Deviations from the exponential decay law in strong decays

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Outline

- Deviations from the exponential decay law in Quantum Mechanics and Quantum Zeno Effect (QZE)
- Experimental observations of QZE
- Deviations from the exponential decay law in Quantum Field Theory: the case of strong decays
- Conclusions

Deviations from the exponential decay law in QM

Let $|a\rangle$ be the state of the system with Hamiltonian H at t=0 (not energy eigenstate)

The survival amplitude and probability read: $P(t) = |\mathcal{A}(t)|^2 = |\langle a|e^{-iHt}|a\rangle|^2$

By expanding around t =0: $P(t) \sim 1 - t^2/\tau_Z^2, \qquad \tau_Z^{-2} \equiv \langle a|H^2|a\rangle - \langle a|H|a\rangle^2$

For a short time interval after the “preparation” of the system the survival probability is not exponential (also at “late” times the survival probability falls off with a power law)

By performing pulsed measurements with period τ

$$P^{(N)}(t) = P(\tau)^N = P\left(t/N\right)^N \sim \exp\left(-t^2/\tau_Z^2 N\right) \stackrel{N\to\infty}{\longrightarrow} 1$$

The decay process is slowed down (and eventually hindered) by measurements!!

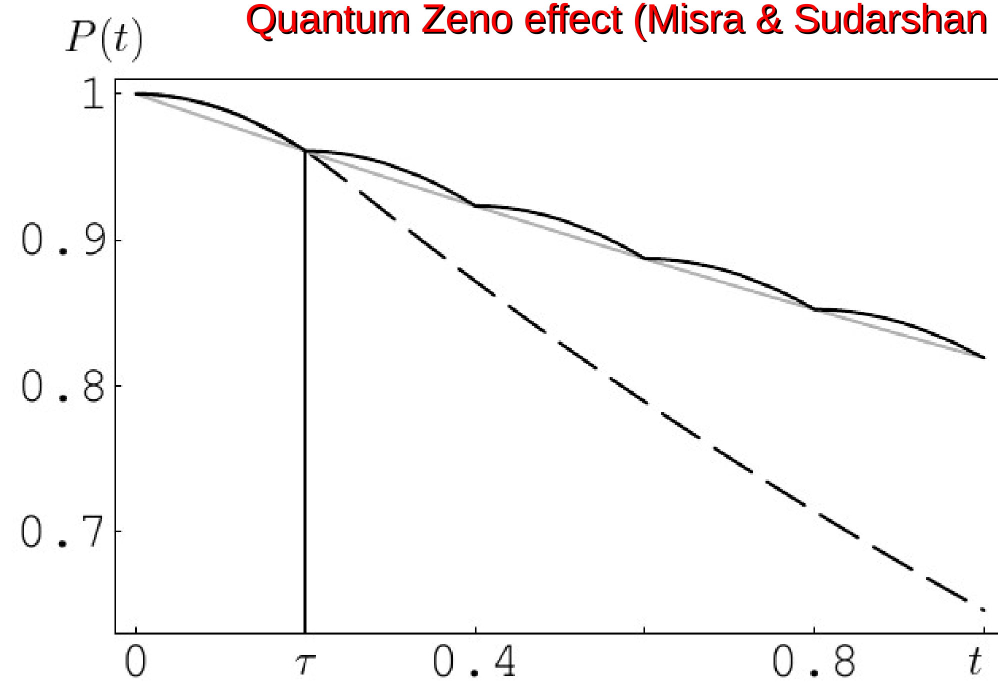
τ_z quantifies the non-exponential regime

$$\gamma_{\text{eff}}(\tau) \equiv -\frac{1}{\tau}\log P(\tau)$$

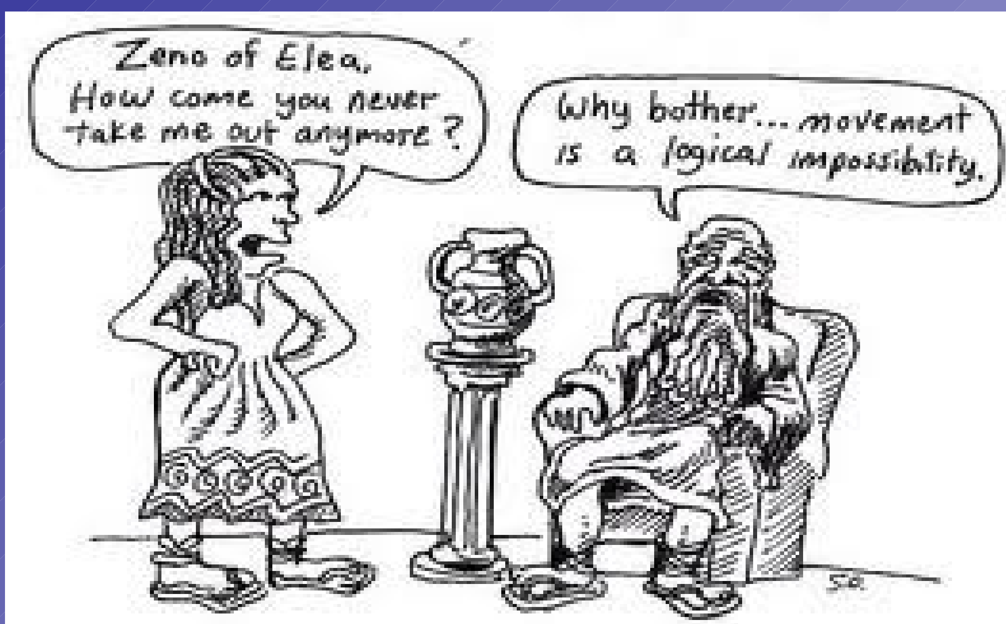
$$\gamma_{\text{eff}}(\tau) \sim \tau/\tau_Z^2 \qquad (\tau \to 0)$$

