Tetraquarks


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Duality invented to give consistency to the description of hadronic reactions

*s-channel* resonances, e.g., \(N^*, \Delta\), etc., in the case of \(\pi N\) scattering

*t-channel* resonances, e.g., \(\rho, \rho'\), etc.

Should be equivalent with *complete* summation (Veneziano model)

Strong *s* channel \(\Rightarrow\) strong *t* channel

Weak *t* channel \(\Rightarrow\) strong *t* channel

Can be formulated at the hadron level

But becomes more convincing with *quark diagrams*
Baryonium

Duality diagrams -1

- $\pi - \pi$ with $I = 0$ vs. $I = 2$
- $\pi - N$
Baryonium
Duality diagrams -2

- Baryon–antibaryon (Rosner)
- Prediction of baryonium, new meson preferentially coupled to baryon–antibaryon channels
See D.P. Roy’s review: baryonium–baryon scattering leads to pentaquark!

Pandora box?
Pre-LEAR: peaks in $p\bar{p} \rightarrow \gamma + X$, bumps in $\bar{p}$ cross-section ($S(1932)$, etc.)

In particular, French et al., narrow peak at 2.9 GeV decaying into another peak at 2.2 or 2.0 GeV,

EVIDENCE FOR A NARROW WIDTH BOSON OF MASS 2.95 GeV

LEAR: no confirmation

Post-LEAR Some enhancements in baryon–antibaryon from charmonium, or $B$ decay

Interpretation of pre-LEAR candidates

- Nucleon–antinucleon molecules (Shapiro, Dover, …)
- $(q^2\bar{q}^2)$ states (Veneziano, Jaffe, Chan et al., …)
**Baryonium**

Quark model

- **Topological structure:** With two junctions, this meson contains an underlying coupling to baryon–antibaryon.

- **Explicit model:** \((qq) - (\bar{q}\bar{q})\) separated by an angular momentum barrier that suppresses the rearrangement into two mesons. By string breaking, decay into baryon antibaryon.

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**Q^2\bar{Q}^2 resonances in the baryon-antibaryon system**

R. L. Jaffe

*Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

(Received 1 September 1977)

Two-quark–two-antiquark mesons which couple strongly to baryon-antibaryon channels are classified. The quantum numbers and masses of prominent states are predicted from the MIT bag model. The couplings of \(Q^2\bar{Q}^2\) states to \(BB\) are estimated using the \(2P_0\) model and peripherality. Though most \(Q^2\bar{Q}^2\) states do not couple strongly to \(BB\), many prominent resonances remain. Important \(Q^2\bar{Q}^2\) resonances in the following processes are enumerated and discussed: elastic \(NN\) scattering, \(NN \to \pi^+\pi^-\), \(NN\) resonances at or below threshold, and exotic isotensor baryon-antibaryon resonances.

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**Multiquarks**
More speculative: mock baryonium. If the diquark has colour 6 instead of $\bar{3}$, even the baryon–antibaryon decay is suppressed, and the state might be very narrow even with high mass.

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PHYSICS LETTERS

COLOUR CHEMISTRY -- A STUDY OF METASTABLE MULTIQUARK MOLECULES

CHAN Hong-Mo, M. FUKUGITA$^1$, T.H. HANSSON$^2$, H.J. HOFFMAN, K. KONISHI
Rutherford Laboratory, Chilton, Didcot, Oxon, OX11 OQX, UK

H. HØGAASEN$^3$
Theory Division, CERN, 1211 Geneva 23, Switzerland

and

TSOU Sheung Tsun
Mathematical Institute, Oxford University, UK

Received 10 April 1978
Baryonium

Other applications

- Ideas developed for baryonium applied to other configurations (as a prediction or a warning)
- Bound states of hidden-charm baryon–antibaryon pairs
- Meso-baryons: \((q\bar{q})_8 - (qqq)_8\)
- Demon-deuteron \((qq) - (qq) - (qq)\), etc.
Constituent models of baryonium

- Sood et al.

