Strong-coupling effective action(s) for $SU(3)$ Yang-Mills

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Outline

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Effective theories for thermal QCD

- Analytical, first-principle treatment of QCD is currently a cherished dream...
- That's why a lot of effective/approximate methods have been devised in the years to gain knowledge on the theory.
- Most notably lattice simulations, but: (1) they tell just *what* happens, and not *why*, and (2) finite baryon density is currently unreachable.
- High-temperature (plasma phase): dimensional reduction (i.e. obtain a 3d effective theory after integrating out the hard scales) $+$ Monte Carlo on the resulting model. But this does not allow the study of the hadron phase and of the transition!

Effective theories - II

- EQCD (Braaten, Nieto '96): weak-coupling + perturbative matching. Works only at $T > T_c$ and is *not* Z_3-symmetric.
- Vuorinen, Yaffe '06, Pisarski '06: Z_3-symmetric effective theory, by matching a rather general Lagrangian to the EQCD in the high-T limit. Can describe down to $\sim T_c$, but tuning all parameters is far from a trivial task.

Effective theories - III

Strong-coupling approach (cf. Svetitski-Yaffe conjecture) !

- Several effective formulations ('80s) in terms of Polyakov loops, with drastic approximations such as neglect of spatial plaquettes.
- "Inverse Monte Carlo" method (Jena group '05–'07): fix all parameters of a general Polyakov-loop Lagrangian via (numerical) evaluation of observables. Pro: works at any coupling and uses just scalars as DOFs. Con: the matching is difficult for $SU(3)$ and requires heavy simulations.

$\Rightarrow$ Our approach: higher order-corrections, scalars as fundamental DOFs, no tuning from the parent theories: good at $T \leq T_c$ and in describing the transition.

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Lattice field theories & Monte Carlo

- Euclidean space-time is discretised (with spacing a)
- Use statistical mechanics Monte Carlo techniques to explore configuration space
- Actions and observables must be reformulated in a lattice-friendly way (e. g. with discrete derivatives)
- Continuum physics is recovered (numerically) at the end by looking for the $a \to 0$ limit, say, of an observable (carefully getting rid of lattice artifacts)

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Strong-coupling expansion for pure gauge

Start from the 4d pure-Yang-Mills lattice partition function

$$Z = \int [\mathcal{D}U_\ell] \exp\left\{ \frac{\beta}{2N} \sum_{\Box} (\mathrm{Tr}\, U_\Box + \mathrm{Tr}\, U_\Box^\dagger) \right\} \quad \text{with} \quad \beta = \frac{2N}{g^2}$$

Integrate out the spatial DOFs and get an effective action

$$Z = \int [\mathcal{D}U_{\mathrm{time}}] \exp(-S_{\mathrm{eff}})$$

Small β: strong-coupling expansion of S_{eff} with effective couplings $\lambda_i(\beta, N_\tau)$:

$$S_{\mathrm{eff}} = \lambda_1 S_1 + \lambda_2 S_2 + \dots$$

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Finding S_{eff}

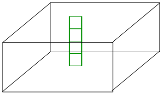
The actions S_i happen to depend only on (traced) Polyakov loops $L_i = \mathrm{Tr} \sum_\tau U_{\text{time}}(i, \tau)$: we will end up in a 3d model with $L(x, y, z)$ as fundamental degrees of freedom.

Find the transition point $\{\lambda_i^{(c)}\} \Rightarrow \beta_c$ found for all N_τ at once!

The S_i are obtained by means of character expansion + moment-cumulant formalism, and is given by *enumerating all graphs to a certain order in* β (actually in $u = a_f(\beta) \sim \beta + \cdots$)

Some contributions (nearest-neighbour interaction)

Leading-order graph:



$$\Rightarrow -\lambda_1 S_1 = u^{N_\tau} \sum_{<ij>} (L_i^* L_j + L_i L_j^*)$$

Subleading corrections, e. g.  $\Rightarrow u^{N_\tau}(4N_\tau u^4) S_1$

Additional decorations give higher orders in u:  $\sim u^{N_\tau+8}$

All of this goes into $\lambda_1(u, N_\tau)$ (known to order u^{10}).