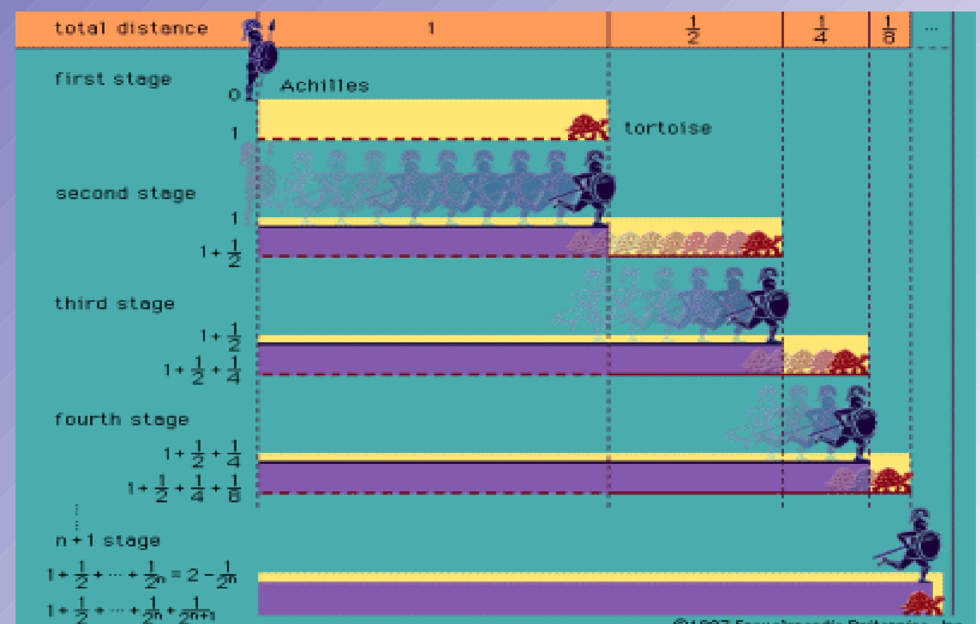
The decay rate is not constant, memory effect

Quantum Zeno effect (Misra & Sudarshan 1977)



See Facchi & Pascazio quant-ph/0202127





A text-book argument:

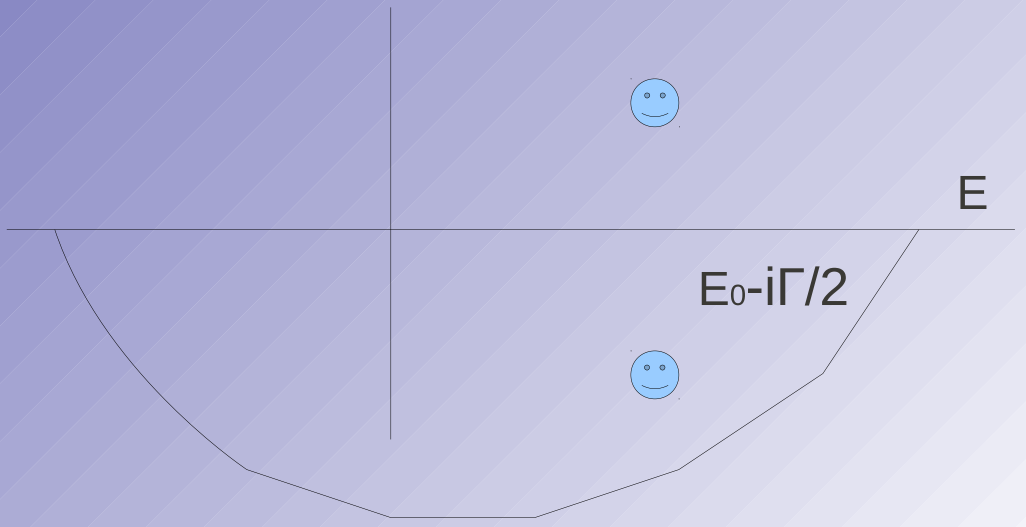
Probability that the state collapses into a H eigenstate, the survival amplitude is the Fourier transform of ρ

$$\rho(E) \; = \; |\langle E|a\rangle|^2$$

$$A(t) \; = \; \int dE\, \rho(E) e^{-itE}$$

By considering a Breit-Wigner distribution the standard exponential law is recovered. The energy of the particle takes an imaginary part.

$$\rho(E)_{BW} \; = \; (\Gamma/2\pi)/((E-E_0)^2+\Gamma^2/4)$$
$$P(t)_{BW} \; = \; e^{-\Gamma t}$$



In general, under the hyp. that:

$$\langle E \rangle = \int dE\, E\rho(E) < +\infty$$

$$\frac{dA(t)}{dt} = \int dE\,(-iE)\rho(E)e^{-itE}$$

$$\frac{dP(t)}{dt} = \frac{dA(t)^*}{dt}A(t) + A(t)^*\frac{dA(t)}{dt}$$

→

$$-\dot{A}(-t)A(t) + A(-t)\dot{A}(t) = 0$$

for t=0

The survival probability cannot be exponential at small times !!

