt{N}} \ \mathrm{fm}$$

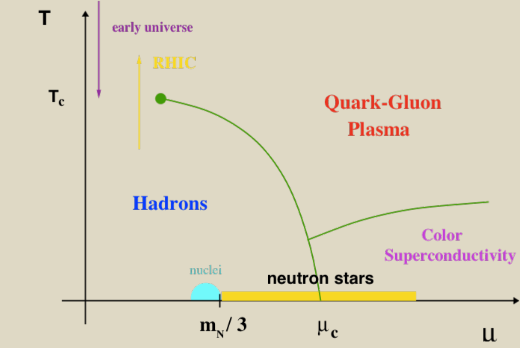
The system is essentially in the thermodynamic limit!

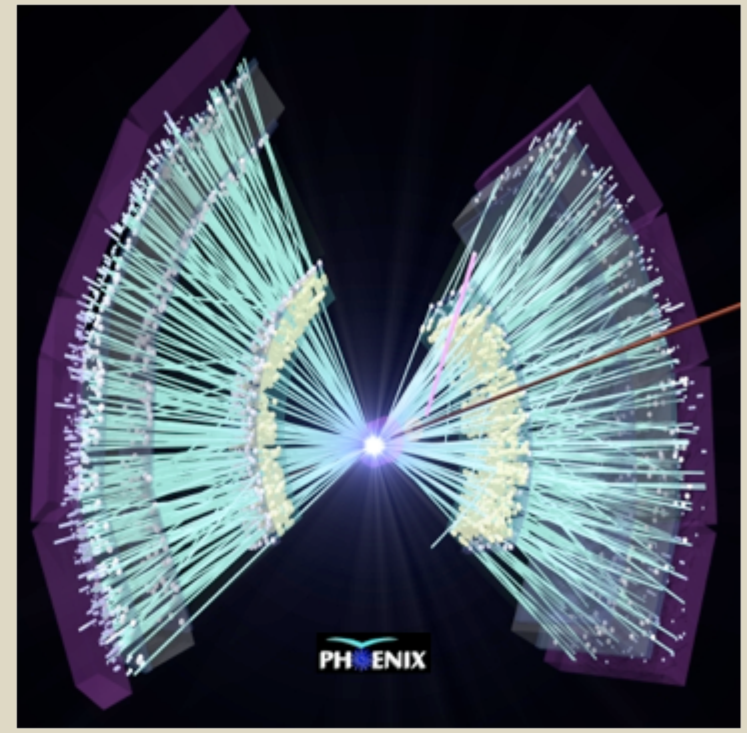
Heavy ion collisions (Little Bang): $L_{QGP} \leq 10 - 15 \ \mathrm{fm}$

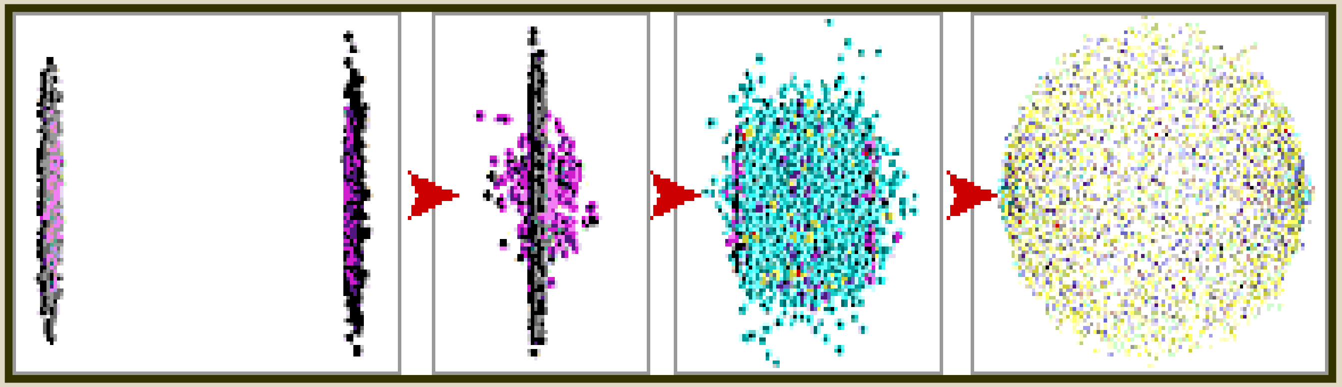
The system can be very small!

Huge differences in (time and length scales) between Big and Little Bangs…

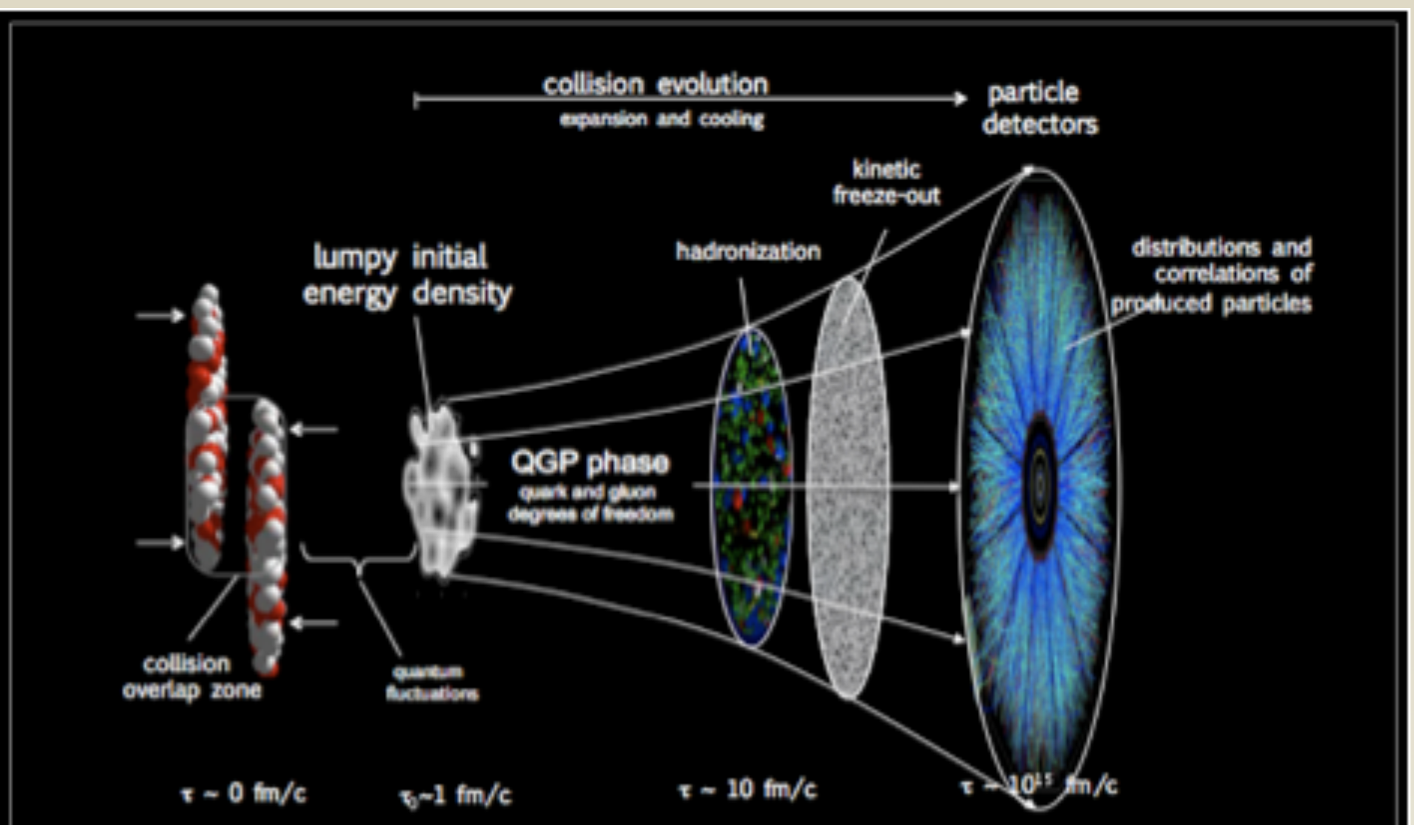
Real life: mapping the phase diagram with heavy ion experiments







[RHIC]



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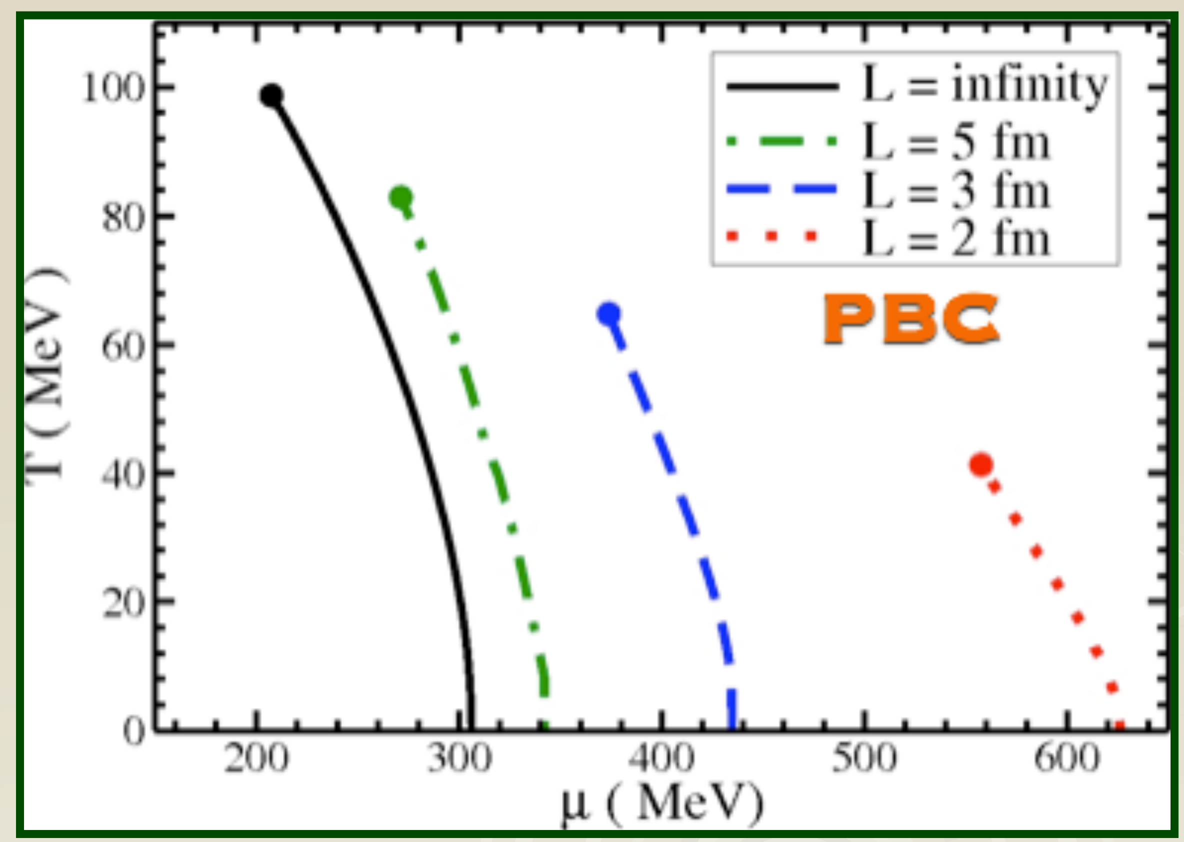
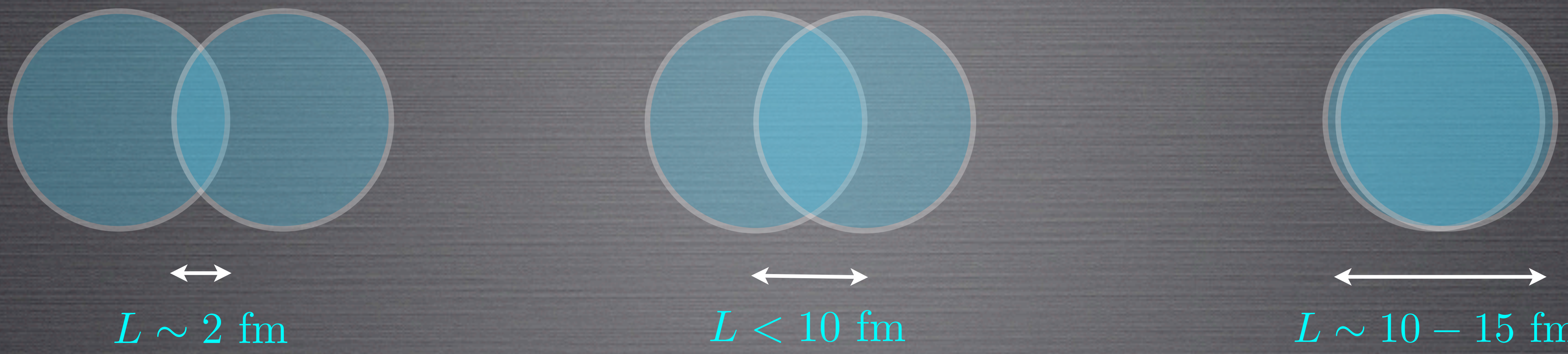


[Palhares, ESF & Kodama (2009)]

The system created in HIC's is **FINITE**, and its size is **CENTRALITY-DEPENDENT**:

$L(N_{\rm part})$



How can this feature influence/affect possible signatures of the CEP?