**EXACT SELECTION RULES FOR “CHROMO-HARMONIC” DIQUONIUM DECAY INTO MESONS**

M.B. GAVELA, A. Le YAOUANC, L. OLIVER, O. PÈNE, J.C. RAYNAL and Sudhir SOOD

*Laboratoire de Physique Théorique et Hautes Energies*, Orsay, France

Received 13 July 1978

Using a “chromo-harmonic” interquark potential we study the stability of “diquoniums” against the decay into mesons. We find an exact selection rule which implies that “true-diquoniums” are uncoupled from open meson channels for orbital excitation greater than 1. “Mock-diquoniums” only appear for an orbital excitation greater than 8 and are coupled to mesons and to pairs of orbitally excited baryons.

- Some postulated properties might get an explanation from simple dynamics

**DO WE NEED “MOCK” DIQUONIUMS TO EXPLAIN NARROW WIDTHS INTO BB CHANNELS?**

J.P. ADER, B. BONNIER and Sudhir SOOD

*Laboratoire de Physique Théorique*, Bordeaux, France

Received 18 April 1979

Baryon–antibaryon widths of the “true” diquoniums are calculated using the quark-pair-creation model, otherwise very successful for the decays of ordinary baryons and mesons. The resulting widths are one order of magnitude smaller than generally believed. Combined with an earlier study of diquonium decays into mesonic channels, this would avoid the need for “mock” configurations and make it difficult to interpret structures with widths ~ 100 MeV as diquonium.

- Similar to (qq) – q structure of orbitally excited baryons.
Tetraquarks
Early attempts

- Forget baryon–antibaryon and stay in the meson sector.
  - Is there room for tetraquarks \((qq\bar{q}\bar{q})\) in the excitation spectrum?
  - Can we find stable \((qq\bar{q}\bar{q})\) states in some flavour sectors?

- **Tetraquarks vs. radial excitations.**

  Yoichi IWASAKI

  Research Institute for Fundamental Physics
  Kyoto University, Kyoto

  (Received January 20, 1975)

  We assign \(\phi(3695)\) to an exotic meson \(c\bar{c}(\rho^+ + n\pi)\) and \(\phi(3105)\) to a vector meson \(c\bar{c}\), respectively. Then we can explain naturally two facts: 1) \(\phi(3695)\) decays strongly to \(\phi(3105) + 2\pi\) and 2) there is very little \(\phi(3695)\) production compared with \(\phi(3105)\) production in \(p\bar{p}\) scattering at Brookhaven. In this model we expect two broad resonances at 3.7~4.1 GeV and at 3.1 GeV.

- **Already for \(\psi'\)**
The same story was repeated for some higher resonances, 

**Hydronic molecules and the charmonium atom**

M. B. Voloshin and L. B. Okun’

_institute of Theoretical and Experimental Physics_  
(February 16, 1976)  
_Pis'ma Zh. Eksp. Teor. Fiz. 23, No. 6, 369–372 (20 March 1976)_

We consider the possible existence of levels in a system consisting of a charmed particle and a charmed antiparticle; these levels result from exchange of ordinary mesons (ω, π, η, φ, etc.). An interpretation of the resonances in e⁺e⁻ annihilation in the region 3.9–4.8 GeV is proposed.
Puzzling BR into $D\overline{D}$, $D^*\overline{D} + c.c.$, $D^*\overline{D}^*$ from the node structure,

1) Why is psi-prime-prime-prime (4.414) SO narrow?

2) Strong Decays of psi-prime-prime (4.028) as a Radial Excitation of Charmonium.

3) Charmonium: Comparison with Experiment.
**Tetraquarks**

**Same story again?**

- \( X(3872) \) was predicted as a \( D^* \bar{D} + c.c. \) molecule (Törnqvist, Voloshin, Manohar and Wise, Ericson and Karl, …) and further described in this framework (see Swanson’s review for refs.)