A more intuitive argument (Ersak 1969):

Suppose we can define the unstable-state wave function

$$\exp(-iHt)|\psi_{UN}\rangle = A(t)|\psi_{UN}\rangle + |\varphi(t)\rangle$$

$$\langle\psi_{UN}|\varphi(t)\rangle = 0$$

Unstable-state wave function

Decay products wave function

Let the system evolve to t':

$$A(t+t') = A(t)A(t') + \langle\psi_{UN}|\exp(-iHt')|\varphi(t)\rangle$$

Without rescattering processes

$$A(t) = \exp(-\alpha t)$$

The possibility of re-forming the unstable state via rescattering of the decay products is responsible for the deviations from the exponential law!!!

Experiments

PHYSICAL REVIEW A VOLUME 41, NUMBER 5 1 MARCH 1990

Quantum Zeno effect

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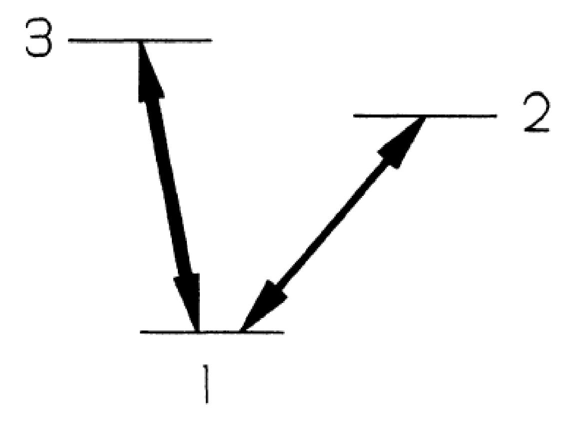
(Received 12 October 1989)

The quantum Zeno effect is the inhibition of transitions between quantum states by frequent measurements of the state. The inhibition arises because the measurement causes a collapse (reduction) of the wave function. If the time between measurements is short enough, the wave function usually collapses back to the initial state. We have observed this effect in an rf transition between two $^9Be^+$ ground-state hyperfine levels. The ions were confined in a Penning trap and laser cooled. Short pulses of light, applied at the same time as the rf field, made the measurements. If an ion was in one state, it scattered a few photons; if it was in the other, it scattered no photons. In the latter case the wave-function collapse was due to a null measurement. Good agreement was found with calculations.



Experimental evidence for non-exponential decay in quantum tunnelling

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An exponential decay law is the universal hallmark of unstable systems and is observed in all fields of science. This law is not, however, fully consistent with quantum mechanics and deviations from exponential decay have been predicted for short as well as long times[1–8]. Such deviations have not hitherto been observed experimentally. Here we present experimental evidence for short-time deviation from exponential decay in a quantum tunnelling experiment. Our system consists of ultra-cold sodium atoms that are trapped in an accelerating periodic optical potential created by a standing wave of light. Atoms can escape the wells by quantum tunnelling, and the number that remain can be measured as a function of interaction time for a fixed value of the well depth and acceleration. We observe that for short times the survival probability is initially constant before developing the characteristics of exponential decay. The conceptual simplicity of the experiment enables a detailed comparison with theoretical predictions.

Nature 1997

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Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System

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(Received 30 March 2001; published 10 July 2001)

We report the first observation of the quantum Zeno and anti-Zeno effects in an unstable system. Cold sodium atoms are trapped in a far-detuned standing wave of light that is accelerated for a controlled duration. For a large acceleration the atoms can escape the trapping potential via tunneling. Initially the number of trapped atoms shows strong nonexponential decay features, evolving into the characteristic exponential decay behavior. We repeatedly measure the number of atoms remaining trapped during the initial period of nonexponential decay. Depending on the frequency of measurements we observe a decay that is suppressed or enhanced as compared to the unperturbed system.

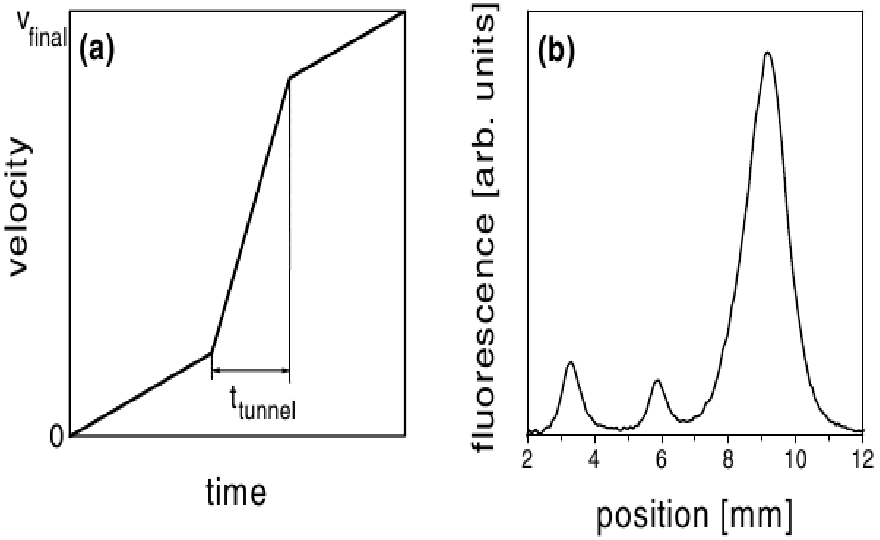


FIG. 1. Part (a) shows a diagram of the acceleration sequence. Part (b) shows a typical integrated spatial distribution of atoms after the time of ballistic expansion. The large peak on the right is the part of the atomic cloud that was not trapped during the initial acceleration. The center peak indicates the atoms that tunneled out of the optical potential during the fast acceleration period. The leftmost peak corresponds to atoms that remained trapped during the entire sequence. The survival probability is the area under the left peak normalized by the sum of the areas under the left and center peak.