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Other terms

Nearest-neighbour terms can be regrouped as

$$\sum_{<ij>} \left[2\lambda_1 \mathrm{Re}(L_i^* L_j) + \ldots \right] \mapsto \sum_{<ij>} \log[1 + 2\lambda_1 \mathrm{Re}(L_i^* L_j)]$$

Subleading contributions for next-to-nearest neighbours:

$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \; = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \; = 2$$

as well as terms from loops in the *adjoint* representation:

$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

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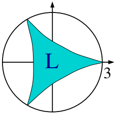
Simulating the effective theories

The "one-coupling" 3d model (λ_1 only) has thus the final form:

$$Z = \left(\prod_x \int \mathrm{d}\, L_x \right) e^{-S_{\mathrm{eff}}} \quad ; \quad S_{\mathrm{eff}} = -\sum_{<ij>} \log(1 + 2\lambda_1 \mathrm{Re} L_i L_j^*) - \sum_x V_x$$

where V_x depends on the parametrisation for L and encodes the Haar group measure:

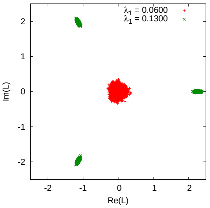
$$\begin{aligned} L(\theta, \phi) &= e^{i\theta} + e^{i\phi} + e^{-i(\theta+\phi)} \quad , \quad -\pi \le \theta, \phi \le +\pi \\ V_x &= \frac{1}{2} \log(27 - 18|L_x|^2 + 8 \mathrm{Re} L_x^3 - |L_x|^4) \end{aligned}$$

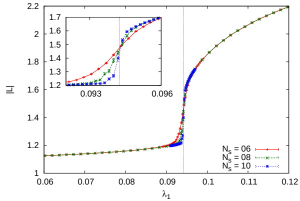
This 

is the domain for $L = \mathrm{Tr}\, W$ in $\mathbb{C}$

Practical implementation

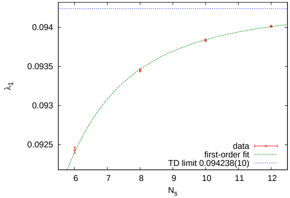
Easy to implement with a standard Metropolis local accept/reject strategy
(in the simulated action the *log*$(1 + \cdots)$ is a truncated expansion, however consistency is recovered at the end)
Expectation ($\lambda_1 \sim \beta^{N_\tau}$): spontaneous Z_3 symmetry breaking for $\lambda_1 > \lambda_{1,c}$:

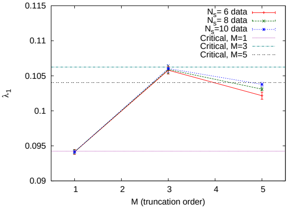


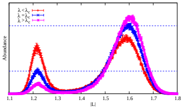


Transition point

- The critical point is found with a scaling analysis from finite-size estimators such as the maximum of $|L|$ susceptibility
- For various truncations M, a critical $\lambda_{1,c}^{(M)}$ is found and they show convergence in M (as suggested by the reference $SU(2)$ case)
- The transition is first-order as it should be (from Binder cumulant analysis)

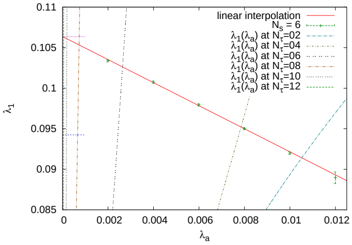






Theories with more than one coupling

2d parameter space: find the critical *line* as before. All couplings depend on $(N_\tau, \beta) \Rightarrow$ final result is a list of $\beta_c(N_\tau)$
Double truncation necessary: $M_1 = 3, M_{2,a} = 1$ for consistency



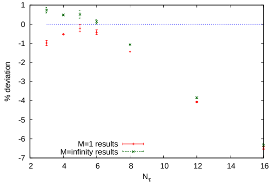
For both (λ_1, λ_2) and (λ_1, λ_a) models, at high enough N_τ only λ_1 is important

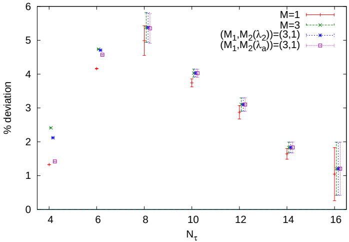
Results

Once a $\lambda_{1,c}$ is found (for a given model with a given truncation M), one uses the maps

$$\lambda_1(\beta, N_\tau) \quad , \quad \lambda_2(\beta, N_\tau) \quad , \quad \lambda_a(\beta, N_\tau) \quad \ldots$$