<table>
<thead>
<tr>
<th>Composite</th>
<th>( J^{PC} )</th>
<th>Deuson</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D \bar{D}^* )</td>
<td>0–+</td>
<td>( \eta_c ) (~3870)</td>
</tr>
<tr>
<td>( D \bar{D}^* )</td>
<td>1++</td>
<td>( \chi_{c1} ) (~3870)</td>
</tr>
<tr>
<td>( D^* \bar{D}^* )</td>
<td>0++</td>
<td>( \chi_{c0} ) (~4015)</td>
</tr>
<tr>
<td>( D^* \bar{D}^* )</td>
<td>0–+</td>
<td>( \eta_c ) (~4015)</td>
</tr>
<tr>
<td>( D^* \bar{D}^* )</td>
<td>1–+</td>
<td>( h_{c0} ) (~4015)</td>
</tr>
<tr>
<td>( D^* \bar{D}^* )</td>
<td>2++</td>
<td>( \chi_{c2} ) (~4015)</td>
</tr>
<tr>
<td>( B \bar{B}^* )</td>
<td>0–+</td>
<td>( \eta_b ) (~10545)</td>
</tr>
</tbody>
</table>

**Acknowledgements.** I thank J.-M. Richard for discussions and for pointing out the numerical method used in this paper. Also comments by T.E.O. Ericson, A.M. Green, G. Karl, J. Paton and D.O. Riska have been useful in this work.

- This picture faces some difficulties. E.g., \( X \rightarrow \psi(2S) + \gamma \) and \( X \rightarrow \psi(1S) + \gamma \)

- It also predicts other molecules, in \( D^* \bar{D}^* \) and in the charm = 2 sector.
Tetraquarks
Tetraquarks with hidden beauty

- \( \Upsilon(10860) \) **significantly broader** than \( \Upsilon(4S) = 10580 \)
- \( \Gamma = 110 \text{ MeV} \) vs. \( 20 \text{ MeV} \)
- Thresholds \( B\bar{B} = 10560 \), \( B^*\bar{B} = 10605 \) \( B^*\bar{B}^* = 10650 \)
- Ali et al.: Tetraquark with maximal isospin violation, \([bu] – [\bar{b}\bar{u}]\) and \([bd] – [\bar{b}\bar{d}]\) as mass eigenstates (charged partners?)
- With predictions such as \( \Upsilon(10860) \to \Upsilon(1S)K^+K^- / \ldots K^0\bar{K}^0 = 4 \)
- Not endorsed by e.g., Bugg [1101.1659], who suggests a strong coupling to \( B^{(*)}\bar{B}^* \) channels
- The model of Törnqvist, if extrapolated here, induces
  - a bound state in \( 0^{++} \) with about \( -50 \text{ MeV} \)
  - a radial excitation very close to the threshold
  - an orbital excitation very close to it
- a coherent \( (b\bar{b}) – \) Tetraquark – molecule very likely
Tetraquarks

Genuine exotics?

∃ tetraquarks that can not be confused with ordinary $q\bar{q}$?

All charm? (Vary et al.)

**All-charm tetraquarks**

Richard J. Lloyd and James P. Vary  
*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*  
(Received 5 November 2003; published 27 July 2004)

We investigate four-body states with only charm quarks. Working in a large but finite oscillator basis, we present a net binding analysis to determine if the resulting states are stable against breakup into a pair of $c\bar{c}$ mesons. We find several close-lying bound states in the two models we examine.