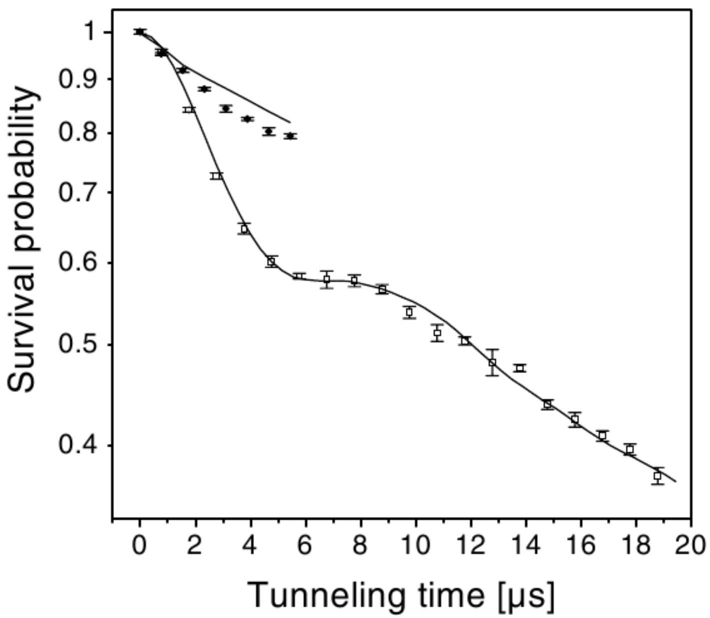


FIG. 3. Probability of survival in the accelerated potential as a function of duration of the tunneling acceleration. The hollow squares show the noninterrupted sequence, and the solid circles show the sequence with interruptions of 50 μs duration every 1 μs. The error bars denote the error of the mean. The data have been normalized to unity at $t_{\mathrm{tunnel}} = 0$ in order to compare with the simulations. The solid lines are quantum mechanical simulations of the experimental sequence with no adjustable parameters. For these data the parameters were $a_{\mathrm{tunnel}} = 15\,000\ \mathrm{m/s^2}$, $a_{\mathrm{interr}} = 2000\ \mathrm{m/s^2}$, $t_{\mathrm{interr}} = 50\ \mu\mathrm{s}$, and $V_0/h = 91$ kHz, where h is Planck's constant.

…and in Quantum field theory?

Let us consider the super-renormalizable Lagrangian

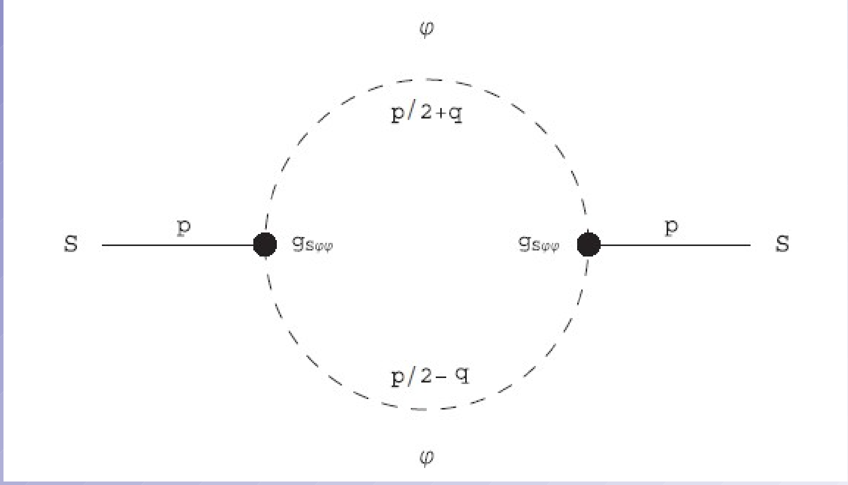
$$\mathcal{L} = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M_0^2 S^2 + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2 + gS\varphi^2$$

S is an unstable particle which decays into 2 φ with on-mass shell (tree-level) decay rate:

$$\Gamma^{\text{t-l}}_{S\varphi\varphi} = \frac{\sqrt{\frac{M_0^2}{4} - m^2}}{8\pi M_0^2}(\sqrt{2}g)^2$$

$$p_{\text{t-l}}(t) = e^{-\Gamma^{\text{t-l}}_{S\varphi\varphi} t}$$

$$\Sigma(p^2) = \int_q \frac{-i}{\left[\left(\frac{p+2q}{2}\right)^2 - m^2 + i\varepsilon\right]\left[\left(\frac{p-2q}{2}\right)^2 - m^2 + i\varepsilon\right]}$$