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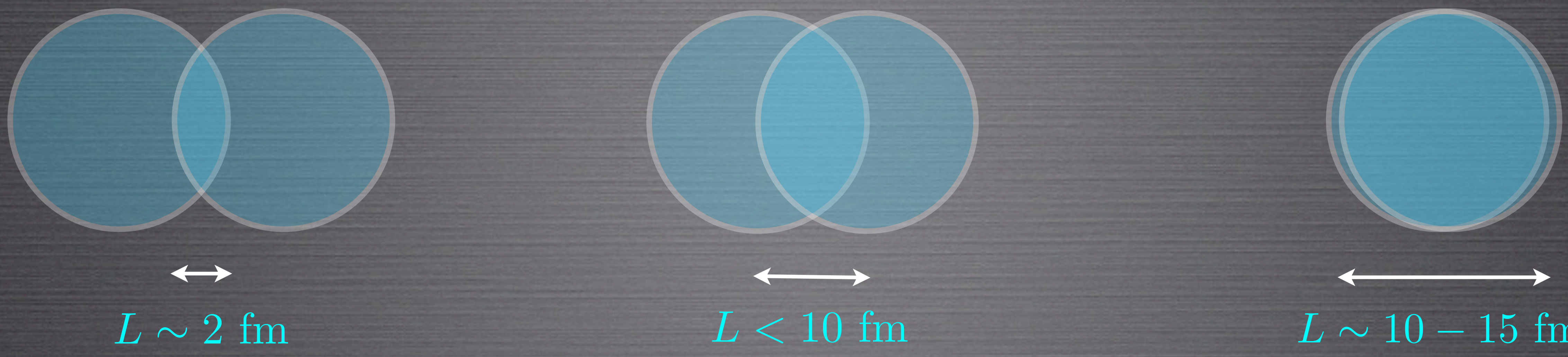

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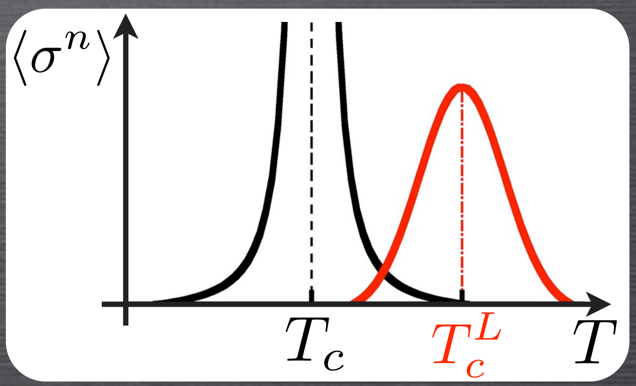
(1) Most signatures will probe pseudocritical quantities, with **smoothened divergences** and **shifted peaks**:

$$\langle \sigma^n \rangle_L \sim \xi^{p_n} \, f_n(\xi/L) \quad \Rightarrow$$



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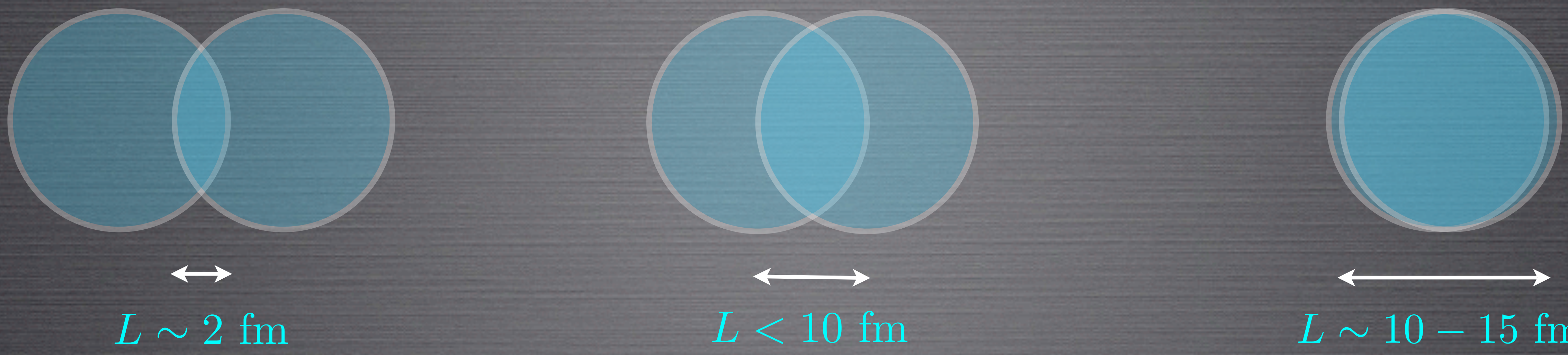

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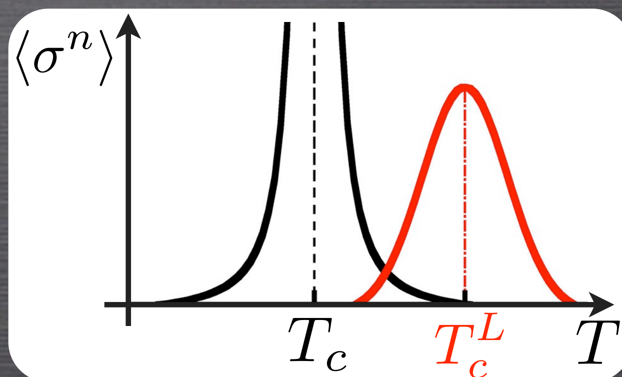


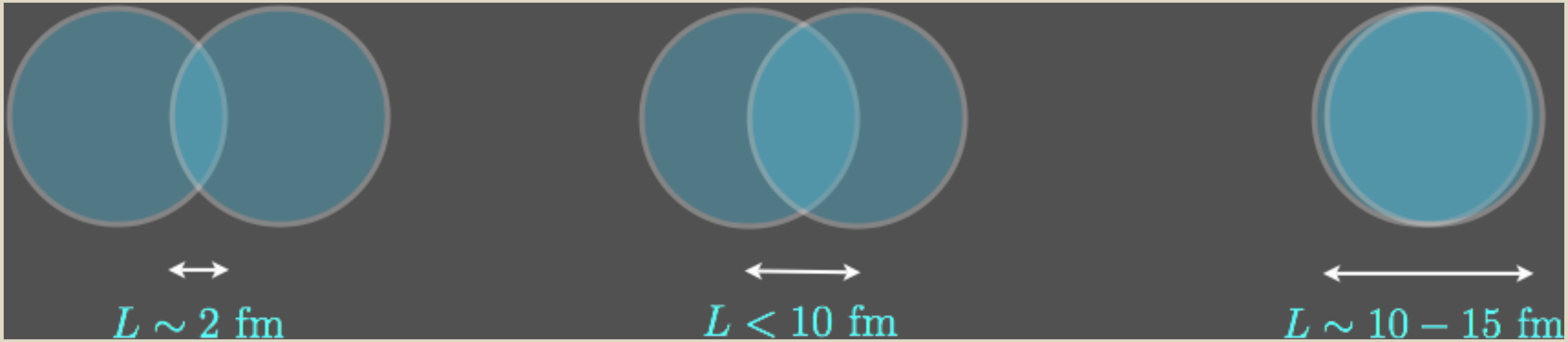
Investigate the importance of the shifts within a chiral model for typical HIC size scales and the consequences for the related signatures.

(2) HIC data as an **ensemble of systems of different sizes.**
Finite-size analysis as a tool to obtain info about the true phase transition, in the thermodynamic limit: location of the CEP and its universality class.

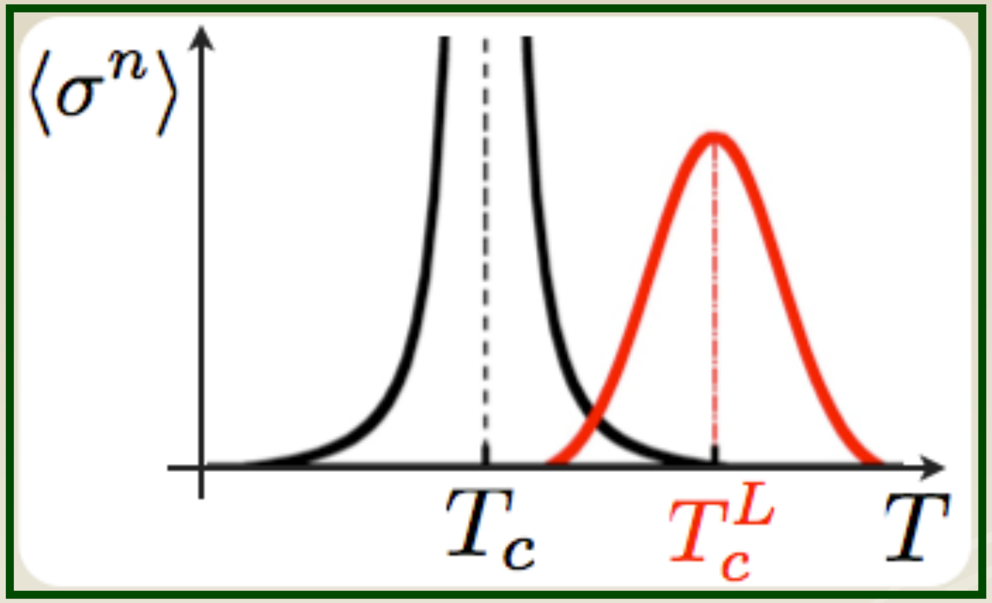
1. Finite size modifications of the phase diagram

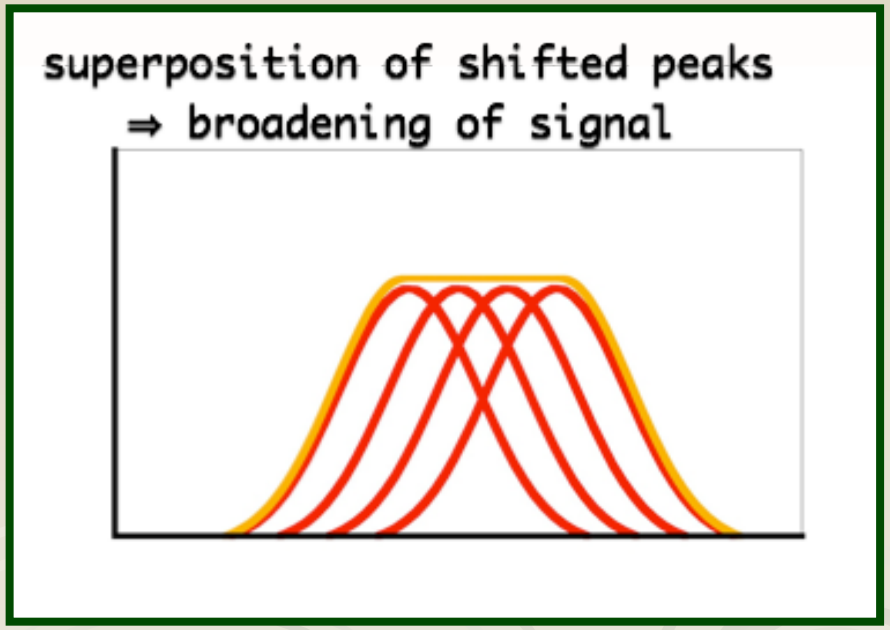
[Palhares, ESF & Kodama (2009)]

- In heavy ion collisions the system size will depend on centrality:



- Measurements will generally probe pseudocritical, smoothened, shifted thermodynamic quantities. Ex. – cumulants:





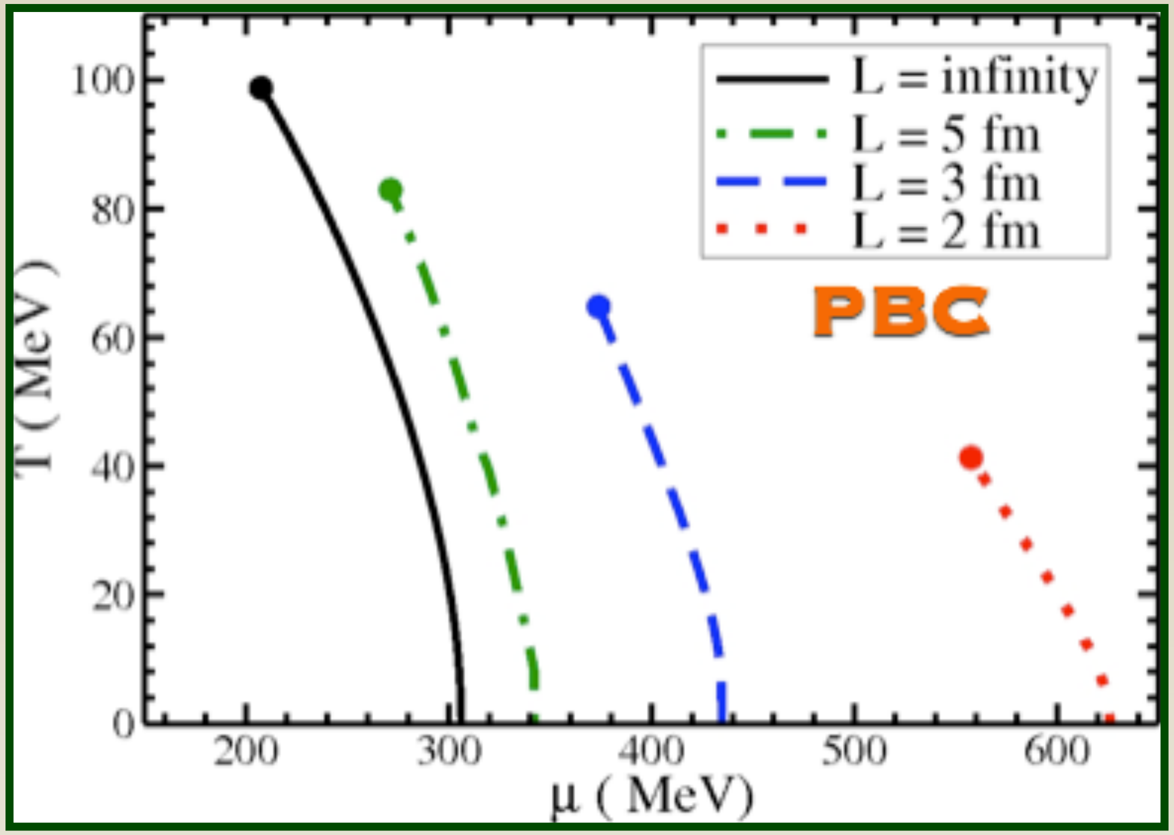
- Most (≈all) signatures based on non-monotonic behavior of observables [e.g.: Stephanov (2009)].

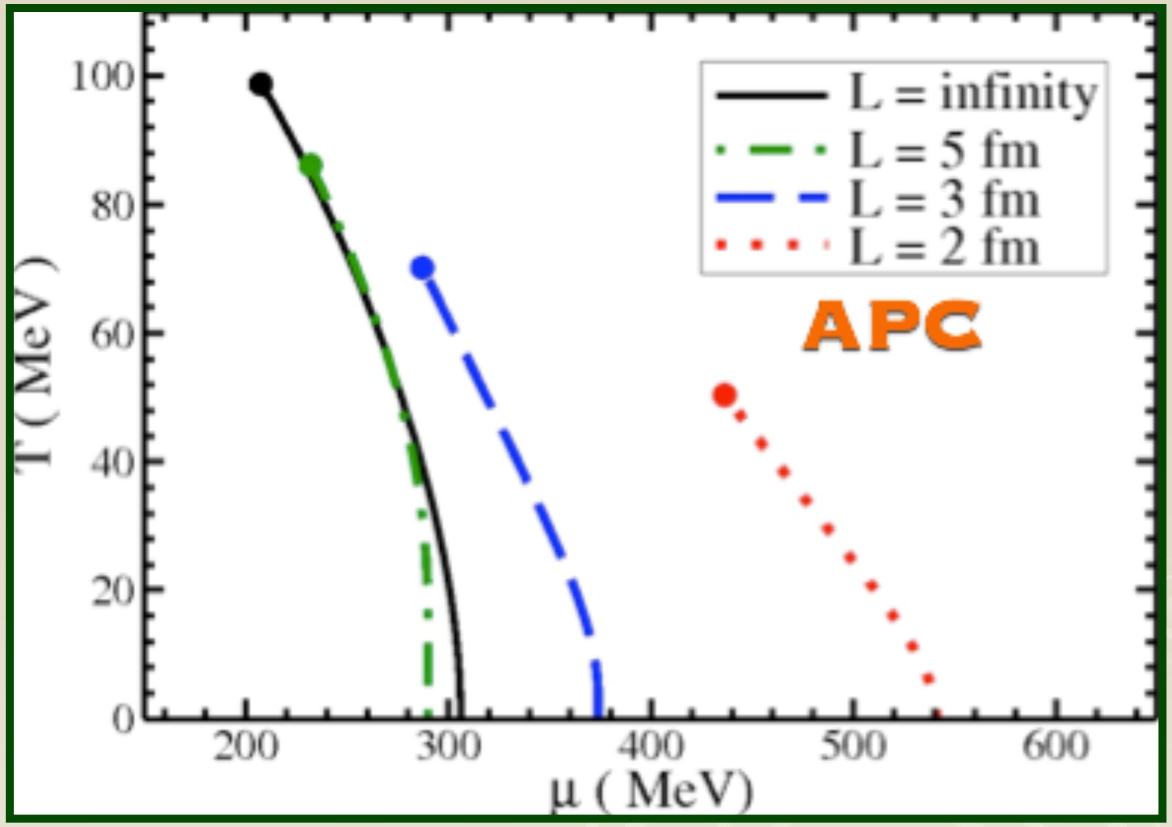
- partially hidden by background. shifts and smoothening.

• Therefore, observables and signatures will be related to a pseudocritical, modified phase diagram. How large are the modifications?

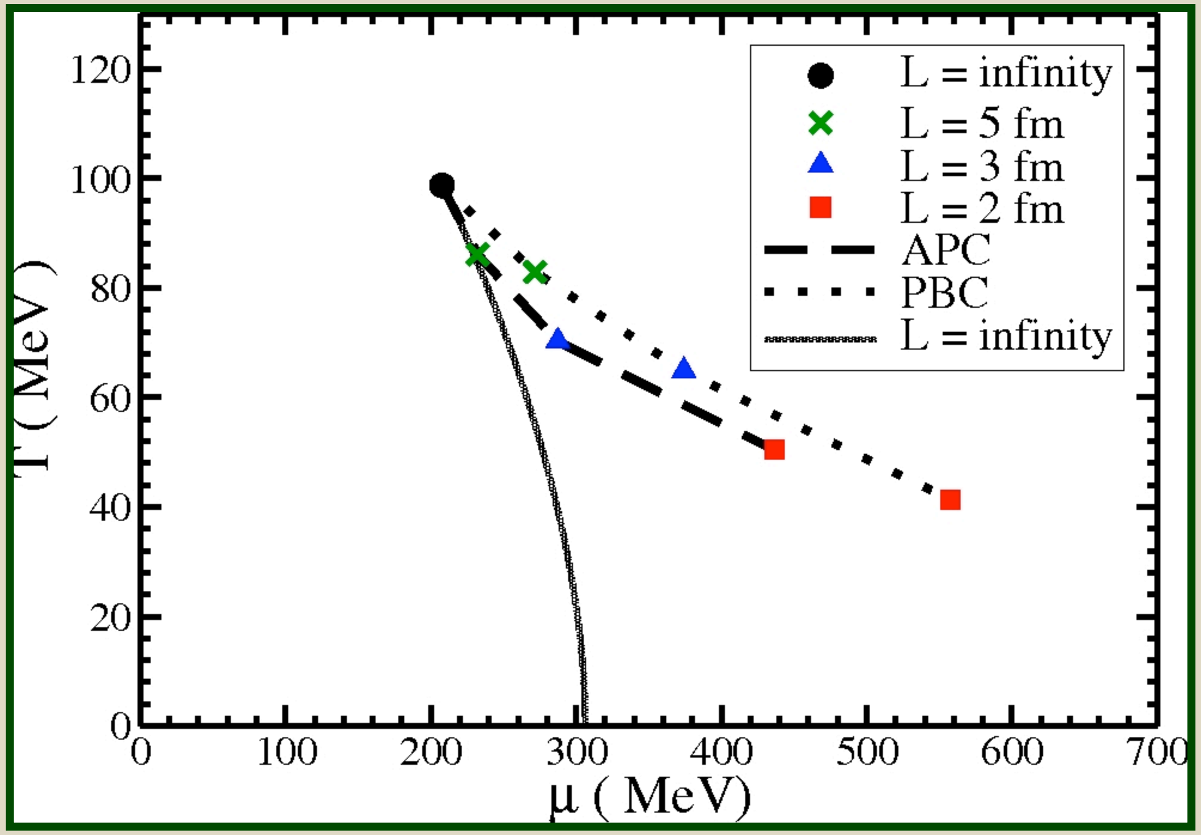
• In the linear sigma model with quarks, one can compute all thermodynamic quantities and provide an estimate of the amplitude of "shifting and rounding" due to finite size corrections [Palhares, ESF & Kodama (2009)]

phase diagram



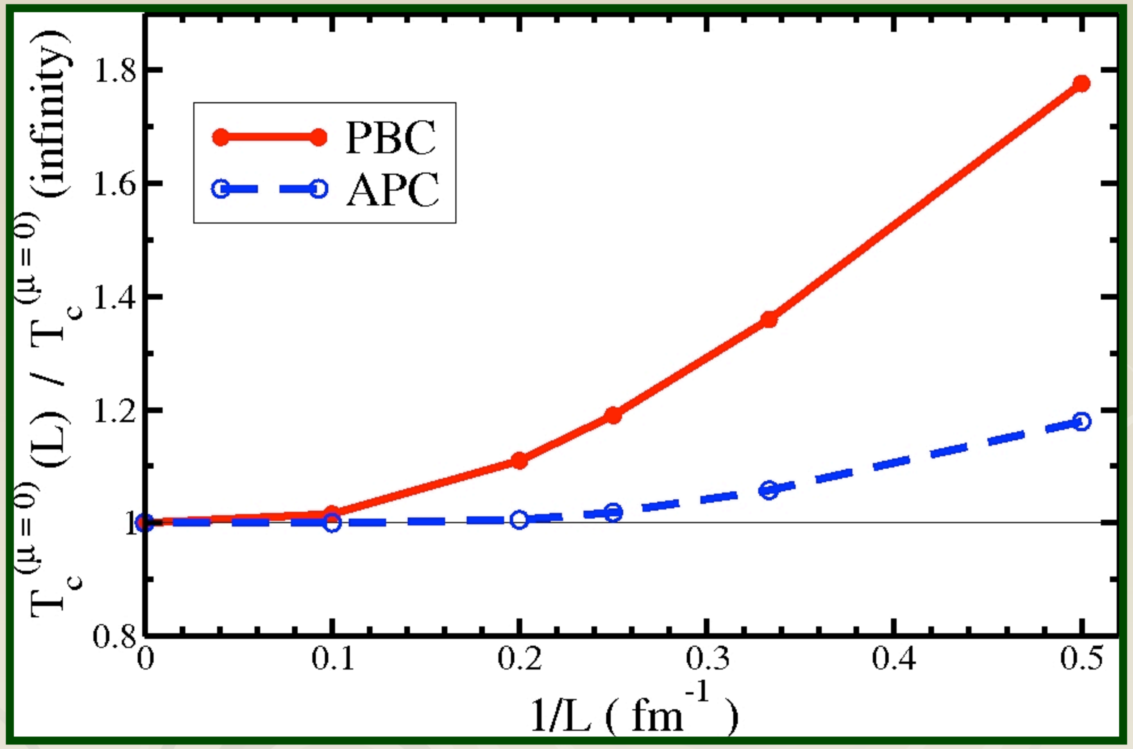


critical endpoint



- Large corrections to the position of the CEP, especially in the μ direction.
- Sensitivity to boundary conditions.
- Critical line shifted appreciably in the range of sizes probed in current experiments.