To be confirmed (Vary, private communication)

In this chromoelectric regime, with colour additive forces ($\propto \sum \tilde{\lambda}_i \tilde{\lambda}_j \nu(r_{ij})$), the system is usually found unbound (see next slides)

and unequal masses are required ($QQ'\bar{q}\bar{q}'$) (next to next slides)
Equal masses

QED vs. QCD-1

- \((e^+, e^+, e^-, e^-)\) proposed by Wheeler in 1945
- Found to be likely unstable by Ore in 1946
  
  **Binding Energy of Polyelectrons**
  
  AADNE ORE
  Slocane Physics Laboratory, Yale University, New Haven, Connecticut
  June 10, 1946
  
  Although the evidence here presented against the stability of the polelectron composed of two electrons and two positrons is not conclusive in a strict mathematical sense, it counsels against the assumption that clusters of this (or even of higher) complexity can be formed.

- Demonstrated to be stable by the same Ore in 1947
  
  **Binding Energy of the Positronium Molecule**
  
  EGIL A. HILLERÅS
  Institute of Theoretical Physics, University of Oslo, Oslo, Norway
  AND
  AADNE ORE
  Slocane Physics Laboratory, Yale University, New Haven, Connecticut
  (Received December 26, 1946)
  
  A system of two electrons and two positrons is shown to possess dynamic stability. The

- Found in 2007
Equal masses
QED vs. QCD-2

- Why the $P_{s2}$ problem of QED and the additive $(..) \sum \tilde{\lambda}_i \cdot \tilde{\lambda}_j v(r_{ij})$ pairwise quark model any different?
- Little to do with Coulomb vs. linear
- in $H = \sum \frac{p_i}{(2m)} + \sum g_{ij} v(r_{ij})$,
- With $\sum g_{ij}$ fixed for both the threshold and the 4-body,
- Due to charge conservation or colour singlet,
- $E = \text{min}(H)$ maximal is all $g_{ij}$ equal,
- $E$ decreases if $\{g_{ij}\}$ more asymmetric, i.e., if $\Delta g$ larger
- Now, if you compare $P_{s2}$ and quark models: $P_{s2}$ favoured.

<table>
<thead>
<tr>
<th>$(abcd)$</th>
<th>$v(r)$</th>
<th>$g_{ij}$</th>
<th>$\bar{g}$</th>
<th>$\Delta g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1,3)+(2,4)$</td>
<td>$-1/r$, $r$</td>
<td>${0, 0, 1, 0, 1, 0}$</td>
<td>$1/3$</td>
<td>$0.22$</td>
</tr>
<tr>
<td>$P_{s2}$</td>
<td>$-1/r$</td>
<td>${-1, -1, 1, 1, 1, 1, 1}$</td>
<td>$1/3$</td>
<td>$0.89$</td>
</tr>
<tr>
<td>$[(qq)\bar{3}(\bar{q}q)_{\bar{3}}]$</td>
<td>$-1/r$, $r$</td>
<td>${1/2, 1/2, 1/4, 1/4, 1/4, 1/4}$</td>
<td>$1/3$</td>
<td>$0.01$</td>
</tr>
<tr>
<td>$[(qq)<em>6(\bar{q}q)</em>{\bar{6}}]$</td>
<td>$-1/r$, $r$</td>
<td>${-1/4, -1/4, 5/8, 5/8, 5/8, 5/8, 5/8}$</td>
<td>$1/3$</td>
<td>$0.17$</td>
</tr>
</tbody>
</table>

Multiquarks
Tetraquarks with unequal masses

In a pure static interaction (spin-independent) $m \uparrow$ means $E \downarrow$, e.g., $E \propto 1/\sqrt{m}$ for HO and $E \propto -1/m$ for Coulomb, but in large systems, the effect often benefits more the threshold than the system, e.g., $(p, e^+, \bar{p}, e^-)$ in QED unstable while $(e^+, e^+, e^-, e^-)$ is stable.

For the same reason, in most models, $(qqqQQQ)$ hardly bound, as the lowest threshold, $(qqq) + (QQQ)$ benefits maximally from the large masses.