From the propagator (with resummed self-energy) it is possible to define the spectral function which expresses the probability that S has a certain value of energy

$$\Delta_S(p^2) = \left[p^2 - M_0^2 + (\sqrt{2}g)^2\Sigma(p^2) + i\varepsilon\right]^{-1}$$

$$d_S(x = \sqrt{p^2}) = \frac{2x}{\pi}\left|\lim_{\varepsilon\to 0}\mathrm{Im}[\Delta_S(p^2)]\right|$$

The normalization is fulfilled (Kaellen-Lehman representation)

$$\int_0^\infty d_S(x)dx = 1$$

Correct limit for g->0

$$d_S(x) = \delta(x - M_0)$$

The survival probability reads:

$$a(t) = \int_{-\infty}^{+\infty} \mathrm{dx}\; d_S(x) e^{-ixt} \;,\; p(t) = |a(t)|^2$$

If the average mass of the particle is finite:

$$\int_0^\infty x\, d_S(x) dx \;\longrightarrow\; p'(t=0) = 0$$

$$\gamma(t) = \frac{-1}{t} \ln p(t)$$

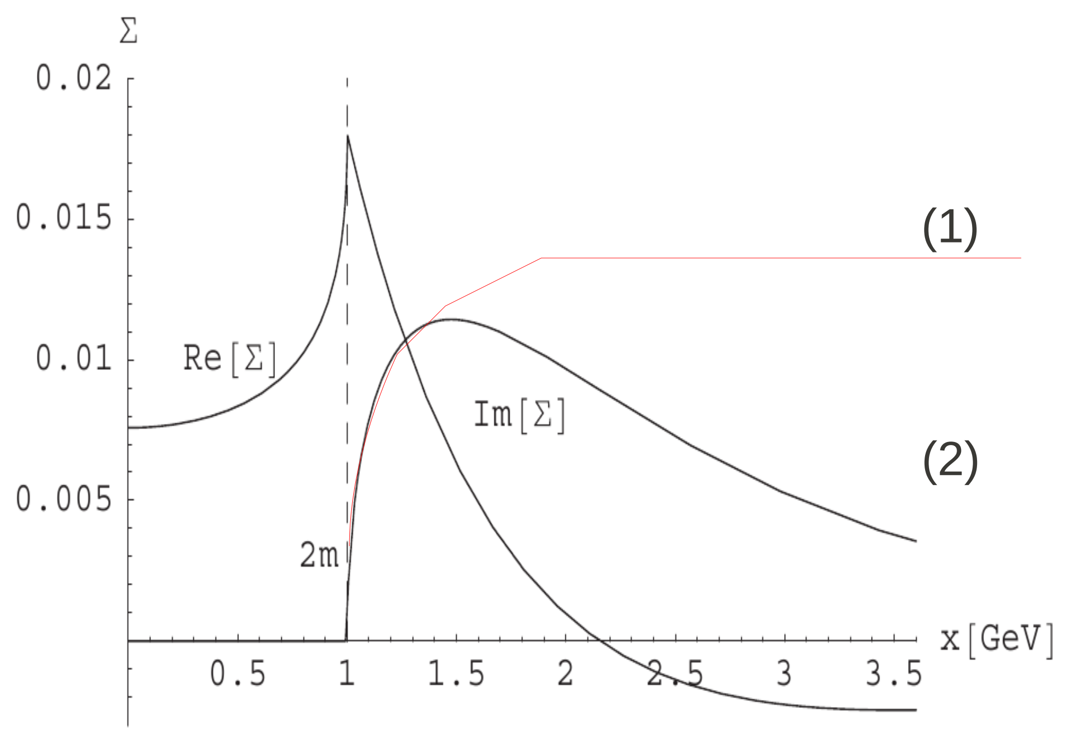
$$\lim_{t\to 0^+} \gamma(t) = -\lim_{t\to 0^+} \frac{p'(t)}{p(t)} = 0$$

The effective decay rate vanishes for t->0 , Quantum Zeno effect is possible also in QFT!

$$\Sigma(x) = \frac{-\sqrt{4m^2 - x^2}}{8\pi^2 x} \arctan\left(\frac{\Lambda x}{\sqrt{\Lambda^2 + x^2}\sqrt{4m^2 - x^2}}\right) - \frac{1}{8\pi^2} \log\left(\frac{m}{\Lambda + \sqrt{\Lambda^2 + m^2}}\right).$$

(1) Regard the model as a "fundamental theory" , $\Lambda \to \infty$

(2) Regard the model as an effective hadronic theory with a cutoff in the energy range of the masses of the particles, 1 GeV



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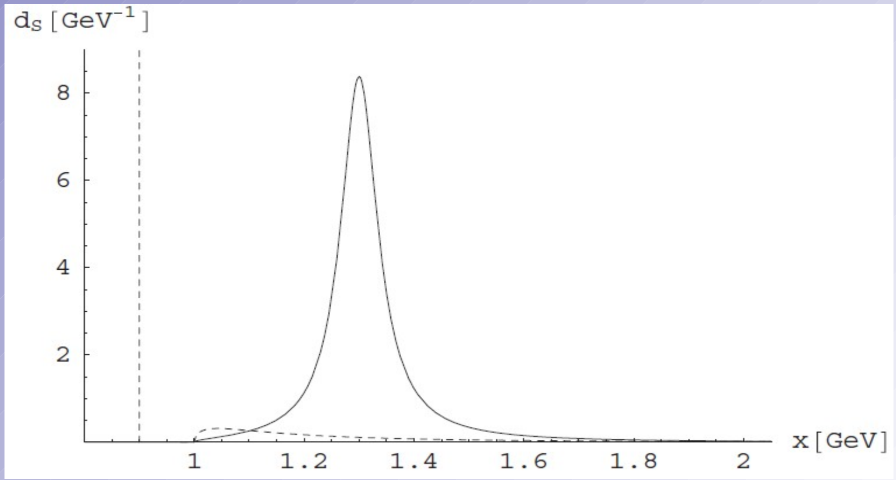
$$d_S(x) = \frac{2x}{\pi} \frac{I(x)}{\left(x^2 - M_0^2 + R(x)\right)^2 + I(x)^2}$$