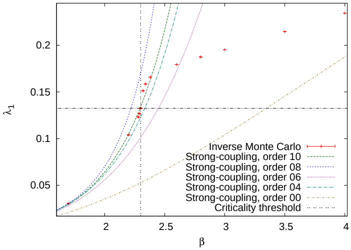
and gets the critical values $\beta_c(N_\tau)$. General outcome: for fine lattices, agreement with 4d Monte Carlo within 2%





Comparison with Inverse Monte Carlo

IMC method: start with the most general theory respecting the desired symmetries and fix all couplings numerically



Our curves match with the IMC values up to the transition (where the series stop converging)

In conclusion. . .

Pure gauge features:

- Dimensionally reduced Z_3-symmetric effective theory
- Systematically derived with strong-coupling techniques
- Captures most features of the transition, and predictive within few percent
- Computationally cheap: in a few days with a modest PC one finds already satisfactory results
- Adjoint- and next-to-nearest-neighbour- interactions are negligible, especially at finer lattices

Outlook: straightforward extension to $SU(N > 3)$, and – most important – adding the effect of fermions and finite density (see next slides).

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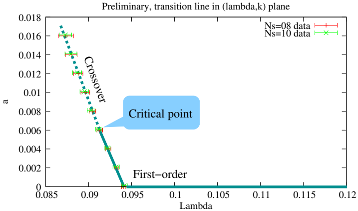
Adding heavy matter (*work in progress*)

Heavy matter: hopping expansion in $\kappa \propto 1/M \Rightarrow$ an "external magnetic field" to the spin model

$$Z_F = \prod_x \det \left[\left(1 + ((2\kappa e^{\mu})^{N_\tau}) W_x \right)^2 \left(1 + ((2\kappa e^{-\mu})^{N_\tau}) W_x^\dagger \right)^2 \right]$$

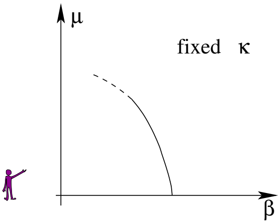
$(\mathrm{Tr}(W_x) = L_x,\ a \equiv 2\kappa)$

No chemical potential: first results support the expected picture



Adding nonzero chemical potential

(This will be the next step!)
With $\mu \neq 0$, complex action $\Rightarrow$ sign problem strikes again...
Possible strategy: reweighting.
First tests show it is a viable approach, at least for moderate μ.



For higher μ, it should be possible to devise a worm-like approach in the "flux representation" (cf. the Z_3 case)

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End of presentation

End of presentation

Thank you all for the attention.

Additional 1: $\lambda_1(u)$ maps

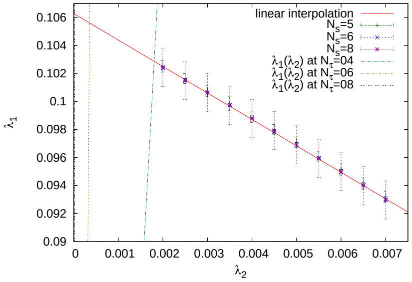
Example for the case $N_\tau = 4$:

$$\lambda_1(u) = u^4 \exp\left[4\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1035317}{5120}u^{10}\right)\right]$$

where

$$u(\beta) = \frac{1}{3}\frac{(\beta/6) + \frac{1}{2}(\beta/6)^2 + \cdots}{1 + (\beta/6)^2 + \cdots}$$

Additional 2: multi-coupling theory



$$\lambda_2(u, N_\tau = 4) = u^8(12u^2 + 26u^4 + 364u^6 + \cdots)$$

is subleading with respect to

$$\lambda_a(u, N_\tau = 4) = \frac{9}{8}u^8 + \cdots$$

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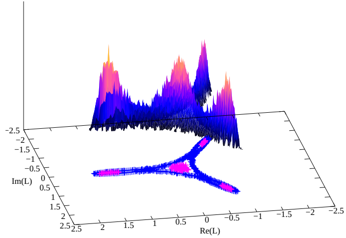
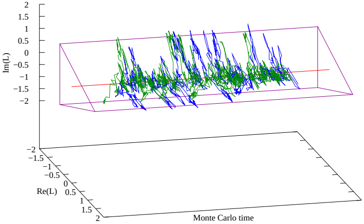
Additional 3: tunnelling and first-order evidences





Additional 4: $SU(2)$, formulas

Here L is real, and (one-coupling case)

$$Z = \left(\prod_x \int_{-2}^{+2} \mathrm{d}\, L_x \sqrt{4 - L_x^2} \right) \prod_{<ij>} (1 + \lambda_1 L_i L_j)$$

(similar expressions hold for multi-coupling versions)
In this case, a second-order transition is expected, within the 3d Ising universality class...