crossover temperature

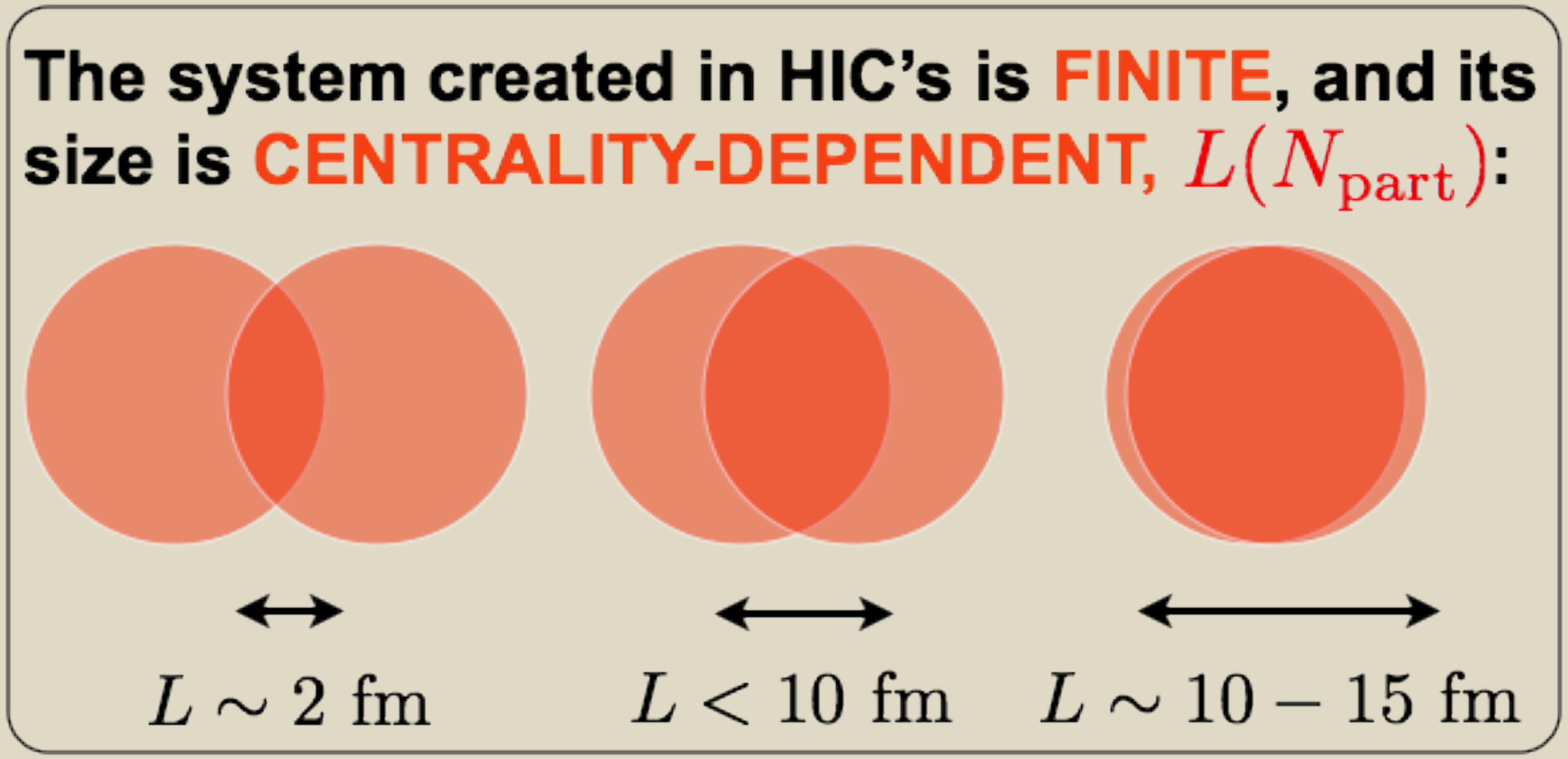


- L ≈ 10 fm is already large, so that the plasma generated in central collisions is essentially in the thermodynamic limit.
- Non-monotonic behavior (signal for the CEP) will be smoothened by finite size effects and by the short lifetime of the system (even more constraining).

2. Scaling plots and the critical endpoint of QCD

[Palhares, ESF & Kodama (2009/2010)]

• Possible bright side of the peculiar finiteness of the system in heavy ion collisions: data comes as an ensemble of systems of different sizes !



• Data can be arranged analogous to realizations in lattices of different sizes...

Finite Size Scaling comes naturally as a possible tool!
(use of scaling plots to search for the CEP)

[Palhares, ESF & Kodama (2009/2010)]

- **In the vicinity of the CEP (criticality):**

✓ the correlation length becomes divergent -> system is scale invariant;

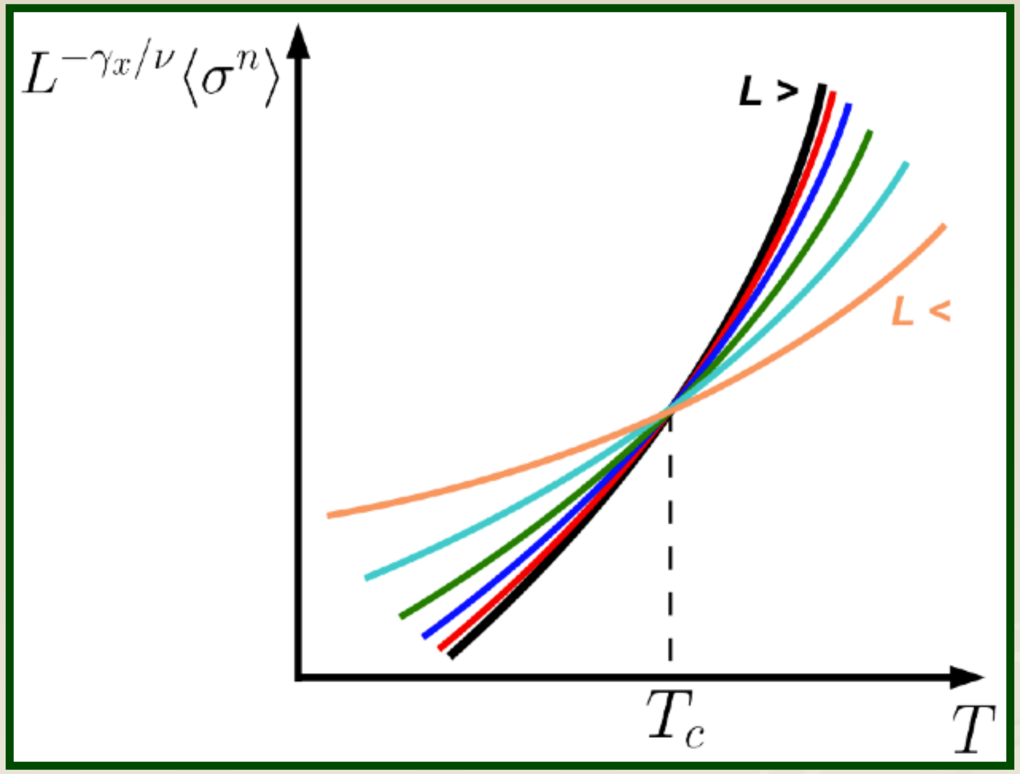
$$\xi_\infty \sim t^{-\nu} \quad ; \quad t \equiv \frac{T - T_{\rm CEP}}{T_{\rm CEP}}$$

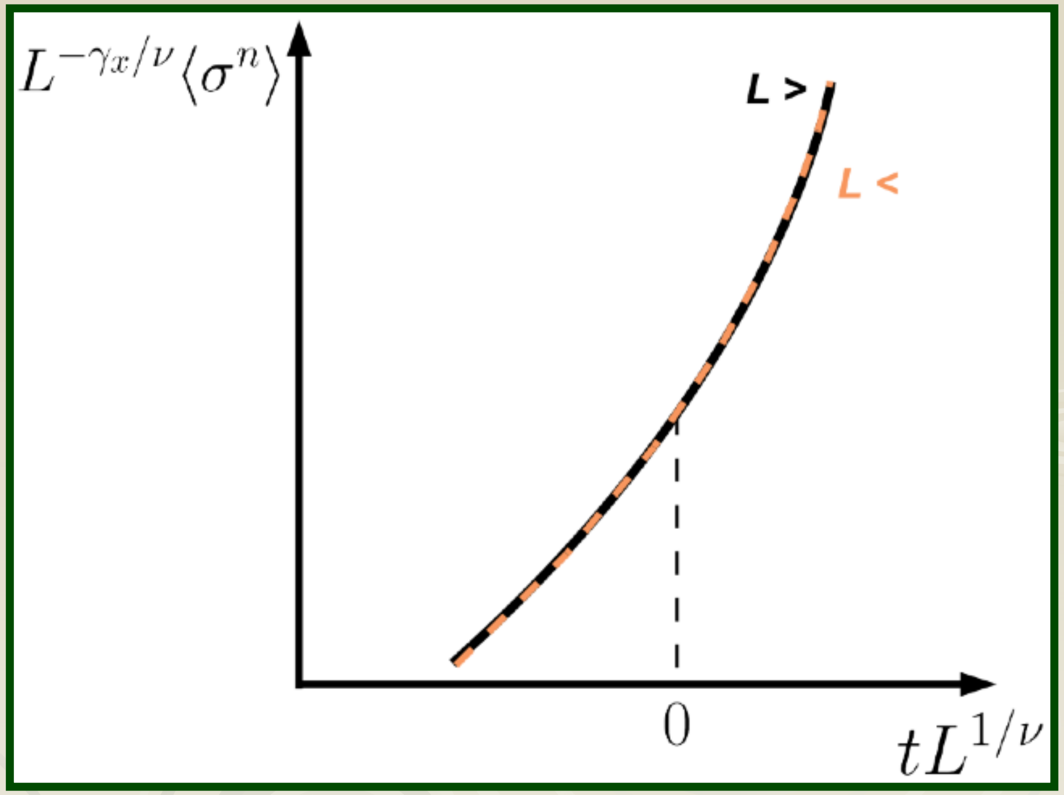
✓ FSS applies as can be demonstrated by a RG analysis;

✓ all lines should collapse in a full scaling plot, i.e., for any correlation function of the order parameter

$$X(t, L) = L^{\gamma_X/\nu} f_X(tL^{1/\nu})$$

Ex.: cumulant scaling plots





3. Comparing to heavy ion data

[ESF, Palhares & Sorensen (in prep.)]

- **Variables:**

✓ Size of the system (L): difference defined by centrality of collisions; measured by HBT analysis (interferometry).

✓ Distance to the CEP ("t"): constrained by freeze-out curve, parametrized either by μ or by center-of-mass energy $\sqrt{s_{nn}}$

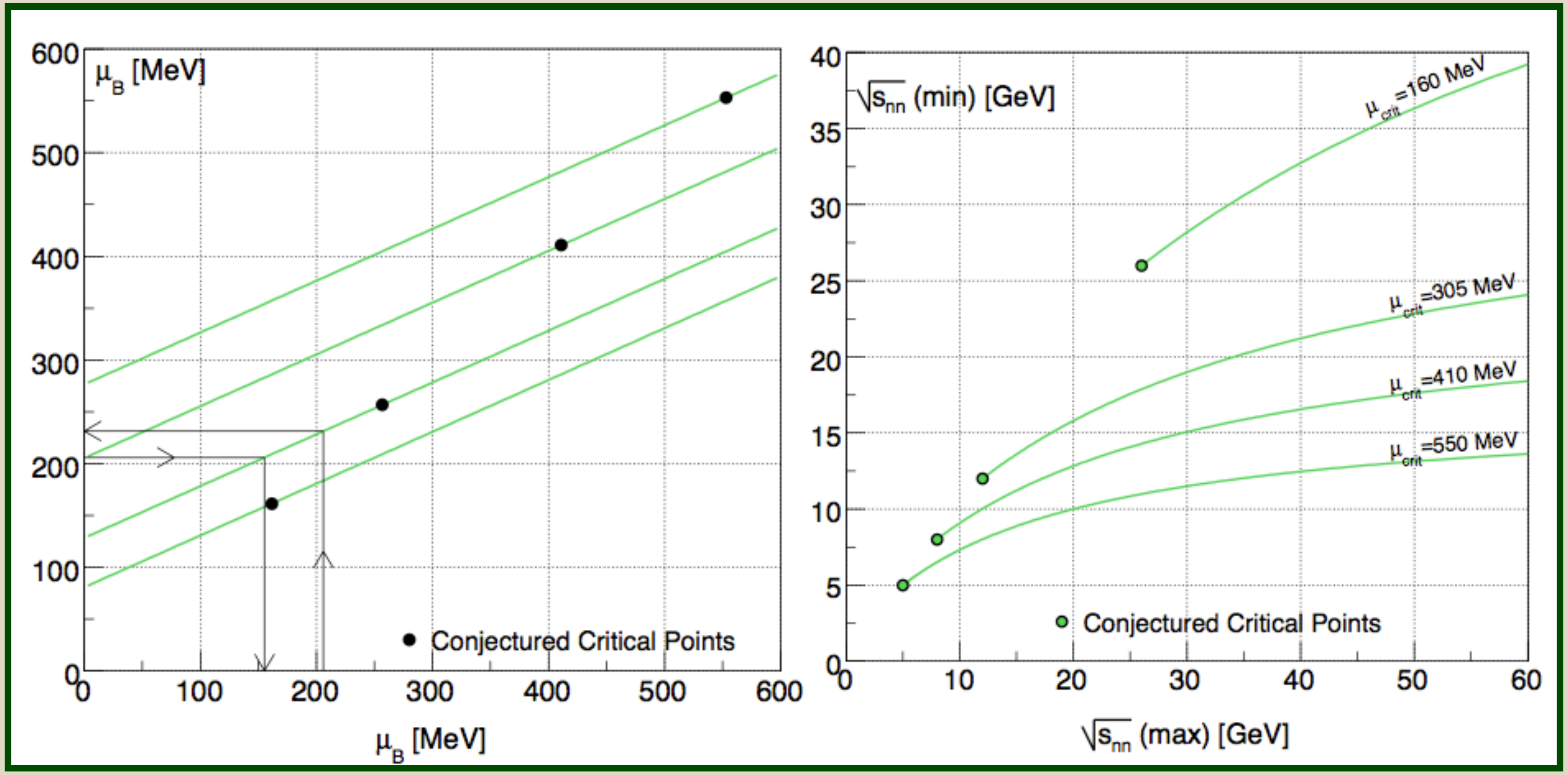
✓ Observables (X): transverse momentum fluctuations, pion multiplicity fluctuations (soft pions), etc.