On the other hand, $(QQ\bar{q}\bar{q})$ takes profit of the heavy–heavy interaction that in absent in the threshold $(Q\bar{q}) + (Q\bar{q})$. It is predicted to be stable by many authors, but was never investigated experimentally.
Improving the pairwise ansatz

The colour-additive model \( V \propto \sum \tilde{\lambda}^{(c)}_i \cdot \tilde{\lambda}^{(c)}_j \nu(r_{ij}) \)
- used for mesons vs. baryons (Stanley and Robson, Lipkin, . . .)
- exact in the quark–diquark limit
- now routinely replaced by the \textit{Y}-shape ansatz

\[ \begin{array}{c}
\bullet \\
\bullet \\
\circ \\
\bullet
\end{array} \]

- as anticipated by Artru, Dosch, Merkuriev, Fabre de la Ripelle, Kogut, Kuti, . . ., and now supported by lattice QCD,
- But the change in baryon spectroscopy not very significant, as compared to the additive model.
Flip-flop and Steiner-tree for tetraquarks

- **Y shape ext. to tetraquarks as**

- **But the dynamics is dominated by**

- **V taken as the minimum at each point**
Flip-flop and Steiner-tree for tetraquarks

- Y shape ext. to tetraquarks as

![Diagram showing Y shape with vertices v1, v2, v3, v4 and edges s1, s2]

- But the dynamics is dominated by

![Diagram showing the Y shape with an additional dashed line between v2 and v4]

- V taken as the minimum at each point

- Picture now supported by lattice QCD and even ADS/QCD, but anticipated (Lenz et al., Carlson et al.)

- More recent: dramatic changes in tetraquark spectroscopy (Vijande et al.)

- If alone, binds most configurations.

- Hence promising future for exp. tetraquark search, especially in the heavy quark sector.
Steiner-tree confinement

Results

- First estimate

  Absence of exotic hadrons in flux-tube quark models

- Second estimate (Vijande et al.)

  Stability of multiquarks in a simple string model

- This corresponds to the Born–Oppenheimer limit, without antisymmetrisation constraint.

- Next step (in progress): coupled channel interaction, of which the lowest eigenvalue is the minimal Steiner-tree.

- Notice: without antisymmetrisation (i.e., different quarks), the same model binds several pentaquark and hexaquark \( q^6 \) or \( q^3\bar{q}^3 \) configurations.
Configuration mixing

- If model $|a\rangle$ has some interesting properties, and $|b\rangle$ some other nice properties
- It is not obvious that $\cos \vartheta |a\rangle + \sin \vartheta |b\rangle$, makes any sense.
- Simple mixing schemes of close neighbours significantly depart from serious coupled-channel calculations.
- For instance, a $(c\bar{c})$ admixture into a $(c\bar{c}q\bar{q})$ with $J^{PC} = 1^{++}$ is not necessarily a radial excitation.
- Hence is not convincingly a solution to the $X \rightarrow \gamma\psi(2S)/X \rightarrow \gamma\psi(1S)$ problem.
- A simpler example is $S - D$ mixing. At first, $\psi(3686) = 2S$ and $\psi(3770) = 1D$.
- If a tensor force is introduced in a pure potential picture, then $\psi(3770)$ acquires a nodeless $S$-wave component.
- While most empirical pictures assume $\cos \vartheta |1D\rangle + \sin \vartheta |2S\rangle$, now, the mixing is probably due also to coupling to meson–meson channels, and the picture becomes even more involved.
Conclusions

- Intense activity in the tetraquark sector,
- Molecules bound with nuclear forces: if it works for $X(3872)$, other configurations predicted,
- Diquark–antidiquark: clustering remains to be justified. Again, other configurations expected (charged states, dibaryons, etc.)
- Constituent models: The Steiner tree confinement gives more attraction than the empirical colour-additive ansatz,
- But its application to configurations with identical quarks remains to be worked out,
- Mixing requires a lot of care.