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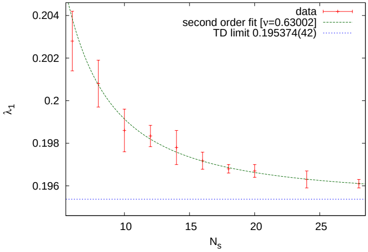
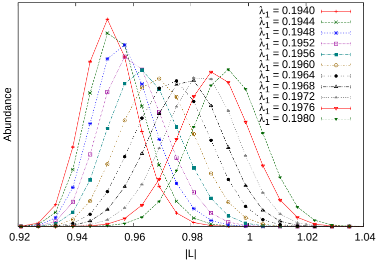
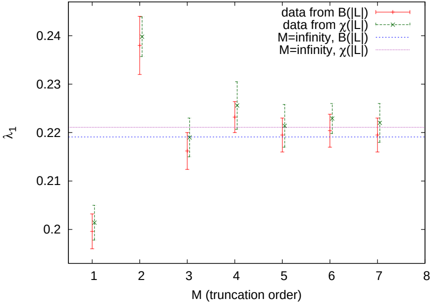
Additional 5: $SU(2)$, transition





Scaling of maximum of en. susceptibility, $\lambda_{N_s} = \lambda_c + \mathcal{O}(N_s^{-1/\nu})$, and $|L|$ histogram across the transition

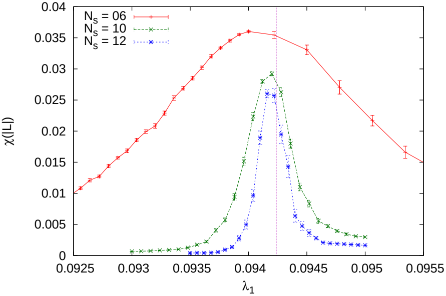
Additional 6: $SU(2)$, truncations and convergence in M



Here the $M = \infty$ transition is directly obtained with simulations: fast convergence in M is suggested

Additional 7: susceptibility of $|L|$

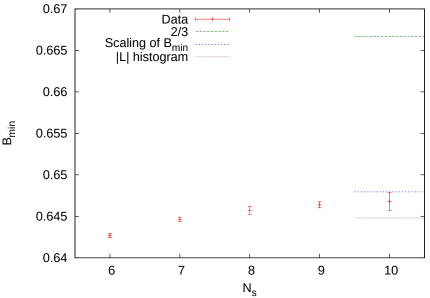
Finite-volume behaviour of $\chi_{|L|} = \left\langle \left(|L| - \langle |L| \rangle \right)^2 \right\rangle$



Additional 8: Binder cumulant of $|L|$

For a first-order transition

$$B_{|L|} = 1 - \frac{\langle |L|^4 \rangle}{3\langle |L|^2 \rangle^2} \to \frac{2}{3} - \frac{1}{12}\left(\frac{|L|_1}{|L|_2} - \frac{|L|_2}{|L|_1}\right)^2 \quad \text{for} \quad N_s \to \infty$$



Additional 9: the predicted β_c

N_τ	$M = 1$	$M = 3$	$\lambda_1, \lambda_2(3, 1)$	$\lambda_1, \lambda_a(3, 1)$	4d YM
4	5.768	5.830	5.813	5.773	5.6925(002)
6	6.139	6.173	6.172	6.164	5.8941(005)
8	6.300	6.324	6.324	6.322	6.0010(250)
10	6.390	6.408	6.408	6.408	6.1600(070)
12	6.448	6.462	6.462	6.462	6.2680(120)
14	6.488	6.500	6.500	6.500	6.3830(100)
16	6.517	6.528	6.528	6.528	6.4500(500)