✓ Basic recipe: for different centralities, plot

$$X(t, L) = L^{\gamma_X/\nu} f_X(tL^{1/\nu})$$

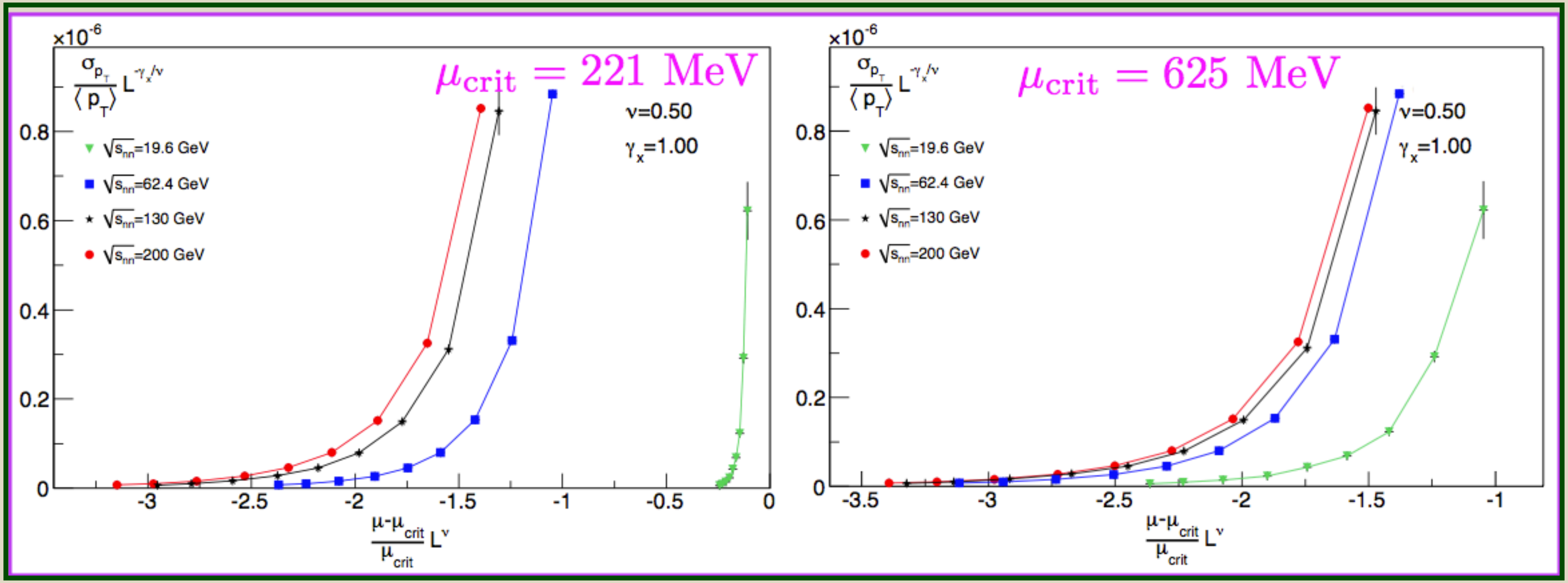
looking for scaling. **In the vicinity of the CEP, one must have FSS. -> preliminary results!**

• Range of applicability:



✓ The range of sizes accessed in the collisions restricts the range one can test FSS: largest system ≈ 15fm ; variation ≈ factor of 4.

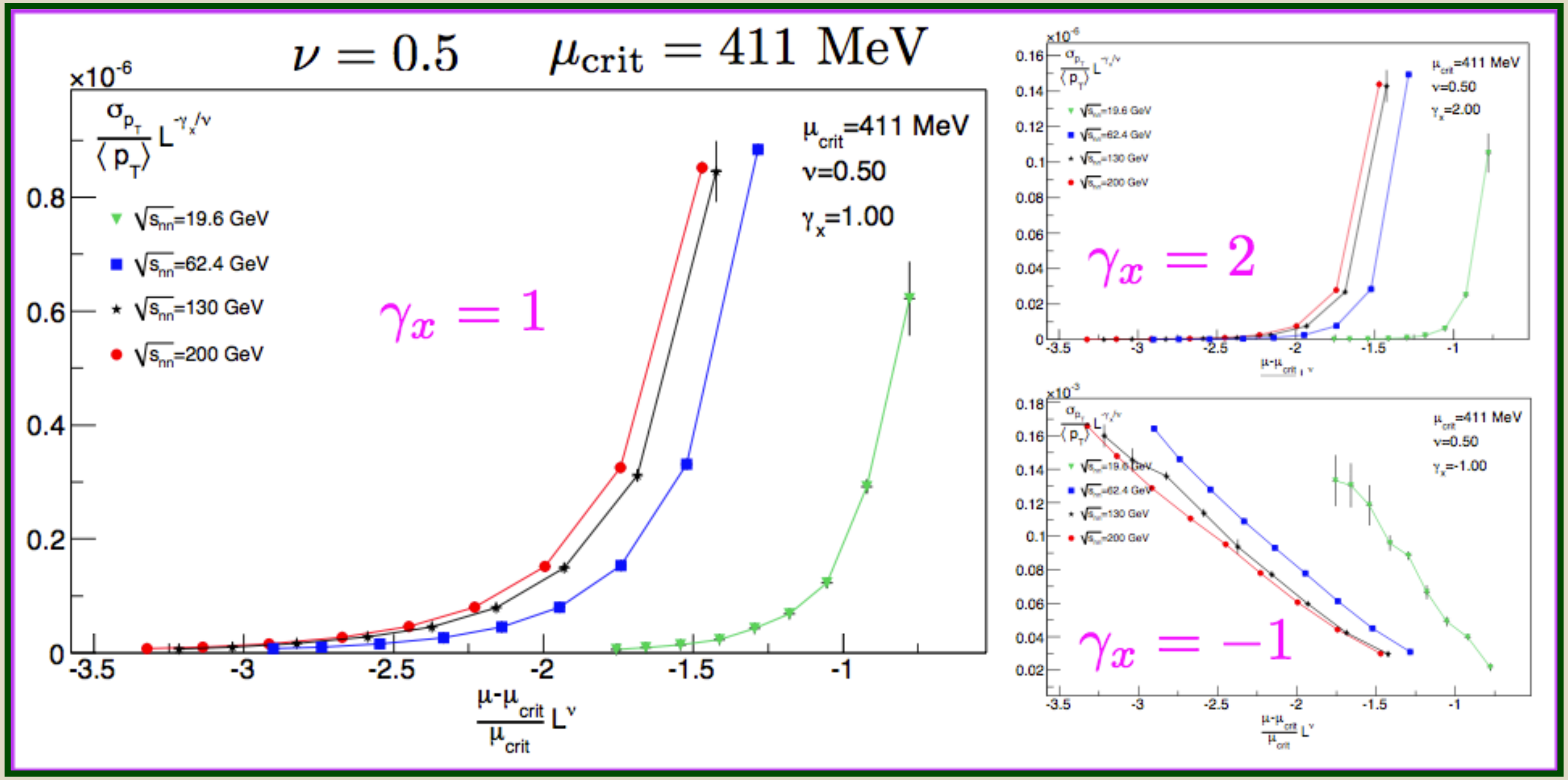
✓ Illustration of values that can be compared for different conjectured CEP locations (ν=1/2 & variation ≈ factor of 4).



✓ Data seems to favor larger values of the critical endpoint chemical potential. In line with expectations from lattice simulations (even if these are rather exploratory yet).

✓ Predicted fluctuations for lower energies (SPS) much larger than expected (CERN-SPS data). To be compared to the Beam Energy Scan data at RHIC.

- Transverse momentum fluctuations from RHIC-BNL and SPS-CERN



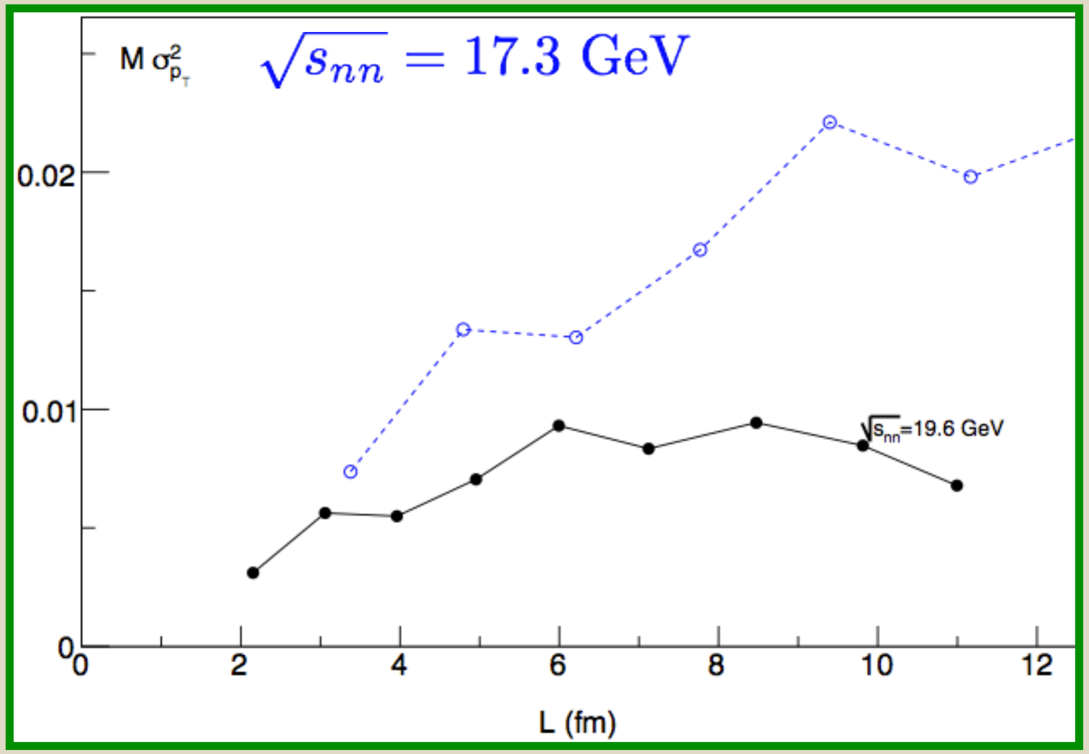
✓ Data in general not compatible with scaling expectation.