(1) $d_s(x)$ scales as 1/x^3 and only the average energy is finite. The variance, which is proportional to the second derivative of p(t) diverges → Zeno time vanishes but the QZE is still possible (p'(t)=0 at t=0).

(2) all the divergences are removed and also the Zeno time can be defined, p(t) is quadratic at small times.

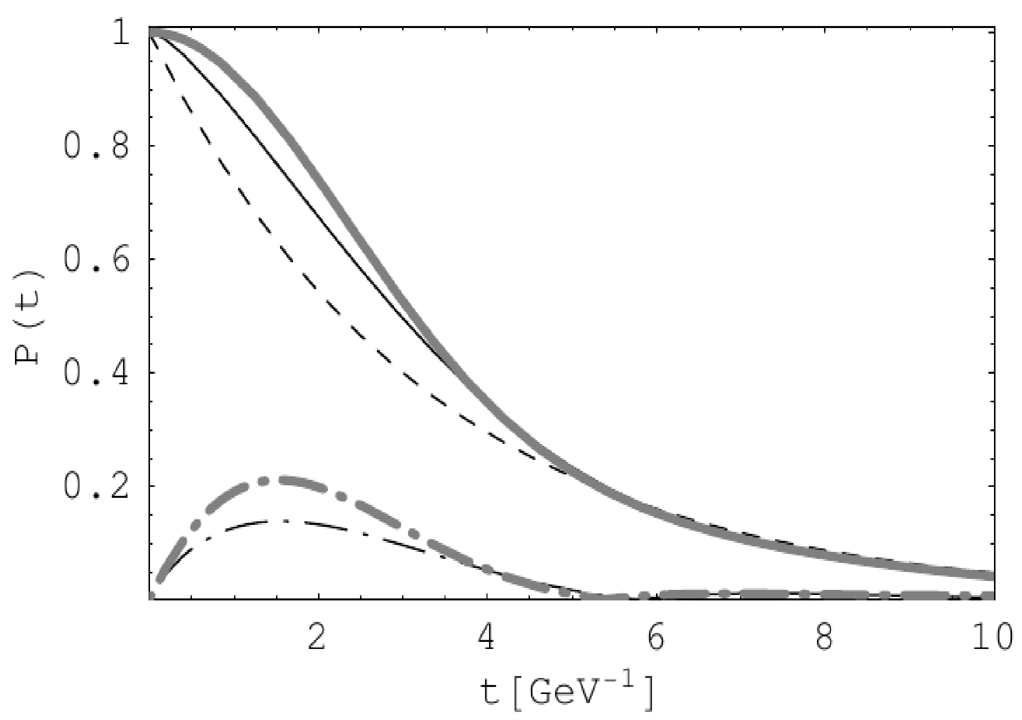


FIG. 1: The survival probability $p(t)$ of Eq. (6) is shown in the case of infinite (thin solid line) and finite, $\Lambda = 1$ GeV, (thick gray line) cutoff. In both cases the non-exponential behavior at short times is clearly visible. The exponential tree-level decay is shown for comparison (dashed line). The quantity $\left|p(t) - e^{-\Gamma t}\right|$ is also displayed by the thin dot-dashed and thick gray dot-dashed lines for the two cases respectively.

A more general quantity which characterizes the deviations from the exponential law