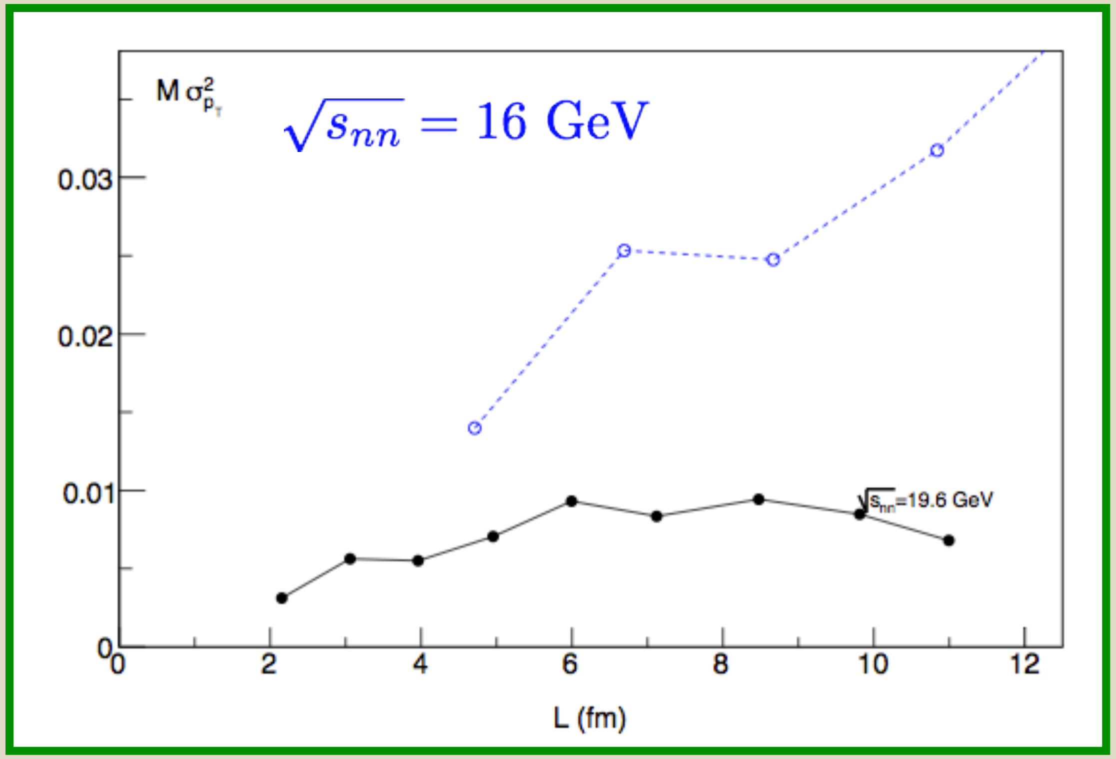
✓ Very sensitive to the dimension exponent γ_x.

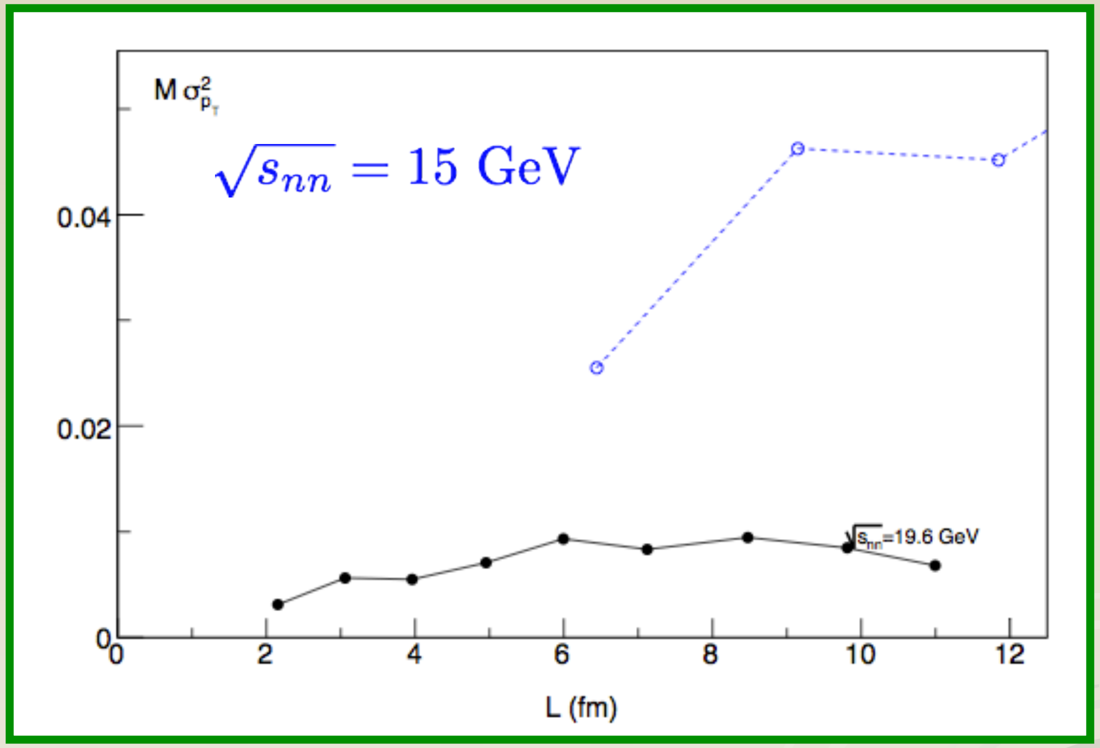
✓ Similar behavior for other observables.

• FSS <u>predictions</u> for different energies based on STAR data for 19.6 GeV

[ESF, Palhares & Sorensen (in prep.); data: Adams et al (2007)]







✓ Predicted fluctuations well above CERN-SPS data for the same range of energies.

✓ We see again the limitation on the range of comparison.

✓ The BES at RHIC will provide data for an interval containing the SPS data.

FSS as a tool for searching the CEP...

... in HIC's

- **HIC data as experimental realization of an ensemble of systems of different sizes/centralities** ⇒ we can investigate the size dependence of the observables

- Since **FSS** ⇔ **CEP**, then identifying **FSS** in the centrality dependence of HIC data

FSS: $X(t,L) = L^{\gamma_x/\nu} f_x(tL^{1/\nu})$ →
- locate CEP
- determine its critical exponents (universality class of QCD)

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FSS: $X(t,L) = L^{\gamma_x/\nu} f_x(tL^{1/\nu})$ ⟶ { locate CEP; determine its critical exponents (universality class of QCD) }

How can we look for FSS in HIC data? Verifying if data is compatible with the FSS relation:

FSS relation in the case of HIC

(we need observables related to t, L, etc only up to normalization consts.)

- distance t to the CEP: $t \mapsto (\sqrt{s} - \sqrt{s}_c)/\sqrt{s}_c$
- size L of the system: $L \mapsto N_{\rm part}^{1/2}$
- Correlation function X of the order parameter: $\longmapsto$ correlations of soft pion fluctuations, e.g.

$$X \; N_{\rm part}^{-\gamma_x/2\nu} = f_x\left(y_{\rm scl}\right)$$

scaling variable: $y_{\rm scl} = \frac{\sqrt{s} - \sqrt{s}_c}{\sqrt{s}_c} N_{\rm part}^{1/2\nu}$

Scaling plots in HIC's

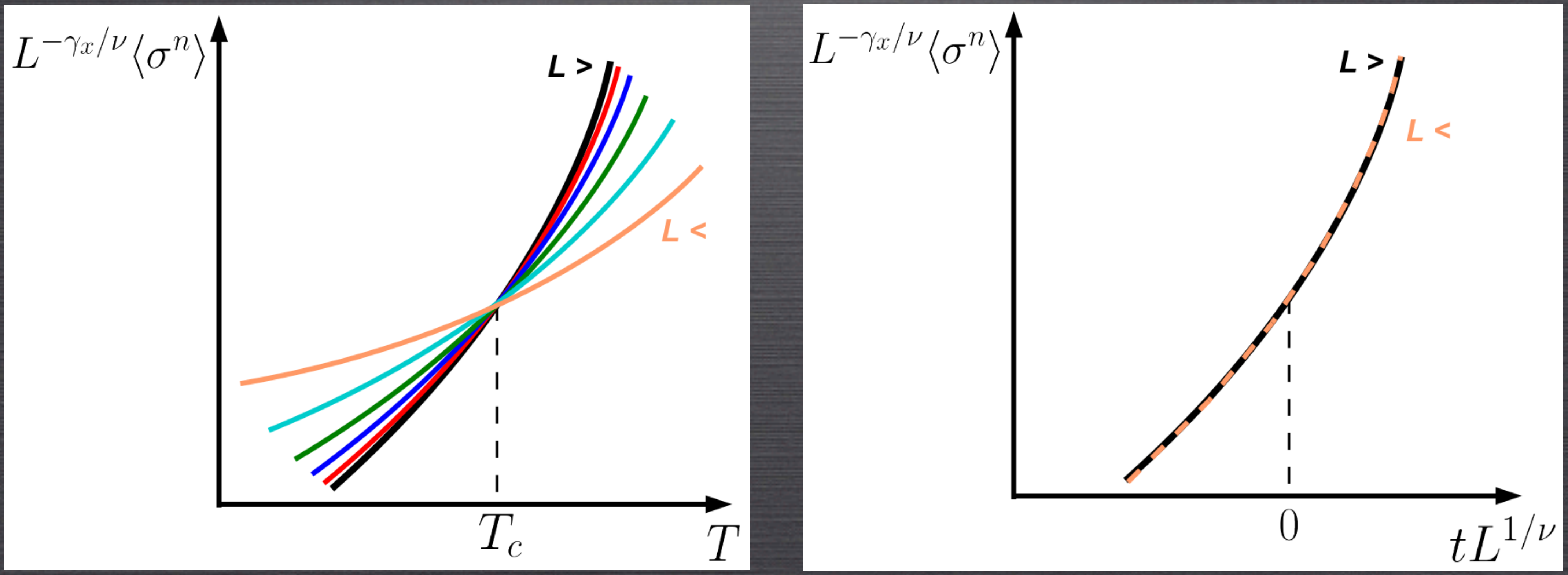
[LFP, Fraga, Kodama (2009)]

- Necessary **and** sufficient condition for FSS (and thus for the presence of the CEP)

- Should be valid in a **larger vicinity** of the CEP and could be tested even if there is data only above **or** below the CEP

PROCEDURE: Search for γ_x, ν, $\sqrt{s_c}$ which **collapse** data from **different centralities** in the associated scaling plot ($X N_{\mathrm{part}}^{-\gamma_x/2\nu} \times y_{\mathrm{scl}}$)

Illustration:



Back-up Slides

[FS-CPOD09+SQM09]

Searching the Critical Endpoint in HIC's

Some of the most popular **signatures** of the **Critical Endpoint (CEP; 2nd order transition;** $\xi \to \infty$ **)** are based on the expected divergent behavior of the correlation functions of the quasi-particle σ, related to the order parameter of the chiral transition $\langle\bar{\psi}\psi\rangle$:

[Stephanov, Rajagopal, Shuryak (98,99); Berdnikov, Rajagopal (2000);Stephanov(2009)]

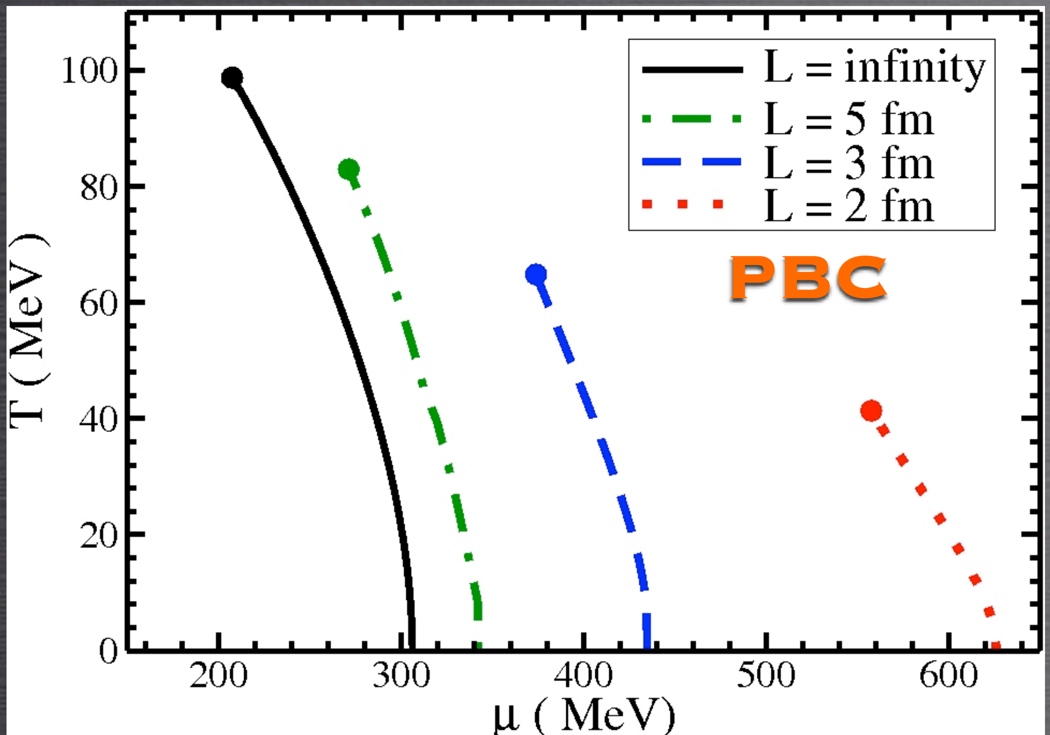
$$\boxed{\langle\sigma^n\rangle \sim \xi^{p_n}}$$

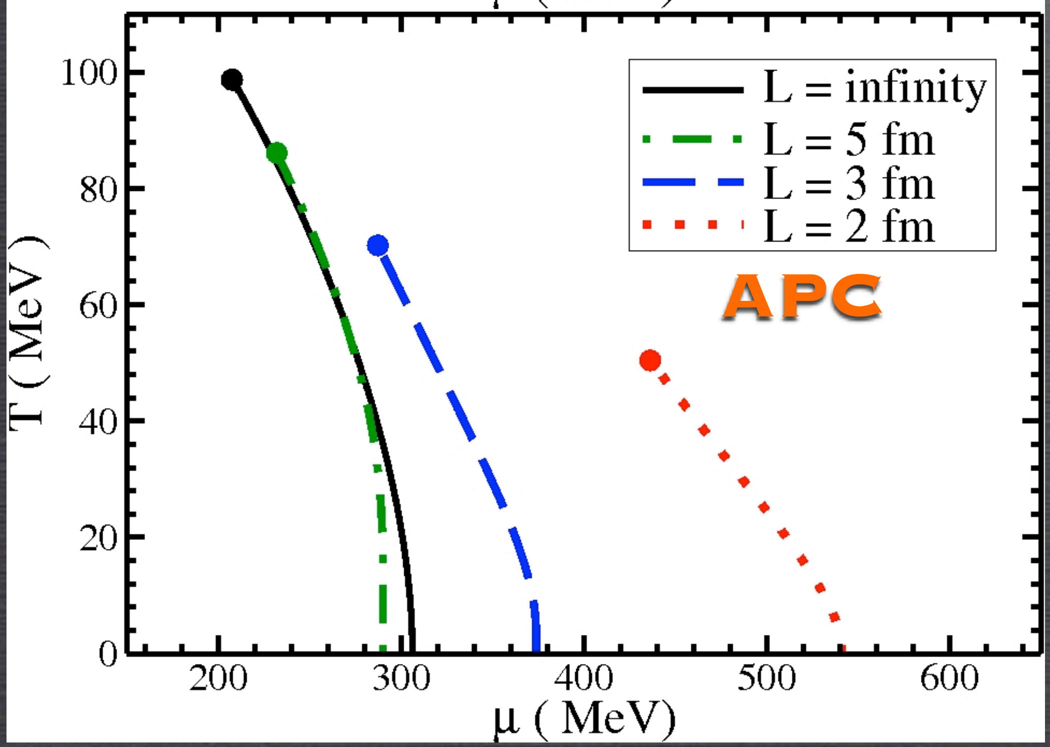
As is well-known, in any real system, the correlation length ξ is always finite, and a nonmonotonic behavior is expected instead.

This feature should be translated into the final observable spectra in HIC via mesonic decays of the sigma field into other particles, especially soft pions (created as soon as the medium-dependent sigma reaches the mass threshold).

The Pseudocritical Phase Diagram of the chiral transition

[LFP, Fraga, Kodama (2009)]



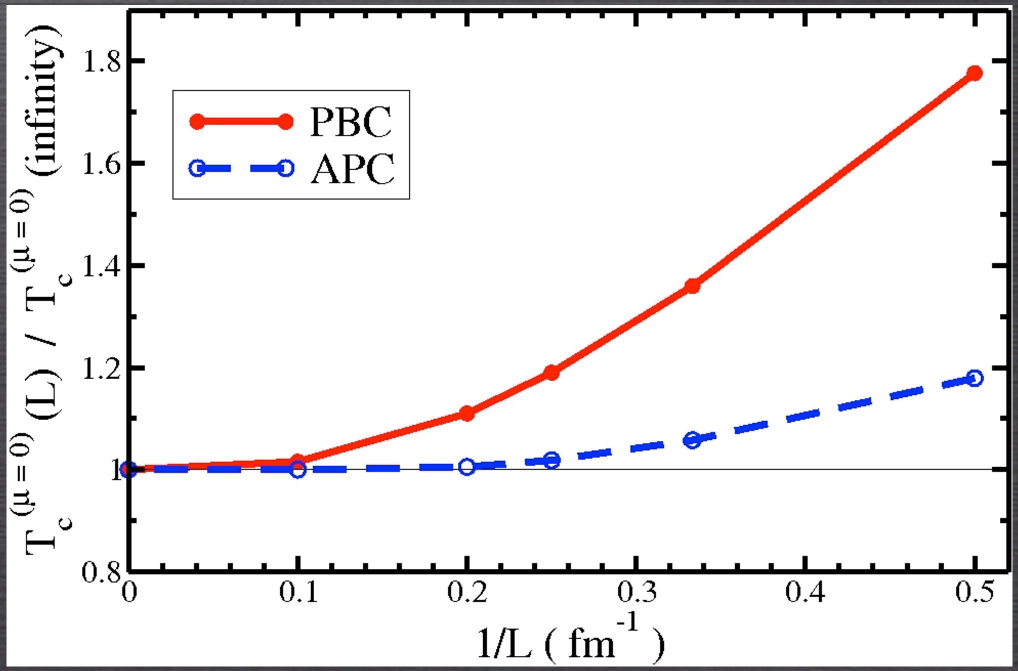


- Large effects for size ranges relevant to current HIC's

- Transition line and CEP significantly shifted to large $\mu \Rightarrow$ outside experimental reach?

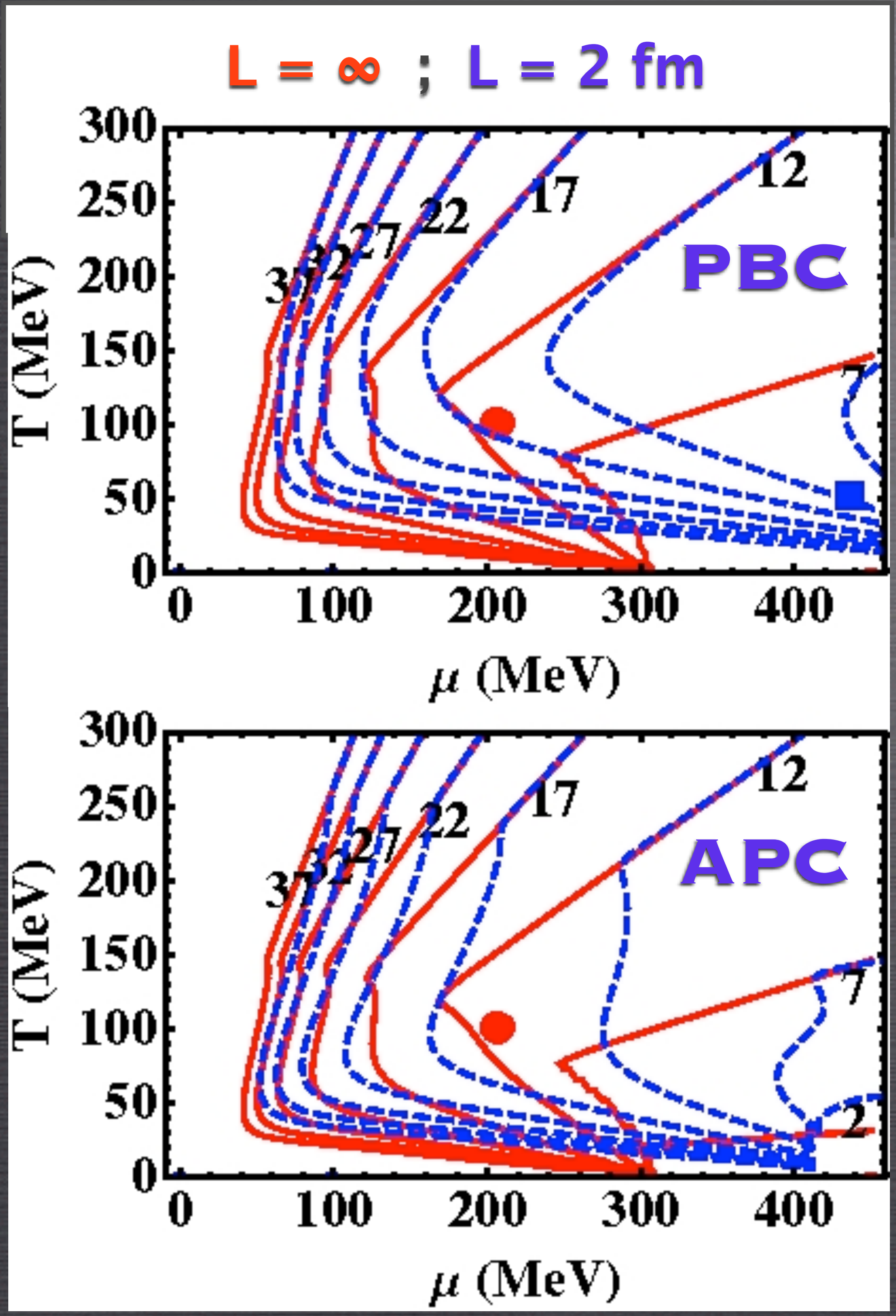
- Strong boundary-condition dependence

Crossover Temperature at zero chemical potential



Isentropic Trajectories at finite volume

[LFP, Fraga, Kodama (2009)]



- large effects in the critical region.
- analysis based on (nearly) isentropic hydro evolutions could be greatly affected.

FSS as a tool for searching the CEP...

CEP $\Rightarrow$ 2nd order phase transition $\rightarrow$
- divergent correlation length
- scale invariance on the criticality

These features imply the existence of Finite-size Scaling for finite systems in the vicinity of the CEP (rigorous proof through RG analysis):

$$X(t,L) \quad = \quad L^{\gamma_x/\nu} f_x(tL^{1/\nu})$$

$t = (T - T_c)/T_c$ (distance to the genuine CEP)

X $\Rightarrow$ (any) correlation function of the order parameter

ν $\Rightarrow$ universal critical exponent (div. of corr. length)

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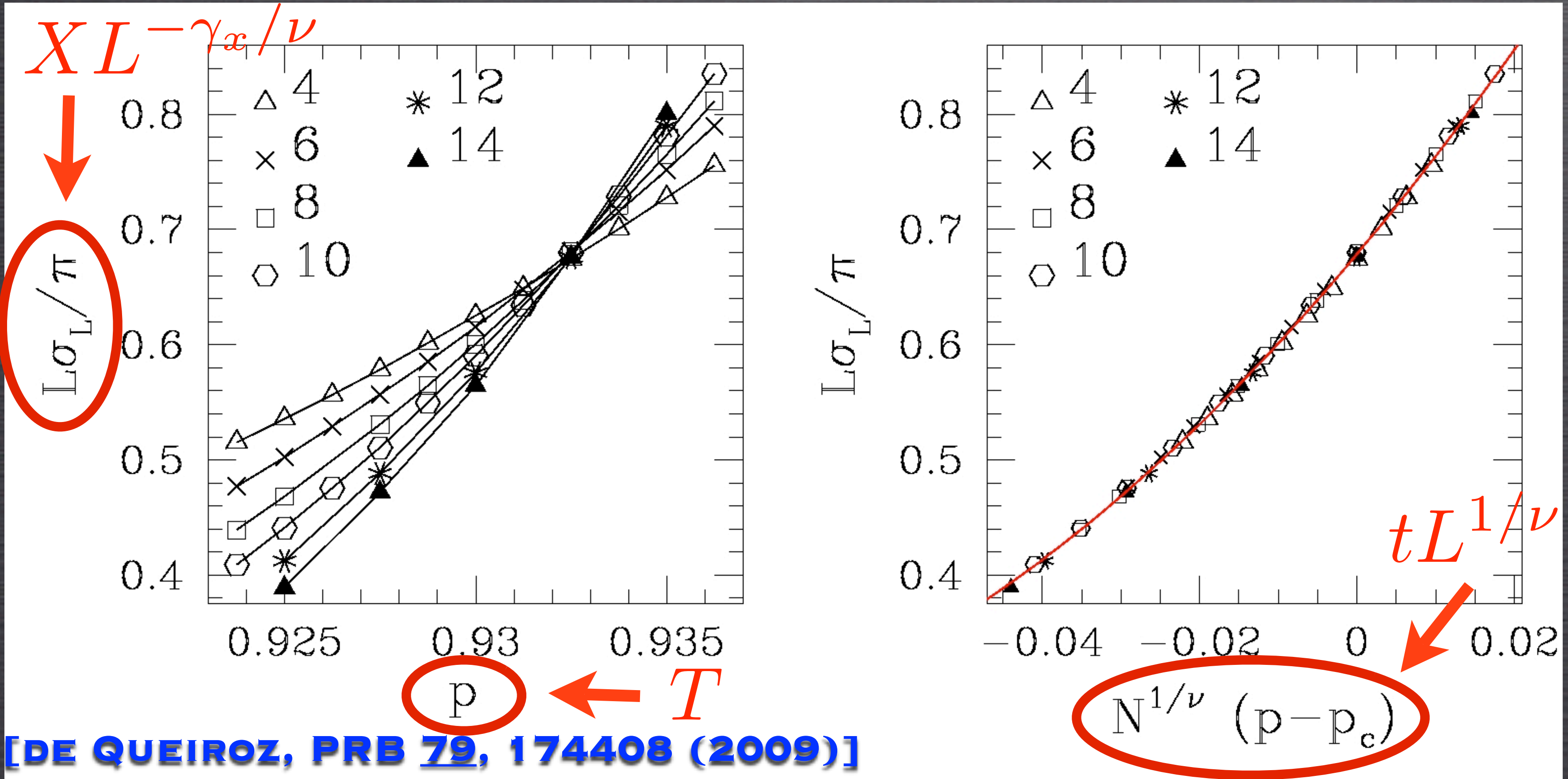
Scaling plots in Condensed Matter: Spin Glass transitions in disordered Ising systems