$$\max\left(p(t) - e^{-\Gamma t}\right) \rightarrow t = \tau_M$$

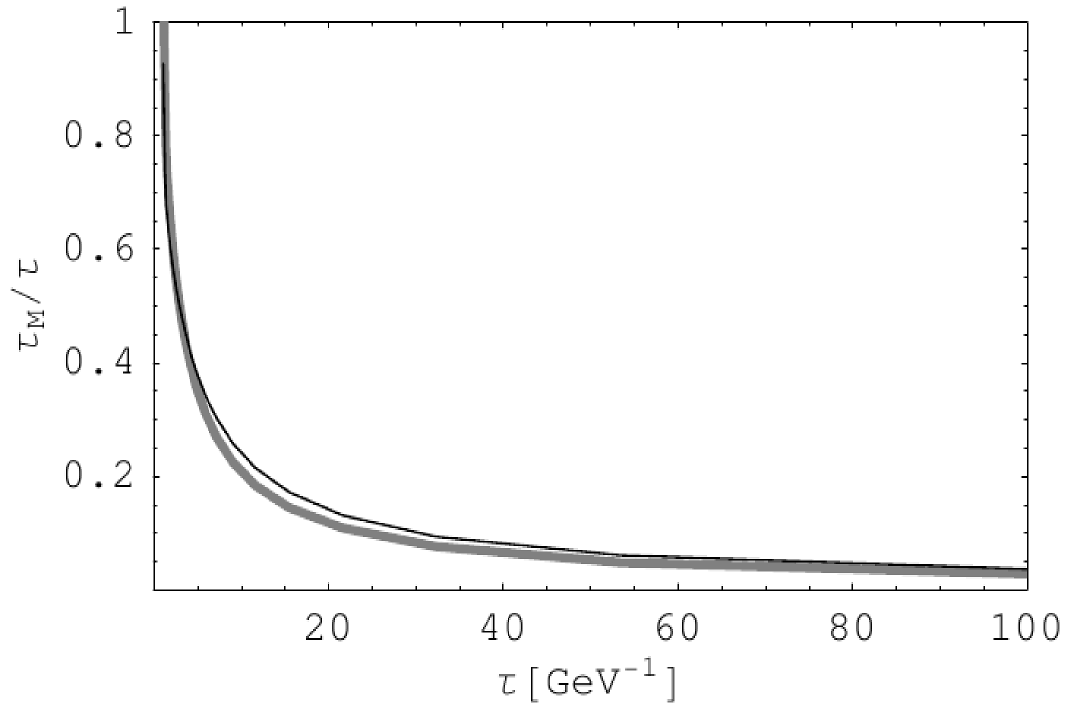
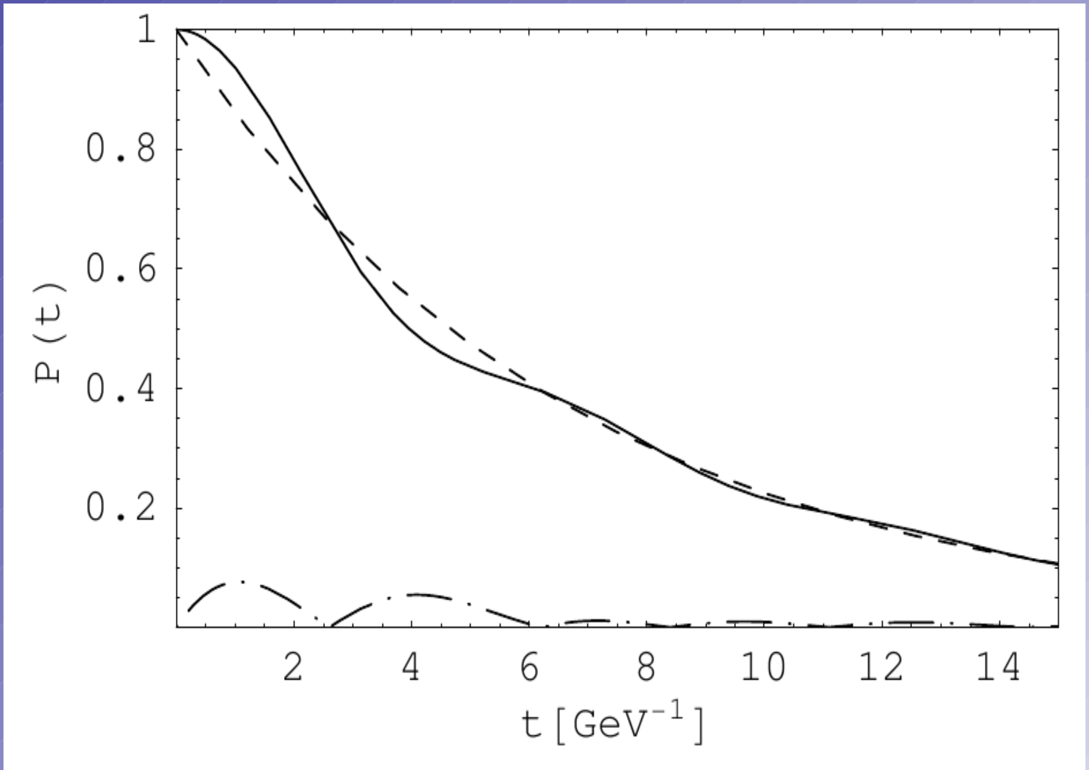


FIG. 2: $\tau_M/\tau_{\text{t-1}}$ as function of $\tau_{\text{t-1}}$ in the cases of infinite (thin line) and finite ($\Lambda = 1$, thick gray line) cutoffs. For short living particles, such as hadronic resonances, the non exponential regime lasts for a time scale of the same order of magnitude of the mean life time of the particle.

An example: the ρ meson

$$d_\rho(x) = \frac{2x}{\pi} \frac{x\Gamma_{\rho\to\pi\pi}(x)}{(x^2 - M_\rho^2)^2 + x^2\Gamma_{\rho\to\pi\pi}(x)^2}$$

$$\Gamma_{\rho\to\pi\pi}(x) = \frac{\left(\frac{x^2}{4} - m^2\right)^{3/2}}{6\pi x^2} g_\rho^2$$



Possible effects in heavy ions physics ?? In microscopic transport calculations (URQMD) the exponential law is usually adopted.

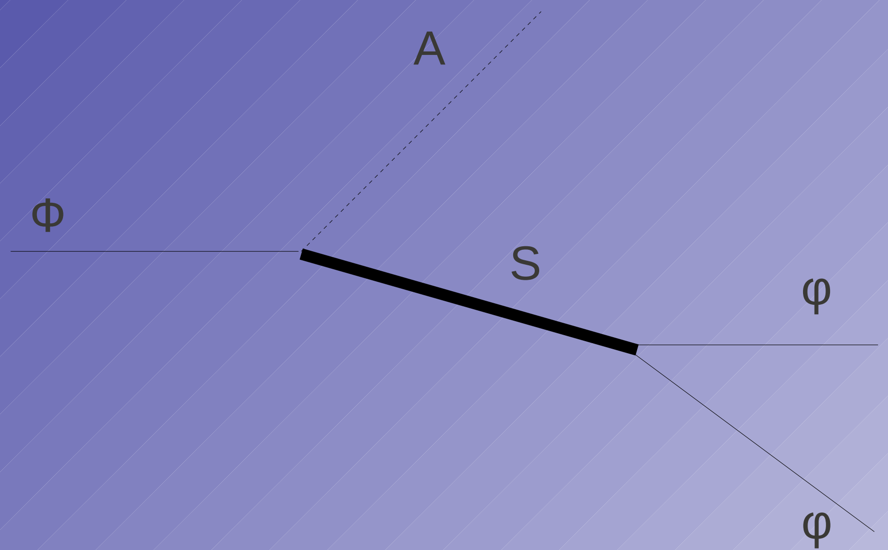
Conclusions

- Deviations from the exponential decay law are necessary in QM. Such deviations have been observed for unstable states.
- Similar results hold also in QFT, with finite & infinite cut-off. The non-exponential regime lasts for a time interval comparable with the mean life-time for some hadronic resonances.
- Further studies: renormalizable and non-renormalizable theories, resonances with more decay channels, applications in heavy ions transport simulations.

Appendix

Considering also the process of formation of the resonance:

$$\phi \to AS \to A\varphi\varphi$$



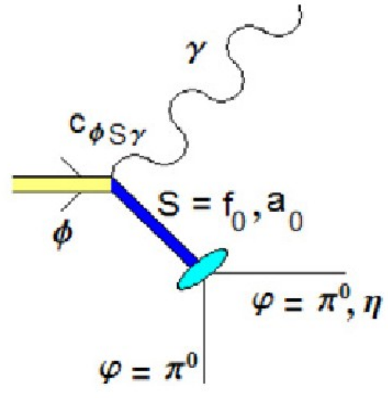
$$e^{-iHt}\left|\phi\right\rangle = e^{-\Gamma_\phi t/2}\left|\phi\right\rangle + \left(1 - e^{-\Gamma_\phi t/2}\right)\left(a(t)\left|S\right\rangle + b(t)\left|\varphi\varphi\right\rangle\right)\left|A\right\rangle$$

The survival aplitude appears in a more general and complicated expression

Radiative Φ decay

$$d\Gamma_{\phi\gamma\pi^0\pi^0}(x) = \Gamma^{\text{t-l}}_{\phi\gamma f_0}(x)\left[d_{f_0}(x)\,dx\right]\left[\frac{\Gamma^{\text{t-l}}_{f_0\pi^0\pi^0}(x)}{\Gamma^{\text{t-l}}_{f_0\pi\pi}(x)+\Gamma^{\text{t-l}}_{f_0KK}(x)}\right],$$

Spectral function



(c) Radiative ϕ-decay

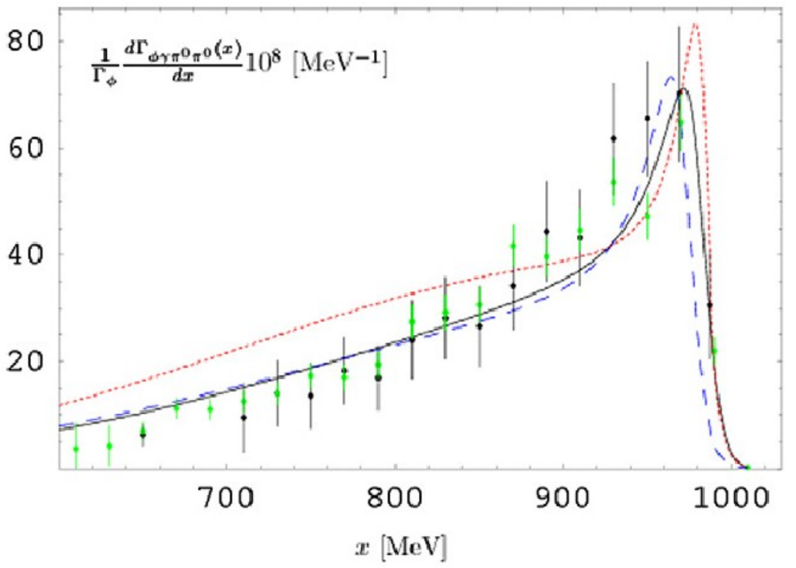


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Fig. 3. Branching ratio $\frac{1}{\Gamma_\phi}\frac{d\Gamma_{\phi\gamma\pi^0\pi^0}(x)}{dx}\times 10^8\ \text{MeV}^{-1}$ as function of the invariant mass x (colors online). $\Gamma_\phi =$ 4.26 MeV is the full width of the ϕ meson. We consider data sets from the SND and KLOE Collaborations [20,21] corresponding respectively to the black and grey (green online) dots. The continuous line is the result of the fit by setting $c_{f_0KK} = 0$, the dashed line corresponds to the case $c_{f_0KK} = 12\ \text{GeV}^{-1}$. Both cases are in Table 1. The dotted line corresponds also to $c_{f_0KK} = 12\ \text{GeV}^{-1}$ but only data points above 0.8 GeV are used in the fit, see Table 2.