[de Queiroz, PRB 79, 174408 (2009)]

Tool for determining T_c and critical exponents (universality class)

NOTE: signature present even in observables that show no nonmonotonic behavior

Scaling plots in HIC's

FSS analysis can be implemented **ON** the CEP (t = 0) **[Lizhu, Chen, Yuanfang (2009); see talk by Y. Wu]** **OR VIA**

FULL SCALING PLOTS [LFP, Fraga, Kodama (2009)]

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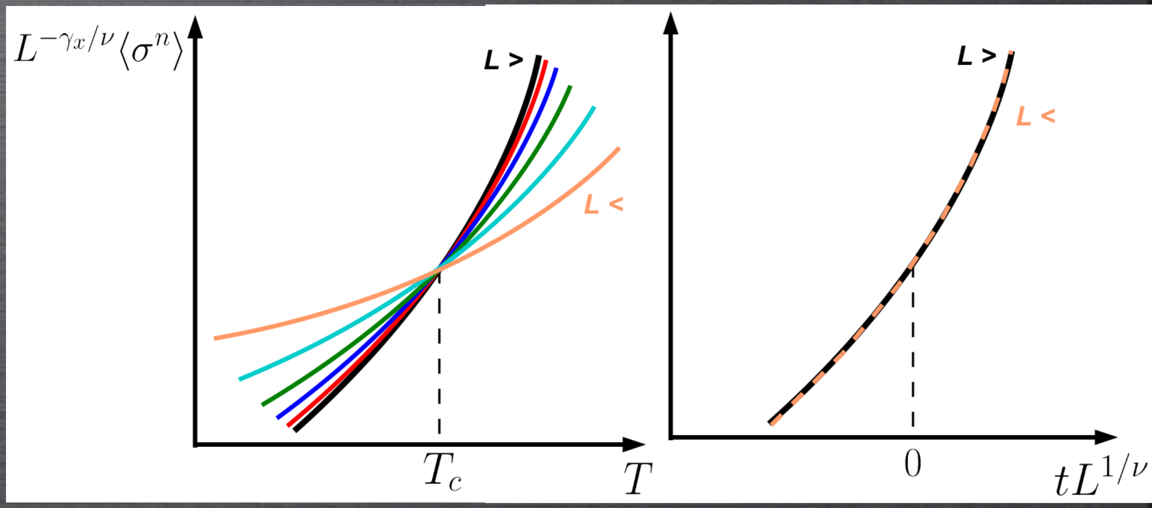
Illustration:



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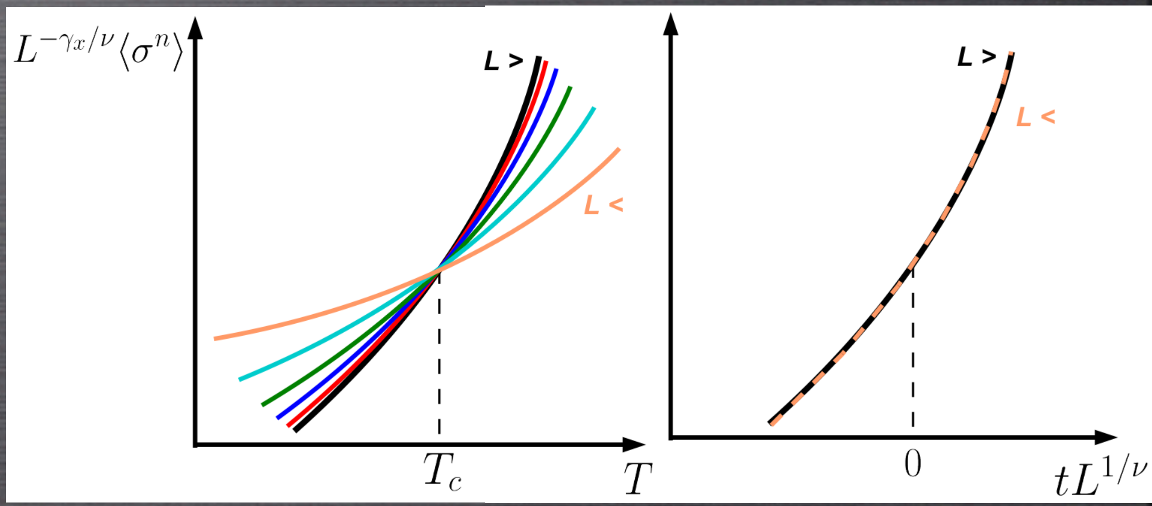


PROCEDURE: Search for γ_x, ν, $\sqrt{s}_c$ which **collapse** data from **different centralities** in the associated scaling plot ($XN_{\text{part}}^{-\gamma_x/2\nu} \times y_{\text{scl}}$)

METHODS FOR DATA ANALYSIS:

Fits (standard method in statistical mechanics)

χ^2 method: minimize the difference between data points associated with the same value of the scaling variable:

$$\chi^2(\nu,\sqrt{s}_c; y_0 = N_{\text{part},0}^{\nu/2}\frac{\sqrt{s}_0-\sqrt{s}_c}{\sqrt{s}_c}) = \sum_{y_{\text{scl}}(\sqrt{s},N_{\text{part}})=y_0}\left(\frac{X(\sqrt{s},N_{\text{part}},\nu)N_{\text{part}}^{-\gamma_x/2\nu}}{X(\sqrt{s}_0,N_{\text{part},0},\nu)N_{\text{part},0}^{-\gamma_x/2\nu}}-1\right)^2$$

Conclusions

Finite-size effects can play a crucial role in the search for the CEP in HIC's in BES-RHIC and FAIR-GSI.

(1) **Nonmonotonic (or sign-change) signatures will probe pseudocritical observables,** that are smoothened and shifted from the genuine criticality in the thermodynamic limit by corrections dependent on size/centrality and boundary conditions.

We show within the LσM that:

- corrections can be large for the size scales involved in current HIC
- the (pseudo)critical line is shrinked and shifted to the higher μ regime, as the size decreases.
- isentropic trajectories change significantly around the critical region

Due to the size/centrality dependent shifts of the pseudocritical peaks, averages over not sufficiently small centrality windows could generate a broader nonmonotonic signature, contributing to wash it out in the thermal background.

⇒ Data analysis in small centrality bins

Conclusions(II)

(2) HIC data can be seen as an ensemble of systems of different sizes

⇒ **Finite-size scaling** can be a useful tool in the search for the CEP in HIC in BES-RHIC and FAIR-GSI. Its presence represents an independent and complementary signature of the 2[nd] order CEP and can give info about the phase diagram in the thermodynamic limit (including the universality class).

We propose the application of full scaling plots in the search for FSS in HIC data, and discuss a χ^2-method as one possible systematic tool for data analysis.

As well as most of the other CEP signatures, the FSS signature relies on the fact that we can connect correlations in final particle spectra to correlation functions of the order parameter of the transition.

Possible tests of the FSS signature:
MC simulations of the evolution of correlations in the thermal background;
FSS analysis in low-energy nuclear CEP data.

Finite size and the QCD phase transitions

- Big Bang *vs* Little Bang: **huge differences in time and length scales**

Early universe (Big Bang):
[at the epoch of the QCD transitions]

$$L_{\text{univ}}(T_{\text{QCD}}) \sim 10^{18}\text{fm}$$

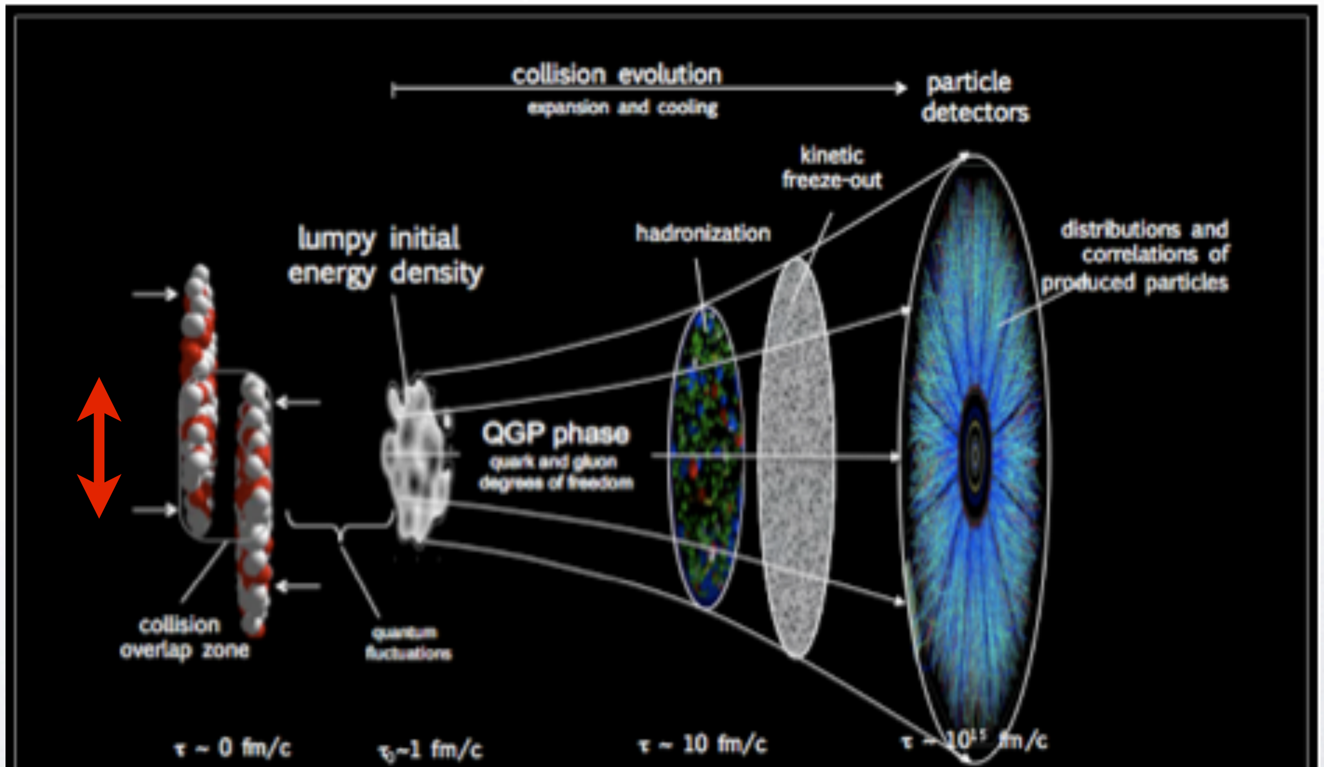
The system is essentially in the thermodynamic limit

Heavy-ion collisions (Little Bang):

$$L_{\text{HICs}} \leq 10 - 15\,\text{fm}$$

The system can be small

→ **Experimental mapping of the QCD phase diagram in a finite system.**



Elliptic Flow: A Study of Space-Momentum Correlations In Relativistic Nuclear Collisions.
P. Sorensen, Quark Gluon Plasma, Vol. 4, World Scientific, arXiv:0905.0174 [nucl-ex]

Remarks on finite-size effects

Finite-size effects can play a crucial role in the search for the CEP in HICs.

(1) The shifts of pseudocritical lines in the chiral phase diagram at typical HIC size scales can be large

➠ the actual phase diagram probed by nonmonotonic signatures in HICs can be quantitatively very different from the usual picture in the thermodynamic limit.

(2) Finite-size scaling can be a useful tool in the search for the CEP in HIC in BES-RHIC, FAIR-GSI and NICA-JINR.

➠ FSS techniques are simple and well defined. Even if it is hard to define the ideal scaling variable in the case of HICs, it provides a pragmatic alternative signature to search for the critical endpoint.